My Precalculus “How-To” Manual

www.waynecc.edu

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Presentation for AMATYC’s Milwaukee 2019 Conference
Do you remember . . . ?
Objectives:
* Provide inventive narratives, memorable one-liners, and clever ditties to solidify mathematical processes and techniques.
* Discuss the challenges of working with “math haters” and how best to equip them with memory assistance.
* Furnish and complete a literal “How To” Manual for Precalculus.

The Plan:
* Keep teaching the content!
* Provide these additional learning strategies to assist those students who are not mathematically inclined: songs, visuals, anecdotes, jokes, repetition, etc.
* PRACTICE, PRACTICE, PRACTICE

Our Focus:

Disclaimer:
No guarantee these techniques will help everyone.
We do guarantee these techniques will help someone.

Challenges from “non-mathematical” students?

Challenges from “old school” lecturers?
1) Absolute Value Equations

\[ 3|x - 4| + 5 = 35 \]

2) Quadratic Formula

\[ 2x^2 - 3x + 4 = 0 \]

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
3) Completing the Square

\[ x^2 - 6x + 14 = 0 \]

4) Simplifying Radicals

\[ \sqrt{75} = \]

\[ \sqrt{98} = \]

\[ \sqrt[3]{125} = \]
5) Families of Functions

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6) Composition of Functions

\[ f(x) = x^2 + 3x \quad g(x) = x - 4 \]

Find \((f \circ g)(x)\).

Find \((f \circ g)(3)\).
7) Domain for Composition of Functions

\[ f(x) = \frac{1}{x + 10} \quad g(x) = x^2 + 7x \]

8) Synthetic Division

Divide: \( x^3 - 7x - 6 \) by \( x + 2 \)
9) Rational Exponents
Source: June Blackwell, Raleigh, NC

\[ \sqrt[5]{x^8} = \quad \quad x^7 = \]

10) Exponential Functions (real-world)
11) Logarithms (properties)

Expand: \( \log_2(x^3 y^4 z) = \)

Condense: \( 4 \ln(x) + 3 \ln(y) - 5 \ln(z) = \)

12) Exponential Equations

Solve: \( 2^x + 5 = 38 \)
13) Logarithmic Equations

Solve: \(2 \ln(x + 3) - 5 = 25\)

14) Difference Quotient

\[ f(x) = x^2 + 3x \quad \frac{f(x + h) - f(x)}{h}, h \neq 0 \]
15) Factoring Trinomials with leading coefficient of 1

*”Car Method” (front seat, back seat)
*T-table for middle linear term

*Constant (loser) ➝ Daddy
-If Daddy is +, Daddy says “DOUBLES”
-If Daddy is -, Daddy says “COMBO”

*Linear term (#x) ➝ Mom
-Mom says “Put MY sign with my OLDEST child!”

16) Factoring Sum/Difference of Cubes
Source: June Blackwell, Raleigh, NC

“Square the Outsides,
Opposite Product in the Middle”

\[ x^3 + 125 = \]
17) Trig Hexagon

![Trig Hexagon Diagram]

18) Bearings

The Titanic II leaves the harbor and sails 110 miles on a bearing of N 54° E.

The bearing from the Lusitania II to the Cape Hatteras lighthouse is S 36° W.
19) Sum & Difference Formulas

A Sine “Sandwich”
Where the Signs Stay the Same

\[
\sin(A + B) = \sin A \cos B + \cos A \sin B
\]
\[
\sin(A - B) = \sin A \cos B - \cos A \sin B
\]
20) Sum & Difference Formulas

Source: Michael Stueben’s

_Twenty Years Before the Blackboard_

As we all know, some of the people to whom we are attracted are not attracted to us.
And it is not unusual for a person who has shown interest in us to later lose interest in us.
Maybe that is a good thing, because it forces us to date a lot of people and to become more experienced in maintaining relationships.

Anyway, this is the story of _Sinbad_ and _Cosette_. _Sinbad_ loved _Cosette_, but _Cosette_ did not feel the same way about _Sinbad_. Naturally, when _Sinbad_ was in charge of their double date, he would put himself with _Cosette_, and he put his brother with her sister. _Sinbad_ loved to tell people that his and _Cosette_’s signs were the same.

\[
\begin{align*}
\sin(A + B) &= \sin A \cos B + \cos A \sin B \\
\sin(A - B) &= \sin A \cos B - \cos A \sin B
\end{align*}
\]

However, when _Cosette_ was in charge of the double date she placed herself with her sister and put _Sinbad_ with his brother. She made sure everyone knew that their signs were NOT the same. Also, notice that _Cosette_ placed herself and her sister BEFORE _Sinbad_ and his brother. This detail was important to _Cosette_. She was very snobby, you know.

\[
\begin{align*}
\cos(A + B) &= \cos A \cos B - \sin A \sin B \\
\cos(A - B) &= \cos A \cos B + \sin A \sin B
\end{align*}
\]
YOUR IDEAS?
TRIGGERS FOR YOUR “HOW-TO” MANUALS

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Do you remember . . . ?
My Very Early Morning Jobs
Start Up Near Philadelphia.

Never Eat
Soggy Waffles
Do you remember . . . ?

Does McDonald’s Sell Burgers?

King Henry Died By Drinking Chocolate Milk.

DIVIDE, MULTIPLY, SUBTRACT, BRING DOWN
Do you remember . . . ?

ROY G BIV

FAN BOYS

FACE (face in the space)
Do you remember . . . ?

Months with 30 days
Months with 31 days

Left vs. Right

Multiplication with 9s

The Nine’s Trick

9 x _
Do you remember . . . ?

ABCs

50 Nifty United States
Why Can’t We Use Similar Techniques for our College Students in Precalculus?
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songs, visuals, anecdotes, jokes, repetition, etc.

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Our Focus:

HOW

WHY
DISCLAIMER:

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Challenges from “non-mathematical” students:

* Excuses?

* Bored?

* Refusal to practice?

* Can’t memorize, too many steps?

* “When will I use this in my real life?”
Challenges from “old school” lecturers:

*Too “cute-sy,” feel silly lecturing.

*Students should be able to figure it out, it’s not that difficult.

*This is not high school!

*”Because that’s the way I’ve always done it.”
1) Absolute Value Equations

\[ 3|x - 4| + 5 = 35 \]
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\[3| x - 4 | = 30\]
1) Absolute Value Equations

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\[ |x - 4| = 10 \]
1) Absolute Value Equations

\[ 3|x - 4| + 5 = 35 \]

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\[ x - 4 = 10 \quad x - 4 = -10 \]
1) Absolute Value Equations

\[3|x-4| + 5 = 35\]

\[3|x-4| = 30\]

\[|x-4| = 10\]

\[x - 4 = 10 \quad x - 4 = -10\]

\[x = 14 \quad x = -6\]
2) Quadratic Formula

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ 2x^2 - 3x + 4 = 0 \]
2) Quadratic Formula

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ 2x^2 - 3x + 4 = 0 \]

A negative boy (-b)
was undecided (+)
whether or not he should go to a radical party \( \sqrt{\ } \).
But, because he’s a square \( b^2 \)
he missed out on 4 awesome chicks \( -4ac \).
The party was all over by 2AM \( \div 2a \).
2) Quadratic Formula

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ 2x^2 - 3x - 4 = 0 \quad \text{Line-Up: } a = 2 \]

\[ b = -3 \]

\[ c = -4 \]
2) Quadratic Formula

\[ 2x^2 - 3x - 4 = 0 \]

