Improving Reading Skills of Developmental Mathematics Students

AMATYC 2019
Milwaukee, WI
Diana Hestwood and Linda Russell
Purposes of this session

• To raise awareness of the challenges of reading in math, especially for developmental students.

• To learn the reading skills that are particular to math.

• To understand the “habits of mind” needed to successfully read and learn math.

• To learn specific techniques and practices for the classroom that will enhance students’ ability to read textbooks and online materials.
Disciplinary literacy

• Disciplinary literacy is based on the premise that students can develop deep conceptual knowledge in a discipline only by using the habits of reading, writing, talking, and thinking which that discipline values and uses.

What is reading?

- making meaning out of text
- interacting with text
- pronouncing words correctly
What is reading?

• interpreting

• inferring

• drawing conclusions about text

• anticipating what will come next in text
Now you try reading...

• Read the paragraph on the front of your half-sheet.

• Try to figure out what it is about.

• How can you interpret the actions?
Paideia pulls to start. They come zone with a three-man cup. Y picks up the disc and centers to Fink, who breaks through the cup to Bumsted. We swing up to Z on the high side, then dishy up field. Huck goes up to Seff, dishy to JJ. We reset to JayTay and Y. Backhand break to Y bidding, it's too far. Paideia centers, dumps and sets into their offense. A big Huck goes off -- sky to score.
Hints

• Paideia and Hopkins are two teams.

• The following are names of people: Y, Z, Bumsted, Fink, Seff, JJ, JayTay.

• Flip over your half sheet to see the vocabulary list on the back. How helpful is it? Can you make any sense yet?
Think about this reading experience.

• What was difficult about the reading?

• Did anything help you figure it out?

• Did having the vocabulary list help you?

• What is the key to understanding the reading?
What is the problem?

• Developmental students don’t read or use math materials well (hard copy or digital).

• Study Skills courses generally work at cross purposes with reading in math.
People usually forget 90 percent of what they learn in a class within 30 days.

-Brain Rules by John Medina
What constitutes “text” in math?

• Words
• Symbols (+, ≈, ∨, ( ))
• Abbreviations and Acronyms (L, in., mm, mm², LCM, GCF, LCD)
• Numbers (digits or words) in formulas, equations, and exercises

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What constitutes “text” in math?

• Logograms (e.g., $, which stands for the word “dollars”)

• Drawings and diagrams

• Graphs, tables, charts

• Arrows, labels, special use of color (such as boxed info)
Let’s compare reading tasks.

Most “other” content

• Text is read from top to bottom.

• A line of text is read from left to right.

Mathematics content

• Text may be read from top to bottom, but not always.

• A line of text may be read in varying directions: Order of Operations.

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Example of math content

Example 2: Applying the Order of Operations

Simplify: \( \frac{1}{30} - \left( -\frac{1}{3} \right)^2 \cdot \frac{3}{5} \)

Solution:

\[
\frac{1}{30} - \left( -\frac{1}{3} \right)^2 \cdot \frac{3}{5} \\
= \frac{1}{30} - \frac{1}{9} \cdot \frac{3}{5} \\
= \frac{1}{30} - \frac{1}{9} \cdot \frac{3}{5} \quad \text{Multiply fractions.}
\]

Simplify exponents. Note: \( \left( -\frac{1}{3} \right) \cdot \left( -\frac{1}{3} \right) = \frac{1}{9} \).

\[
= \frac{1}{30} - \frac{1}{15} \\
= \frac{1}{30} - \frac{2}{30} \\
= \frac{1}{30} + \left( -\frac{2}{30} \right) \\
= -\frac{1}{30}
\]

Your eyes begin in the middle

Then they move to the right

Finally, they go back to the left and work to the right

Plus, the student needs to keep things lined up vertically.

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Spacing is important!

• The items in a bulleted list have a relationship to content above it.
  • Such as listing examples of a concept;
  • Providing more explanatory detail;
  • Providing reasoning or evidence to support an argument.

• Lining up the equal signs will help keep the rest of the parts of a problem in line.

• Large or uneven writing
  • characteristic of learning disability.
  • Will need more space to write;
  • a good accommodation is providing extra paper for exams.
Let’s compare reading tasks.

Most “other” content
• The reader may have to decide when to read insets, boxed material, and graphics, relative to the main text.

Mathematics content
• Students may read, move to the margin and do problems, look at the bottom for answers, resume reading display material.

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How your students feel sometimes...
When text is dense.

\[ \text{Density} = \text{many new terms} + \text{new meanings for known words} + \text{new concepts} \]
Density Example 1

“When a letter can stand for various numbers, we call the letter a variable. A number or a letter that stands for just one number is called a constant.

Let \( c \) = the speed of light.

Then \( c \) is a constant. Let \( a \) = the speed of a car. Then \( a \) is a variable since the value of \( a \) can vary.”

Circle all the stand alone letter “a’s” in the above paragraph. How many did you find?
Density Example 2

Example 1(c)

\[
125\% = 125 \times \frac{1}{100}
\]
\[
125\% = \frac{125}{100}
\]

Note that 100\% = 100 \times \frac{1}{100} = 1. That is, 100\% represents one whole unit. In Example 1(c) \(125\% = \frac{5}{4}\) or 1\(\frac{1}{4}\). This illustrates that any percent greater than 100\% represents a quantity greater than one whole. Therefore, its fractional form may be expressed as an improper fraction or as a mixed number (p. 336).

There are both concepts and terminology in this short paragraph that, together, can make it quite densely packed.

Examples:

- Shifting from \% to fractions to mixed numbers.
- Concept of one whole (or whole unit) vs. fractional part vs. greater than one whole.
- Improper fraction (term) and mixed number (term).
Reading demands IN MATH.

“Precision reading.”

Example from Order of Operations: “Do the remaining multiplication and division in the order they occur from left to right.”

• The word “and” is very important, otherwise the students may do all multiplications before any divisions.
Reading demands IN MATH.

A much more detailed, careful, exactness that we don’t emphasize in many other subjects (especially at introductory levels).

• Ex: In the Order of Operations, if you do the order differently in any way, the answer may be wrong (the division might come first; the subtraction might come first).
Reading demands IN MATH.

• If a student has “the gist,” they may be partly right in a social science, and just plain WRONG in math.
  • Compare: 10 divided by 5 vs. 10 divided into 5. Is the word “by” important? Or “into”?
  • What’s the difference between 7 and 4? (In subtraction, it is 3; it could also mean that 7 is odd and 4 is even, or 7 is prime and 4 is not.)
"A bushel is a unit of weight equal to four pecks."

What's a peck?

A quick smooch.

You know, I don't understand math at all.
Examples of “confusing little words.”

• All (not some, many, or most)

• “The number” vs. “a number”

• Find (meaning solve)

• One(s) (place value, as opposed to “one item”) 

• Write or Rewrite (as in “Rewrite mixed numbers as improper fractions”) vs Right
Examples of “confusing little words.”

- **Each** (usually tells you to divide)
- **Do** (a question word)
- **Of versus Off** (percent of vs. percent off)
- **Of** (indicating multiplication, when following a fraction)
- **Divided by vs. divided into**
“Loose” definitions vs. math definitions

Non-Math

• **Line**: like this ______ .

• **Slope**: could just indicate a change of height, say, of a road.

• **Positive**: can be a good thing.

• **Minus**: often used like “it’s minus 10 degrees out there.”

• **Negative**: can be a bad thing.

Math

• **Line** has a precise definition.

• **Slope** is how much tilt a line has and can be positive or negative.

• **Positive** is to the right of zero on a number line.

• **Minus** means subtraction.

• **Negative** is to the left of zero on a number line.