Line-Up: \( a = 2 \)

\( b = -3 \)

\( c = -4 \)

\[ x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-4)}}{2(2)} \]
2) Quadratic Formula

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ 2x^2 - 3x - 4 = 0 \]

Line-Up: \[ a = 2 \]

\[ b = -3 \]

\[ c = -4 \]

\[ x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-4)}}{2(2)} = \frac{3 \pm \sqrt{41}}{4} \]
3) Completing the Square (quadratics)

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\[ x^2 - 6x + 14 = 0 \]

\[ x^2 - 6x \underline{\quad} + 14 = 0 \]

\[ \left( \frac{-6}{2} \right)^2 = (-3)^2 = 9 \]
3) Completing the Square (quadratics)

\[ x^2 - 6x + 14 = 0 \]

\[ x^2 - 6x + 9 - 9 + 14 = 0 \]

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\[ \left( \frac{-6}{2} \right)^2 = (-3)^2 = 9 \]

\[ (x - 3)^2 + 5 = 0 \]
3) Completing the Square (quadratics)

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\[ \left( \frac{-6}{2} \right)^2 = (-3)^2 = 9 \]

\[ (x - 3)^2 + 5 = 0 \]

\[ (x - 3)^2 = -5 \]

\[ x - 3 = \pm i\sqrt{5} \]

\[ x = 3 \pm i\sqrt{5} \]
4) Simplifying Radicals

Squares
1
4
9
16
25
36
49
64
81
100
4) Simplifying Radicals

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Simplify: \( \sqrt{75} \)
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Simplify:

\[ \sqrt{75} = \sqrt{25 \cdot 3} \]
4) Simplifying Radicals

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Simplify:

\[ \sqrt{75} = \sqrt{25 \cdot 3} = \pm 5\sqrt{3} \]
4) Simplifying Radicals

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Simplify:

\[
\sqrt{75} = \sqrt{25 \cdot 3} = \pm 5\sqrt{3}
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\sqrt{98}
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**Simplify:**

\[ \sqrt{75} = \sqrt{25 \cdot 3} = \pm 5\sqrt{3} \]

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Simplify:

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\sqrt{75} = \sqrt{25 \cdot 3} = \pm 5 \sqrt{3}
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Simplify:

\[ \sqrt{75} = \sqrt{25 \cdot 3} = \pm 5\sqrt{3} \]

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\[ \sqrt[3]{250} \]
Squares

1  4  9  16  25  36  49  64  81  100

Cubes

1  8  27  81  125  216  343

Simplify:

\[ \sqrt{75} = \sqrt{25 \cdot 3} = \pm 5\sqrt{3} \]

\[ \sqrt{98} = \sqrt{49 \cdot 2} = \pm 7\sqrt{2} \]

\[ \sqrt[3]{250} = \sqrt[3]{125 \cdot 2} \]
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\sqrt{75} = \sqrt{25 \cdot 3} = \pm 5\sqrt{3}
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\sqrt{98} = \sqrt{49 \cdot 2} = \pm 7\sqrt{2}
\]

\[
\sqrt[3]{250} = \sqrt[3]{125 \cdot 2} = 5\sqrt[3]{2}
\]
5) Families of Functions

Name: **Linus the Line**

*In a Relationship* with Penny the Parabola

Likes:
- Line dancing
- In-line skating
- Laser shows
- Completing Connect the Dots activities

Name: **Penny the Parabola**

*In a Relationship* with Linus the Line

Likes:
- The St. Louis Arch
- Rollercoasters
- Shooting basketballs
- Hitting a pop-fly
- McDonald’s

Name: **Sid the Square Root**

SINGLE

Likes:
- Skating boarding on a halfpipe
- Trimming his sketchy goatee
- Inappropriate staring while at the beach

Name: **Valerie the Absolute Value**

*It’s Complicated* with Quinton the Cubic

Likes:
- Making the “peace” sign
- Cheering (“V” for victory)
- Watching geese fly in a V-formation

Name: **Quinton the Cubic**

*It’s Complicated* with Valerie the Absolute Value

Likes:
- Snakes
5) Functions — What family member?

\[
f(x) = -\frac{1}{2}|x + 3| + 5
\]

Valerie, the Absolute Value

V-shape

\[
g(x) = 3(x + 4)^3 - 6
\]

Quinton, the Cubic

Snake shape

\[
h(x) = -5\sqrt{x + 4}
\]

Sid, the Square Root

Half hill
5) Functions — What family member?

\[ f(x) = -\frac{1}{2}|x + 3| + 5 \]

VALERIE, the Absolute Value
V-shape

\[ g(x) = 3(x + 4)^3 - 6 \]

QUINTON, the Cubic
Snake shape

\[ h(x) = -5\sqrt{x + 4} \]

SID, the Square Root
Half hill
6) Composition of Functions

\[ f(x) = x^2 + 3x \quad \quad g(x) = x - 4 \]

Find \((f \circ g)(x)\).

Find \((f \circ g)(3)\).
6) Composition of Functions

\[ f(x) = x^2 + 3x \quad g(x) = x - 4 \]

Find \((f \circ g)(x)\).

Find \((f \circ g)(3)\).

\[(f \circ g)(x) = (x - 4)^2 + 3(x - 4)\]
6) Composition of Functions

\[ f(x) = x^2 + 3x \quad g(x) = x - 4 \]

Find \((f \circ g)(x)\).
Find \((f \circ g)(3)\).

\[ (f \circ g)(x) = (x - 4)^2 + 3(x - 4) \]
\[ (f \circ g)(x) = x^2 - 8x + 16 + 3x - 12 \]
6) Composition of Functions

\[ f(x) = x^2 + 3x \quad g(x) = x - 4 \]

Find \((f \circ g)(x)\).

Find \((f \circ g)(3)\).

\[
(f \circ g)(x) = (x - 4)^2 + 3(x - 4)
\]

\[
(f \circ g)(x) = x^2 - 8x + 16 + 3x - 12
\]

\[
(f \circ g)(x) = x^2 - 5x + 4
\]
6) Composition of Functions

\[ f(x) = x^2 + 3x \quad g(x) = x - 4 \]

Find \((f \circ g)(x)\).

Find \((f \circ g)(3)\).

First: Find \(g(3)\)

\[ (f \circ g)(x) = (x - 4)^2 + 3(x - 4) \]  

\[ (f \circ g)(x) = x^2 - 8x + 16 + 3x - 12 \]  

\[ (f \circ g)(x) = x^2 - 5x + 4 \]
6) Composition of Functions

\[ f(x) = x^2 + 3x \quad g(x) = x - 4 \]

Find \((f \circ g)(x)\).

Find \((f \circ g)(3)\).

First: Find \(g(3)\)

\[ g(3) = 3 - 4 = -1 \]

\[ (f \circ g)(x) = (x - 4)^2 + 3(x - 4) \]

\[ (f \circ g)(x) = x^2 - 8x + 16 + 3x - 12 \]

\[ (f \circ g)(x) = x^2 - 5x + 4 \]
6) Composition of Functions

\[ f(x) = x^2 + 3x \quad g(x) = x - 4 \]

\[ (f \circ g)(x) = (x - 4)^2 + 3(x - 4) \]

\[ (f \circ g)(x) = x^2 - 8x + 16 + 3x - 12 \]

\[ (f \circ g)(x) = x^2 - 5x + 4 \]

Find \( (f \circ g)(x) \).

Find \( (f \circ g)(3) \).

First: Find \( g(3) \)

\[ g(3) = 3 - 4 = -1 \]

Second: Find \( f(-1) \)

\[ f(-1) = (-1)^2 + 3(-1) = -2 \]
6) Composition of Functions

\( f(x) = x^2 + 3x \quad g(x) = x - 4 \)

Find \((f \circ g)(x)\).

Find \((f \circ g)(3)\).

First: Find \(g(3)\)

\( g(3) = 3 - 4 = -1 \)

Second: Find \(f(-1)\)

\( f(-1) = (-1)^2 + 3(-1) = -2 \)

\((f \circ g)(3) = -2 \)
7) Domain for Composition of Functions

\[ f(x) = \frac{1}{x+10} \quad g(x) = x^2 + 7x \]
7) Domain for Composition of Functions

\[ f(x) = \frac{1}{x+10} \quad g(x) = x^2 + 7x \]

\[ (f \circ g)(x) = \frac{1}{(x^2 + 7x) + 10} = \frac{1}{(x+2)(x+5)} \]
7) Domain for Composition of Functions

\[ f(x) = \frac{1}{x+10} \quad g(x) = x^2 + 7x \]

\[ (f \circ g)(x) = \frac{1}{(x^2 + 7x) + 10} = \frac{1}{(x+2)(x+5)} \]

Check for "allergies" (problems) with INSIDE function?
7) Domain for Composition of Functions

\[ f(x) = \frac{1}{x + 10} \quad g(x) = x^2 + 7x \]

\[(f \circ g)(x) = \frac{1}{(x^2 + 7x) + 10} = \frac{1}{(x + 2)(x + 5)}\]

Check for "allergies" (problems) with \text{INSIDE} function? D:\((-\infty, \infty)\)

Check for "allergies" with \text{COMPOSITION}?
7) Domain for Composition of Functions

\[ f(x) = \frac{1}{x + 10} \quad g(x) = x^2 + 7x \]

\[ (f \circ g)(x) = \frac{1}{(x^2 + 7x) + 10} = \frac{1}{(x + 2)(x + 5)} \]

Check for "allergies" (problems) with **INSIDE** function? \( D:(-\infty,\infty) \)

Check for "allergies" with **COMPOSITION**? \( D:(-\infty,-5) \cup (-5,-2) \cup (-2,\infty) \)
7) Domain for Composition of Functions

\[ f(x) = \frac{1}{x+10}, \quad g(x) = x^2 + 7x \]

\[(f \circ g)(x) = \frac{1}{(x^2 + 7x) + 10} = \frac{1}{(x+2)(x+5)}\]

Check for "allergies" (problems) with INSIDE function? D:(-\(\infty\),\(\infty\))

Check for "allergies" with COMPOSITION? D:(-\(\infty\),-5) \(\cup\) (-5,-2) \(\cup\) (-2, \(\infty\))

*DOMAIN (must not make anyone sick!) D: (-\(\infty\),-5) \(\cup\) (-5,-2) \(\cup\) (-2, \(\infty\))
8) Synthetic Division

Divide: $x^3 - 7x - 6$ by $x + 2$
8) Synthetic Division

Divide: $x^3 - 7x - 6$ by $x + 2$

\[
\begin{array}{c|ccccc}
-2 & 1 & 0 & -7 & -6 \\
\hline
 & & & & \\
\end{array}
\]
8) Synthetic Division

Divide: $x^3 - 7x - 6$ by $x + 2$

\[
\begin{array}{c|ccccc}
2 & 1 & 0 & -7 & -6 \\
\hline
& & -2 & 4 & 6 \\
\end{array}
\]

\[
\begin{array}{cccc}
1 & -2 & -3 & 0 \\
\end{array}
\]
8) Synthetic Division

Divide: \( x^3 - 7x - 6 \) by \( x + 2 \)

\[
\begin{array}{r|llll}
-2 & 1 & 0 & -7 & -6 \\
    &   & -2 & 4  & 6  \\
\hline
    & 1x^2 & -2x & -3 & \boxed{0}
\end{array}
\]
9) Rational Exponents

“Flower Power” → Ms. June Blackwell, Raleigh, NC
*Flowers have ROOTS in the GROUND (low)
*Flowers get their POWER from the SUN (high)
9) Rational Exponents

“Flower Power” → Ms. June Blackwell, Raleigh, NC

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*Flowers get their POWER from the SUN (high)

\[ \sqrt[5]{x^8} \]
9) Rational Exponents

“Flower Power” → Ms. June Blackwell, Raleigh, NC

*Flowers have ROOTS in the GROUND (low)
*Flowers get their POWER from the SUN (high)

\[
\sqrt[5]{x^8} = x^{\frac{8}{5}}
\]
9) Rational Exponents

“Flower Power” → Ms. June Blackwell, Raleigh, NC

*Flowers have ROOTS in the GROUND (low)
*Flowers get their POWER from the SUN (high)

\[ \sqrt[5]{x^8} = x^{\frac{8}{5}} \quad \frac{6}{x^7} \]
9) Rational Exponents

“Flower Power” → Ms. June Blackwell, Raleigh, NC

*Flowers have **ROOTS** in the **GROUND** (low)
*Flowers get their **POWER** from the **SUN** (high)

\[
5\sqrt{x^8} = x^{\frac{8}{5}} \quad \quad \quad x^{\frac{6}{7}} = \sqrt[7]{x^6}
\]
10) Exponential Functions (real-world)
11) Logarithms (properties)

Expand: $\log_2 (x^3 y^4 z)$
11) Logarithms (properties)

Expand: $\log_2(x^3y^4z) = \log_2(x^3) + \log_2(y^4) + \log_2(z)$
11) Logarithms (properties)

Expand: \[ \log_2(x^3y^4z) = \log_2(x^3) + \log_2(y^4) + \log_2(z) \]
\[ = 3\log_2(x) + 4\log_2(y) + \log_2(z) \]

*No kites/balloons, everyone in solitary confinement!*
11) Logarithms (properties)

Expand: \( \log_2(x^3 y^4 z) = \log_2(x^3) + \log_2(y^4) + \log_2(z) \)
\[ = 3 \log_2(x) + 4 \log_2(y) + \log_2(z) \]

*No kites/balloons, everyone in solitary confinement!*

Condense: \( 4 \ln(x) + 3 \ln(y) - 5 \ln(z) \)
11) Logarithms (properties)

Expand: \( \log_2(x^3 y^4 z) = \log_2(x^3) + \log_2(y^4) + \log_2(z) \)

\[ = 3\log_2(x) + 4\log_2(y) + \log_2(z) \]

*No kites/balloons, everyone in solitary confinement!

Condense: \( 4\ln(x) + 3\ln(y) - 5\ln(z) = \ln(x^4) + \ln(y^3) - \ln(z^5) \)
11) Logarithms (properties)

Expand: $\log_2(x^3 y^4 z) = \log_2(x^3) + \log_2(y^4) + \log_2(z)$

$= 3\log_2(x) + 4\log_2(y) + \log_2(z)$

*No kites/balloons, everyone in solitary confinement!

Condense: $4\ln(x) + 3\ln(y) - 5\ln(z) = \ln(x^4) + \ln(y^3) - \ln(z^5)$

$= \ln\left(\frac{x^4 y^3}{z^5}\right)$

*Block Party! Kites/Balloons for everyone!
12) Exponential Equations

Solve: $2^x + 5 = 38$
12) Exponential Equations

Solve: $2^x + 5 = 38$

$2^x = 33$
12) Exponential Equations

Solve: $2^x + 5 = 38$

$2^x = 33$

$\ln(2^x) = \ln(33)$
12) Exponential Equations

Solve: $2^x + 5 = 38$

$2^x = 33$

$\ln(2^x) = \ln(33)$

$x \ln(2) = \ln(33)$
12) Exponential Equations

Solve: \( 2^x + 5 = 38 \)

\[
2^x = 33
\]

\[
\ln(2^x) = \ln(33)
\]

\[
x \ln(2) = \ln(33)
\]

\[
x = \frac{\ln(33)}{\ln(2)}
\]
13) Logarithmic Equations

Solve: \[ 2 \ln(x + 3) - 5 = 25 \]
13) Logarithmic Equations

Solve: $2 \ln(x + 3) - 5 = 25$

$2 \ln(x + 3) = 30$
13) Logarithmic Equations

Solve: $2 \ln(x + 3) - 5 = 25$

$2 \ln(x + 3) = 30$

$\ln(x + 3) = 15$
13) Logarithmic Equations

Solve: \( 2 \ln(x + 3) - 5 = 25 \)

\[
2 \ln(x + 3) = 30
\]

\[
\ln(x + 3) = 15
\]

\[
e^{15} = x + 3 \quad \text{(exponential form)}
\]
13) Logarithmic Equations

Solve: 2\ln(x + 3) - 5 = 25

\[ 2\ln(x + 3) = 30 \]
\[ \ln(x + 3) = 15 \]

\[ e^{15} = x + 3 \quad \text{(exponential form--cursive "e")} \]

\[ x = e^{15} - 3 \]
14) Difference Quotient

\[ f(x) = x^2 + 3x \]

\[ \frac{f(x+h) - f(x)}{h}, h \neq 0 \]
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\[ f(x) = x^2 + 3x \]

\[ (x + h)^2 + 3(x + h) \]

\[ \frac{f(x + h) - f(x)}{h}, \quad h \neq 0 \]
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\[ f(x) = x^2 + 3x \]

\[ \frac{f(x + h) - f(x)}{h}, \ h \neq 0 \]

\[ (x + h)^2 + 3(x + h) \]

\[ (x + h)(x + h) + 3(x + h) \]
14) Difference Quotient

\[ f(x) = x^2 + 3x \]

\[ \frac{f(x + h) - f(x)}{h}, h \neq 0 \]

\[ (x + h)^2 + 3(x + h) \]

\[ (x + h)(x + h) + 3(x + h) \]

\[ x^2 + 2xh + h^2 + 3x + 3h \]
14) Difference Quotient

\[ f(x) = x^2 + 3x \]

\[ \frac{f(x + h) - f(x)}{h}, \quad h \neq 0 \]

\[ (x + h)^2 + 3(x + h) \]

\[ (x + h)(x + h) + 3(x + h) \]

\[ x^2 + 2xh + h^2 + 3x + 3h - (x^2 + 3x) \]
14) Difference Quotient

\[ f(x) = x^2 + 3x \]

\[ \frac{f(x + h) - f(x)}{h}, h \neq 0 \]

\[ (x + h)^2 + 3(x + h) \]

\[ (x + h)(x + h) + 3(x + h) \]

\[ x^2 + 2xh + h^2 + 3x + 3h - (x^2 + 3x) \]

\[ x^2 + 2xh + h^2 + 3x + 3h - x^2 - 3x \]
14) Difference Quotient

\[ f(x) = x^2 + 3x \]

\[ \frac{f(x + h) - f(x)}{h}, \quad h \neq 0 \]

\[ (x + h)^2 + 3(x + h) \]

\[ (x + h)(x + h) + 3(x + h) \]

\[ x^2 + 2xh + h^2 + 3x + 3h - (x^2 + 3x) \]

\[ x^2 + 2xh + h^2 + 3x + 3h - x^2 - 3x \]
14) Difference Quotient

\[ f(x) = x^2 + 3x \]

\[ \frac{f(x + h) - f(x)}{h}, h \neq 0 \]

\[ (x + h)^2 + 3(x + h) \]

\[ (x + h)(x + h) + 3(x + h) \]

\[ x^2 + 2xh + h^2 + 3x + 3h - \left( x^2 + 3x \right) \]

\[ 2xh + h^2 + 3h \]
14) Difference Quotient

\[ f(x) = x^2 + 3x \]
\[ f(x + h) = (x + h)^2 + 3(x + h) \]
\[ (x + h)^2 + 3(x + h) \]
\[ (x + h)(x + h) + 3(x + h) \]
\[ x^2 + 2xh + h^2 + 3x + 3h - (x^2 + 3x) \]
\[ x^2 + 2xh + h^2 + 3x + 3h - x^2 - 3x \]
\[ 2xh + h^2 + 3h \]
\[ \frac{f(x + h) - f(x)}{h}, h \neq 0 \]
14) Difference Quotient

\[ f(x) = x^2 + 3x \]

\[ \frac{f(x+h) - f(x)}{h}, \quad h \neq 0 \]

\[ (x + h)^2 + 3(x + h) \]

\[ (x + h)(x + h) + 3(x + h) \]

\[ x^2 + 2xh + h^2 + 3x + 3h - (x^2 + 3x) \]

\[ x^2 + 2xh + h^2 + 3x + 3h - x^2 - 3x \]

\[ 2xh + h^2 + 3h \]

\[ \frac{2xh + h^2 + 3h}{h} \]
14) Difference Quotient

\[ f(x) = x^2 + 3x \]

\[
\frac{f(x+h) - f(x)}{h}, \ h \neq 0
\]

\[
(x + h)^2 + 3(x + h)
\]

\[
(x + h)(x + h) + 3(x + h)
\]

\[
x^2 + 2xh + h^2 + 3x + 3h - (x^2 + 3x)
\]

\[
x^2 + 2xh + h^2 + 3x + 3h - x^2 - 3x
\]

\[
2xh + h^2 + 3h
\]

\[
= 2x + h + 3, \ h \neq 0
\]
15) Factoring Trinomials  
(Leading Coefficient = 1)

*”Car Method” (front seat, back seat)
*T-table for middle linear term
*Constant (loser) → Daddy
*If Daddy is +, Daddy says “DOUBLES”
*If Daddy is -, Daddy says “COMBO”

*Linear term (#x) → Mom
*Mom says “Put MY sign with my OLDEST child!”
16) Factoring Sum/Difference of Cubes

Source: Ms. June Blackwell, Raleigh, NC

“Square the Outsides, Opposite Product in the Middle”

Factoring Sum and Difference of Two Cubes

\[ x^3 + y^3 = (x+y)(x^2 - xy + y^2) \]
\[ x^3 - y^3 = (x-y)(x^2 + xy + y^2) \]

\[ x^3 + 125 = (x)^3 + (5)^3 \]
\[ = (x+5)[x^2 - (x)(5) + 5^2] \]
\[ = (x+5)(x^2 - 5x + 25) \]
17) Trig Hexagon
18) Bearings

The Titanic II leaves the harbor and sails 110 miles on a bearing of $\text{N} 54^\circ \text{E}$. 

The bearing from the Lusitania II to the Cape Hatteras lighthouse is $\text{S} 36^\circ \text{W}$. 
19) Sum & Difference Formulas

A Sine “Sandwich”
Where the Signs Stay the Same

\[
\sin(A + B) = \sin A \cos B + \cos A \sin B
\]
\[
\sin(A - B) = \sin A \cos B - \cos A \sin B
\]
20) Sum & Difference Formulas

Michael Stueben’s Trig Mnemonic from *Twenty Years Before the Blackboard*:

As we all know, some of the people to whom we are attracted are not attracted to us. And it is not unusual for a person who has shown interest in us to later lose interest in us. Maybe that is a good thing, because it forces us to date a lot of people and to become more experienced in maintaining relationships.

Anyway, this is the story of Sinbad and Cosette. Sinbad loved Cosette, but Cosette did not feel the same way about Sinbad. Naturally, when Sinbad was in charge of their double date, he would put himself with Cosette, and he put his brother with her sister.

Sinbad loved to tell people that his and Cosette’s signs were the same.

\[
\sin(A + B) = \sin A \cos B + \cos A \sin B
\]

\[
\sin(A - B) = \sin A \cos B - \cos A \sin B
\]

However, when Cosette was in charge of the double date she placed herself with her sister and put Sinbad with his brother. She made sure everyone knew that their signs were NOT the same.

\[
\cos(A + B) = \cos A \cos B - \sin A \sin B
\]

\[
\cos(A - B) = \cos A \cos B + \sin A \sin B
\]

Also, notice that Cosette placed herself and her sister BEFORE Sinbad and his brother. This detail was important to Cosette. She was very snobby, you know.
YOUR IDEAS . . .
Our Information . . .
Triggers for your “How-To” Manuals

Elizabeth King
Michael McKenna
Katie Mullins
Wayne Community College, Goldsboro NC
www.waynecc.edu
Do you remember . . . ?

My Very Early Morning Jobs
Start Up Near Philadelphia.

Never Eat
Soggy Waffles

North East South West!
Do you remember . . . ?

Does McDonald’s Sell Burgers?

King Henry Died By Drinking Chocolate Milk.
Do you remember . . . ?

ROY G BIV

FAN BOYS

FACE
(face in the space)
Do you remember . . . ?

Months with 30 days
January, March, May, July, August, October, December

Months with 31 days
February, April, June, September, November

Left vs. Right

Multiplication with 9s
The Nine’s Trick
Do you remember . . . ?

ABCs

50 Nifty United States
Why Can’t We Use Similar Techniques for our College Students in Precalculus?
Objectives:

*Provide inventive narratives, memorable one-liners, and clever ditties to solidify mathematical processes and techniques.

*Discuss the challenges of working with “math haters” and how best to equip them with memory assistance.

*Furnish and complete a literal “How To” Manual for Precalculus.
The plan . . .

*Keep teaching the content!

*Provide these additional learning strategies to assist those students who are not mathematically inclined:
  songs, visuals, anecdotes, jokes, repetition, etc.

*PRACTICE, PRACTICE, PRACTICE
Our Focus:

HOW

NOT WHY
DISCLAIMER:

No guarantee these techniques will help everyone.

We do guarantee these techniques will help someone.
Challenges from “non-mathematical” students:

* Excuses?

* Bored?

* Refusal to practice?

* Can’t memorize, too many steps?

* ”When will I use this in my real life?”
Challenges from “old school” lecturers:

*Too “cute-sy,” feel silly lecturing.

*Students should be able to figure it out, it’s not that difficult.

*This is not high school!

*”Because that’s the way I’ve always done it.”
1) Absolute Value Equations

\[3|x - 4| + 5 = 35\]
1) Absolute Value Equations

\[ 3\left|x - 4\right| + 5 = 35 \]

\[ 3\left|x - 4\right| = 30 \]
1) Absolute Value Equations

\[3\left|x - 4\right| + 5 = 35\]

\[3\left|x - 4\right| = 30\]

\[\left|x - 4\right| = 10\]
1) Absolute Value Equations

\[3|x - 4| + 5 = 35\]

\[3|x - 4| = 30\]

\[|x - 4| = 10\]

\[x - 4 = 10 \quad x - 4 = -10\]
1) Absolute Value Equations

\[ 3\,|x - 4| + 5 = 35 \]
\[ 3\,|x - 4| = 30 \]
\[ |x - 4| = 10 \]

\[ x - 4 = 10 \quad x - 4 = -10 \]

\[ x = 14 \quad x = -6 \]
2) Quadratic Formula

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ 2x^2 - 3x + 4 = 0 \]
2) Quadratic Formula

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ 2x^2 - 3x + 4 = 0 \]

A negative boy (-b)
was undecided (±)
whether or not he should go to a radical party \( \sqrt{} \).
But, because he’s a square (b²)
he missed out on 4 awesome chicks (-4ac).
The party was all over by 2AM ( ÷ 2a).
2) Quadratic Formula

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ 2x^2 - 3x - 4 = 0 \]

Line-Up: \[ a = 2 \]
\[ b = -3 \]
\[ c = -4 \]
2) Quadratic Formula

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ 2x^2 - 3x - 4 = 0 \quad \text{Line-Up:} \quad a = 2 \]

\[ b = -3 \]

\[ c = -4 \]

\[ x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-4)}}{2(2)} \]
2) Quadratic Formula

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ 2x^2 - 3x - 4 = 0 \quad \text{Line-Up: } a = 2 \]

\[ b = -3 \]

\[ c = -4 \]

\[ x = \frac{(-3) \pm \sqrt{(-3)^2 - 4(2)(-4)}}{2(2)} = \frac{3 \pm \sqrt{41}}{4} \]
3) Completing the Square (quadratics)

\[ x^2 - 6x + 14 = 0 \]
3) Completing the Square (quadratics)

\[ x^2 - 6x + 14 = 0 \]

\[ x^2 - 6x \quad \text{__________} + 14 = 0 \]
3) Completing the Square (quadratics)

\[ x^2 - 6x + 14 = 0 \]

\[ x^2 - 6x \underline{\quad} + 14 = 0 \]

\[ \left( \frac{-6}{2} \right)^2 = (-3)^2 = 9 \]
3) Completing the Square (quadratics)

\[ x^2 - 6x + 14 = 0 \]

\[ x^2 - 6x + 9 - 9 + 14 = 0 \]

\[ \left( \frac{-6}{2} \right)^2 = (-3)^2 = 9 \]
3) Completing the Square (quadratics)

\[ x^2 - 6x + 14 = 0 \]

\[ x^2 - 6x + 9 - 9 + 14 = 0 \]

\[ \left(\frac{-6}{2}\right)^2 = (-3)^2 = 9 \]
3) Completing the Square (quadratics)

\[ x^2 - 6x + 14 = 0 \]
\[ x^2 - 6x + 9 - 9 + 14 = 0 \]
\[ \left( \frac{-6}{2} \right)^2 = (-3)^2 = 9 \]
\[ (x - 3)^2 + 5 = 0 \]
3) Completing the Square (quadratics)

\[ x^2 - 6x + 14 = 0 \]
\[ x^2 - 6x + 9 - 9 + 14 = 0 \]
\[ \left( \frac{-6}{2} \right)^2 = (-3)^2 = 9 \]
\[ (x - 3)^2 + 5 = 0 \]
\[ (x - 3)^2 = -5 \]
\[ x - 3 = \pm i\sqrt{5} \]
\[ x = 3 \pm i\sqrt{5} \]
4) Simplifying Radicals

Squares

1
4
9
16
25
36
49
64
81
100
4) Simplifying Radicals

<table>
<thead>
<tr>
<th>Squares</th>
<th>Cubes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>27</td>
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<td>81</td>
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</table>
### 4) Simplifying Radicals

**Squares**

<table>
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<th>1</th>
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<td>4</td>
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</table>

**Simplify:**

\[ \sqrt{75} \]
4) Simplifying Radicals

<table>
<thead>
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<th>Squares</th>
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</table>

Simplify:
\[ \sqrt{75} = \sqrt{25 \times 3} \]
4) Simplifying Radicals

<table>
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Simplify:

\[
\sqrt{75} = \sqrt{25 \cdot 3} = \pm 5\sqrt{3}
\]
4) Simplifying Radicals

<table>
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</table>

Simplify:

\[ \sqrt{75} = \sqrt{25 \cdot 3} = 5\sqrt{3} \]

\[ \sqrt{98} \]
4) Simplifying Radicals

<table>
<thead>
<tr>
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<th>1</th>
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<td>64</td>
<td></td>
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<td>343</td>
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</table>

Simplify:

\[ \sqrt{75} = \sqrt{25 \times 3} = \pm 5\sqrt{3} \]

\[ \sqrt{98} = \sqrt{49 \times 2} \]
4) Simplifying Radicals

Squares

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Simplify:

\[ \sqrt{75} = \sqrt{25 \times 3} = \pm 5\sqrt{3} \]

\[ \sqrt{98} = \sqrt{49 \times 2} = \pm 7\sqrt{2} \]
4) Simplifying Radicals

<table>
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<td>( \sqrt{98} = \sqrt{49 \times 2} = \pm 7\sqrt{2} )</td>
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## 4) Simplifying Radicals

### Squares

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### Cubes

| 1 | 8 | 27 | 81 | 125 | 216 |

### Simplify:

\[ \sqrt{75} = \sqrt{25 \cdot 3} = \pm 5\sqrt{3} \]

\[ \sqrt{98} = \sqrt{49 \cdot 2} = \pm 7\sqrt{2} \]

\[ \sqrt[3]{250} = \sqrt[3]{125 \cdot 2} \]
4) Simplifying Radicals

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5) Families of Functions

Name: **Linus the Line**

In a Relationship with Penny the Parabola

Likes:
- Line dancing
- In-line skating
- Laser shows
- Completing Connect the Dots activities

Name: **Penny the Parabola**

In a Relationship with Linus the Line

Likes:
- The St. Louis Arch
- Rollercoasters
- Shooting basketballs
- Hitting a pop-fly
- McDonald’s

Name: **Sid the Square Root**

Single

Likes:
- Skating boarding on a halfpipe
- Trimming his sketchy goatee
- Inappropriate staring while at the beach

Name: **Valerie the Absolute Value**

It’s Complicated with Quinton the Cubic

Likes:
- Making the “peace” sign
- Cheering (“V” for victory)
- Watching geese fly in a V-formation

Name: **Quinton the Cubic**

It’s Complicated with Valerie the Absolute Value

Likes:
- Snakes
5) Functions — What family member?

\[ f(x) = -\frac{1}{2}|x + 3| + 5 \]

VALERIE, the Absolute Value
V-shape

\[ g(x) = 3(x + 4)^3 - 6 \]

QUINTON, the Cubic
Snake shape

\[ h(x) = -5\sqrt{x + 4} \]

SID, the Square Root
Half hill
5) Functions — What family member?

\[ f(x) = -\frac{1}{2}|x + 3| + 5 \]

VALERIE, the Absolute Value

V-shape

\[ g(x) = 3(x + 4)^3 - 6 \]

QUINTON, the Cubic

Snake shape

\[ h(x) = -5\sqrt{x + 4} \]

SID, the Square Root

Half hill
6) Composition of Functions

\[ f(x) = x^2 + 3x \quad g(x) = x - 4 \]

Find \((f \circ g)(x)\).

Find \((f \circ g)(3)\).
6) Composition of Functions

\[ f(x) = x^2 + 3x \quad g(x) = x - 4 \]

Find \((f \circ g)(x)\).
Find \((f \circ g)(3)\).

\[ (f \circ g)(x) = (x - 4)^2 + 3(x - 4) \]
6) Composition of Functions

\[ f(x) = x^2 + 3x \quad g(x) = x - 4 \]

Find \((f \circ g)(x)\).

Find \((f \circ g)(3)\).

\[
(f \circ g)(x) = (x - 4)^2 + 3(x - 4)
\]

\[
(f \circ g)(x) = x^2 - 8x + 16 + 3x - 12
\]
6) Composition of Functions

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(f \circ g)(x) = x^2 - 5x + 4
\]
6) Composition of Functions

\[ f(x) = x^2 + 3x \quad g(x) = x - 4 \]

Find \((f \circ g)(x)\).

Find \((f \circ g)(3)\).

First: Find \(g(3)\)

\[ (f \circ g)(x) = (x - 4)^2 + 3(x - 4) \]

\[ (f \circ g)(x) = x^2 - 8x + 16 + 3x - 12 \]

\[ (f \circ g)(x) = x^2 - 5x + 4 \]
6) Composition of Functions

\[ f(x) = x^2 + 3x \quad g(x) = x - 4 \]

Find \((f \circ g)(x)\).

Find \((f \circ g)(3)\).

First: Find \(g(3)\)

\[ g(3) = 3 - 4 = -1 \]

\[ (f \circ g)(x) = (x - 4)^2 + 3(x - 4) \]

\[ (f \circ g)(x) = x^2 - 8x + 16 + 3x - 12 \]

\[ (f \circ g)(x) = x^2 - 5x + 4 \]
6) Composition of Functions

\[ f(x) = x^2 + 3x \quad g(x) = x - 4 \]

Find \((f \circ g)(x)\).

Find \((f \circ g)(3)\).

First: Find \(g(3)\)

\[ g(3) = 3 - 4 = -1 \]

Second: Find \(f(-1)\)

\[ f(-1) = (-1)^2 + 3(-1) = -2 \]
6) Composition of Functions

\[ f(x) = x^2 + 3x \quad g(x) = x - 4 \]

\[ (f \circ g)(x) = (x - 4)^2 + 3(x - 4) \]
\[ (f \circ g)(x) = x^2 - 8x + 16 + 3x - 12 \]
\[ (f \circ g)(x) = x^2 - 5x + 4 \]

Find \((f \circ g)(x)\).

First: Find \(g(3)\)

\[ g(3) = 3 - 4 = -1 \]

Second: Find \(f(-1)\)

\[ f(-1) = (-1)^2 + 3(-1) = -2 \]

\[ (f \circ g)(3) = -2 \]
7) Domain for Composition of Functions

\[ f(x) = \frac{1}{x + 10} \quad g(x) = x^2 + 7x \]
7) Domain for Composition of Functions

\[ f(x) = \frac{1}{x + 10} \quad g(x) = x^2 + 7x \]

\[ (f \circ g)(x) = \frac{1}{(x^2 + 7x) + 10} = \frac{1}{(x + 2)(x + 5)} \]
7) Domain for Composition of Functions

\[ f(x) = \frac{1}{x+10} \quad g(x) = x^2 + 7x \]

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Check for "allergies" (problems) with INSIDE function?
7) Domain for Composition of Functions

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Check for "allergies" (problems) with **INSIDE** function? D: \((-\infty, \infty)\)

Check for "allergies" with **COMPOSITION**?
7) Domain for Composition of Functions

\[ f(x) = \frac{1}{x+10} \quad g(x) = x^2 + 7x \]

\[ (f \circ g)(x) = \frac{1}{(x^2 + 7x) + 10} = \frac{1}{(x+2)(x+5)} \]

Check for "allergies" (problems) with INSIDE function? D:\((-\infty,\infty)\)
Check for "allergies" with COMPOSITION? D:\((-\infty,-5) \cup (-5,-2) \cup (-2,\infty)\)
7) Domain for Composition of Functions

\[ f(x) = \frac{1}{x + 10} \quad g(x) = x^2 + 7x \]

\[(f \circ g)(x) = \frac{1}{(x^2 + 7x) + 10} = \frac{1}{(x + 2)(x + 5)}\]

Check for "allergies" (problems) with **INSIDE** function? D: \((-\infty, \infty)\)

Check for "allergies" with **COMPOSITION**? D: \((-\infty, -5) \cup (-5, -2) \cup (-2, \infty)\)

* **DOMAIN** (must not make anyone sick!) D: \((-\infty, -5) \cup (-5, -2) \cup (-2, \infty)\)
8) Synthetic Division

Divide: $x^3 - 7x - 6$ by $x + 2$
8) Synthetic Division

Divide: $x^3 - 7x - 6$ by $x + 2$

\[\begin{array}{c|ccccc}
-2 & 1 & 0 & -7 & -6 \\
\hline
 & & & & & \\
\end{array}\]

\[\begin{array}{cccccc}
 & & & \underline{\text{+}} & & \\
\end{array}\]
8) Synthetic Division

Divide: \( x^3 - 7x - 6 \) by \( x + 2 \)

\[
\begin{array}{c|cccc}
-2 & 1 & 0 & -7 & -6 \\
\hline
& & -2 & 4 & 6 \\
\end{array}
\]

\[
\begin{array}{cccc}
1 & -2 & -3 & |0
\end{array}
\]
8) Synthetic Division

Divide: \( x^3 - 7x - 6 \) by \( x + 2 \)

\[
\begin{array}{c|cccc}
-2 & 1 & 0 & -7 & -6 \\
\hline 
 & & & & \\
\end{array}
\]

\[
\begin{array}{cccc}
+ & -2 & 4 & 6 \\
\hline 
1x^2 & -2x & -3 & | 0 \\
\end{array}
\]
9) Rational Exponents

“Flower Power” → Ms. June Blackwell, Raleigh, NC

*Flowers have ROOTS in the GROUND (low)
*Flowers get their POWER from the SUN (high)
“Flower Power” → Ms. June Blackwell, Raleigh, NC

*Flowers have ROOTS in the GROUND (low)
*Flowers get their POWER from the SUN (high)

\[ \sqrt[5]{x^8} \]
9) Rational Exponents

“Flower Power” → Ms. June Blackwell, Raleigh, NC

*Flowers have ROOTS in the GROUND (low)
*Flowers get their POWER from the SUN (high)

\[
\sqrt[5]{x^8} = x^{\frac{8}{5}}
\]
9) Rational Exponents

“Flower Power” → Ms. June Blackwell, Raleigh, NC

*Flowers have ROOTS in the GROUND (low)
*Flowers get their POWER from the SUN (high)

\[
\sqrt[5]{x^8} = x^{\frac{8}{5}} \quad \frac{6}{x^7}
\]
9) Rational Exponents

“Flower Power” → Ms. June Blackwell, Raleigh, NC

*Flowers have ROOTS in the GROUND (low)
*Flowers get their POWER from the SUN (high)

\[5\sqrt{x^8} = x^{\frac{8}{5}}\]
\[x^7 = \sqrt[7]{x^6}\]
10) Exponential Functions (real-world)
11) Logarithms (properties)

Expand: $\log_2 (x^3 y^4 z)$
11) Logarithms (properties)

Expand: \( \log_2(x^3 y^4 z) = \log_2(x^3) + \log_2(y^4) + \log_2(z) \)
11) Logarithms (properties)

Expand: \( \log_2(x^3y^4z) = \log_2(x^3) + \log_2(y^4) + \log_2(z) \)

\[ = 3 \log_2(x) + 4 \log_2(y) + \log_2(z) \]

*No kites/balloons, everyone in solitary confinement!*
11) Logarithms (properties)

Expand: \( \log_2(x^3 y^4 z) = \log_2(x^3) + \log_2(y^4) + \log_2(z) \)
\[ = 3 \log_2(x) + 4 \log_2(y) + \log_2(z) \]

*No kites/balloons, everyone in solitary confinement!

Condense: \( 4 \ln(x) + 3 \ln(y) - 5 \ln(z) \)
11) Logarithms (properties)

Expand: $\log_2(x^3 y^4 z) = \log_2(x^3) + \log_2(y^4) + \log_2(z)$

$= 3 \log_2(x) + 4 \log_2(y) + \log_2(z)$

*No kites/balloons, everyone in solitary confinement!

Condense: $4 \ln(x) + 3 \ln(y) - 5 \ln(z) = \ln(x^4) + \ln(y^3) - \ln(z^5)$
11) Logarithms (properties)

Expand: \( \log_2(x^3 y^4 z) = \log_2(x^3) + \log_2(y^4) + \log_2(z) \)
\[= 3 \log_2(x) + 4 \log_2(y) + \log_2(z) \]

*No kites/balloons, everyone in solitary confinement!

Condense: \( 4 \ln(x) + 3 \ln(y) - 5 \ln(z) = \ln(x^4) + \ln(y^3) - \ln(z^5) \)
\[= \ln\left(\frac{x^4 y^3}{z^5}\right) \]

*Block Party! Kites/Balloons for everyone!
12) Exponential Equations

Solve: $2^x + 5 = 38$
12) Exponential Equations

Solve: \(2^x + 5 = 38\)

\[2^x = 33\]
12) Exponential Equations

Solve: \( 2^x + 5 = 38 \)

\[ 2^x = 33 \]

\[ \ln(2^x) = \ln(33) \]
12) Exponential Equations

Solve: \( 2^x + 5 = 38 \)

\[ 2^x = 33 \]

\[ \ln(2^x) = \ln(33) \]

\[ x \ln(2) = \ln(33) \]
12) Exponential Equations

Solve: \( 2^x + 5 = 38 \)

\[ 2^x = 33 \]

\[ \ln(2^x) = \ln(33) \]

\[ x \ln(2) = \ln(33) \]

\[ x = \frac{\ln(33)}{\ln(2)} \]
13) Logarithmic Equations

Solve: $2 \ln(x + 3) - 5 = 25$
13) Logarithmic Equations

Solve: \[2 \ln(x + 3) - 5 = 25\]
\[2 \ln(x + 3) = 30\]
13) Logarithmic Equations

Solve: $2 \ln(x + 3) - 5 = 25$

$2 \ln(x + 3) = 30$

$\ln(x + 3) = 15$
13) Logarithmic Equations

Solve: \(2 \ln(x + 3) - 5 = 25\)

\[2 \ln(x + 3) = 30\]

\[\ln(x + 3) = 15\]

\[e^{15} = x + 3 \quad \text{(exponential form)}\]
13) Logarithmic Equations

Solve: \(2 \ln(x + 3) - 5 = 25\)

\[
\begin{align*}
2 \ln(x + 3) &= 30 \\
\ln(x + 3) &= 15 \\
e^{15} &= x + 3 \quad \text{(exponential form--cursive "e")} \\
x &= e^{15} - 3
\end{align*}
\]
14) Difference Quotient

\[ f(x) = x^2 + 3x \]

\[ \frac{f(x + h) - f(x)}{h}, h \neq 0 \]
14) Difference Quotient

\[ f(x) = x^2 + 3x \]

\[ \frac{f(x + h) - f(x)}{h}, \quad h \neq 0 \]
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14) Difference Quotient

\[ f(x) = x^2 + 3x \]

\[ \frac{f(x + h) - f(x)}{h}, h \neq 0 \]
14) Difference Quotient

\[ f(x) = x^2 + 3x \]

\[ f(x + h) = (x + h)^2 + 3(x + h) \]

\[ \frac{f(x + h) - f(x)}{h}, \quad h \neq 0 \]
14) Difference Quotient

\[
f(x) = x^2 + 3x
\]
\[
\frac{f(x + h) - f(x)}{h}, \quad h \neq 0
\]

\[
(x + h)^2 + 3(x + h)
\]
\[
(x + h)(x + h) + 3(x + h)
\]
14) Difference Quotient

\[ f(x) = x^2 + 3x \]

\[ \frac{f(x + h) - f(x)}{h}, h \neq 0 \]

\[ (x + h)^2 + 3(x + h) \]

\[ (x + h)(x + h) + 3(x + h) \]

\[ x^2 + 2xh + h^2 + 3x + 3h \]
14) Difference Quotient

\[ f(x) = x^2 + 3x \]

\[ \frac{f(x + h) - f(x)}{h}, \quad h \neq 0 \]

\[ (x + h)^2 + 3(x + h) \]

\[ (x + h)(x + h) + 3(x + h) \]

\[ x^2 + 2xh + h^2 + 3x + 3h - (x^2 + 3x) \]
14) Difference Quotient

\[ f(x) = x^2 + 3x \]

\[ \frac{f(x + h) - f(x)}{h}, h \neq 0 \]

\[(x + h)^2 + 3(x + h)\]

\[(x + h)(x + h) + 3(x + h)\]

\[x^2 + 2xh + h^2 + 3x + 3h - (x^2 + 3x)\]

\[x^2 + 2xh + h^2 + 3x + 3h - x^2 - 3x\]
14) Difference Quotient

\[ f(x) = x^2 + 3x \]
\[ \frac{f(x + h) - f(x)}{h}, h \neq 0 \]
\[ (x + h)^2 + 3(x + h) \]
\[ (x + h)(x + h) + 3(x + h) \]
\[ x^2 + 2xh + h^2 + 3x + 3h - (x^2 + 3x) \]
\[ x^2 + 2xh + h^2 + 3x + 3h - x^2 - 3x \]
14) Difference Quotient

\[ f(x) = x^2 + 3x \]

\[ \frac{f(x + h) - f(x)}{h}, h \neq 0 \]

\[ (x + h)^2 + 3(x + h) \]

\[ (x + h)(x + h) + 3(x + h) \]

\[ x^2 + 2xh + h^2 + 3x + 3h - (x^2 + 3x) \]

\[ x^2 + 2xh + h^2 + 3x + 3h - x^2 - 3x \]

\[ 2xh + h^2 + 3h \]
14) Difference Quotient

\[ f(x) = x^2 + 3x \]

\[ \frac{f(x + h) - f(x)}{h}, \quad h \neq 0 \]

\[ (x + h)^2 + 3(x + h) \]

\[ (x + h)(x + h) + 3(x + h) \]

\[ x^2 + 2xh + h^2 + 3x + 3h - (x^2 + 3x) \]

\[ \frac{x^2 + 2xh + h^2 + 3x + 3h}{h} \]
14) Difference Quotient

\[ f(x) = x^2 + 3x \]

\[ \frac{f(x + h) - f(x)}{h}, \ h \neq 0 \]

\[ (x + h)^2 + 3(x + h) \]

\[ (x + h)(x + h) + 3(x + h) \]

\[ x^2 + 2xh + h^2 + 3x + 3h - (x^2 + 3x) \]

\[ 2x + 2h + h^2 + 3h \]

\[ \frac{2xh + h^2 + 3h}{h} \]
14) Difference Quotient

\[ f(x) = x^2 + 3x \]

\[ \frac{f(x + h) - f(x)}{h}, h \neq 0 \]

\[
(x + h)^2 + 3(x + h) \\
(x + h)(x + h) + 3(x + h) \\
x^2 + 2xh + h^2 + 3x + 3h - (x^2 + 3x) \\
2xh + h^2 + 3h \\
= 2x + h + 3, h \neq 0
\]
15) Factoring Trinomials
(Leading Coefficient = 1)

*”Car Method” (front seat, back seat)

*T-table for middle linear term

*Constant (loser) → Daddy
*If Daddy is +, Daddy says “DOUBLES”
*If Daddy is -, Daddy says “COMBO”

*Linear term (#x) → Mom
*Mom says “Put MY sign with my OLDEST child!”
16) Factoring Sum/Difference of Cubes

Source: Ms. June Blackwell, Raleigh, NC

“Square the Outsides, Opposite Product in the Middle”

Factoring Sum and Difference of Two Cubes

\[ x^3 + y^3 = (x+y)(x^2 - xy + y^2) \]

\[ x^3 - y^3 = (x-y)(x^2 + xy + y^2) \]

\[ x^3 + 125 = (x)^3 + (5)^3 \]

\[ = (x+5)[x^2 -(x)(5)+5^2] \]

\[ = (x+5)(x^2 - 5x + 25) \]
17) Trig Hexagon

http://mathandmultimedia.com
18) Bearings

The Titanic II leaves the harbor and sails 110 miles on a bearing of $\text{N 54}^\circ \text{E}$. 

The bearing from the Lusitania II to the Cape Hatteras lighthouse is $\text{S 36}^\circ \text{W}$. 

![Diagram of compass directions]
19) Sum & Difference Formulas

A Sine “Sandwich”
Where the Signs Stay the Same

\[
\sin(A + B) = \sin A \cos B + \cos A \sin B \\
\sin(A - B) = \sin A \cos B - \cos A \sin B
\]
20) Sum & Difference Formulas

Michael Stueben’s Trig Mnemonic from *Twenty Years Before the Blackboard*:

As we all know, some of the people to whom we are attracted are not attracted to us. And it is not unusual for a person who has shown interest in us to later lose interest in us. Maybe that is a good thing, because it forces us to date a lot of people and to become more experienced in maintaining relationships. Anyway, this is the story of Sinbad and Cosette. Sinbad loved Cosette, but Cosette did not feel the same way about Sinbad. Naturally, when Sinbad was in charge of their double date, he would put himself with Cosette, and he put his brother with her sister.

Sinbad loved to tell people that his and Cosette’s signs were the same. However, when Cosette was in charge of the double date she placed herself with her sister and put Sinbad with his brother. She made sure everyone knew that their signs were NOT the same.

Also, notice that Cosette placed herself and her sister BEFORE Sinbad and his brother. This detail was important to Cosette. She was very snobby, you know.

\[
\begin{align*}
\sin(A + B) &= \sin A \cos B + \cos A \sin B \\
\sin(A - B) &= \sin A \cos B - \cos A \sin B \\
\cos(A + B) &= \cos A \cos B - \sin A \sin B \\
\cos(A - B) &= \cos A \cos B + \sin A \sin B
\end{align*}
\]
YOUR IDEAS . . .
Our Information . . .