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Familiar words...specialized meanings

An employee was using this password:

MickeyMinniePlutoHueyLouieDeweyDonaldGoofySacramento

Why?
When asked why she had such a long password, she rolled her eyes and said: "Hello! It has to be at least 8 characters and include at least one capital."
Habits of Mind for Math

• **Accurate reading** of every word. Correct representation of words, numbers, etc. No “looseness.” Pronunciation needs to be correct too. (Commutative has nothing to do with Communication.)

• **Precision reading**, which indicates a degree of exactness that may be well beyond the usual experience of students. (8.0, 8, and 8.000 are not the same if rounding is involved.)
Habits of Mind for Math

• **Meticulousness** in terms of having a systematic process. This dovetails nicely with the problem solving steps that math texts always include, as well as for following other important steps, like order of operations.

• **Persistence.** Reading a sentence or paragraph several times if it doesn’t make sense.

• **Valuing** all of the above traits and habits. This can be a tough sell, but it is really important to change attitudes.
Conclusions: what can you do?

- Talk with your reading colleagues about confusions that students have on your syllabus, course schedule, or test items.
- Explicitly teach the habits of mind. Reinforce how important it is to be precise and accurate. Use current events that reinforce the role these play in construction, accounting, etc. when appropriate.
- Model the habits overtly.

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Conclusions: what can you do?

✓ When showing worked examples, mention when you are being precise, reinforce important terminology, and point out those “little words.”

✓ Say aloud your own thought process when working examples.

✓ Provide missing background information when possible.

✓ Look for math materials that are less dense and use more “plain language.”
How less dense text handles terminology.

- All math terms are introduced one at a time.
- Terms are introduced in a non-technical way, relating them to things students already know.
- Terms are compared to common English words.
- Examples are given, as well as counter examples.
- The text includes frequent, short reminders of the meanings when the term is encountered pages or chapters later.

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Improving Reading Skills of Developmental Mathematics Students
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These are the activities we referenced in our presentation. They are designed to help your students:

- Use textbooks or online materials more effectively
- Take better lecture notes
- Create more useful study cards

We also include a brief description of disciplinary literacy in four disciplines (including math) and an extensive bibliography as a resource for you.

We hope you enjoyed learning about the reading aspects of learning mathematics! Please contact either of us if you have questions.
ACTIVITY OUTLINE FOR TEXTBOOKS:

Objectives
1. Explain the meaning of text features such as section numbering, objectives, cautions.
2. Locate the Answers, Solutions, and Index sections.

A textbook can be very helpful. Find out what our book has to offer. First, let’s look at some general features that will help in all chapters. Explain what the feature is, list a page number showing one, and tell how you might use it. [Make a short activity in class for this, or assign it outside of class the first week. Students can work in pairs and turn in one paper, for example.]

Table of Contents

Section Numbering

Chapter Features
Four sample features that appear in the chapters of many books.

- Objectives.
- Periodic Check-Understanding Exercises.
- Cautions, Pointers, Notes, Calculator Tips, or other “Tips.”

- End-of-Chapter Features
  - Chapter Summary
  - Quick Review
  - Review Exercises. Make sure you do the Mixed Review Exercises to practice for tests.
  - Chapter Test
  - Cumulative Review

Answers
Flag the Answers Section with a sticky note or other device, so that you can turn to it quickly.

Solutions

Index
SAMPLE FOR ONLINE MATERIALS

Objectives
1. Explore the features of the online materials; analyze when each part might be a useful exercise.
2. Locate the practice quizzes and tests.

There are multiple online homework systems which have many helpful features. What are they? When would students use them? [Many students ONLY use the “Homework” section because that is what their instructor assigns. How can you get them using the other ones?]

Homework (features inside this may include “Ask My Instructor,” “Connect to a Tutor,” “Show me an example,” etc.)

Quizzes and Tests

Study Plan/Time Management

Gradebook

Chapter Contents

Multimedia Library

Course Tools
Study Skills  
**Taking Lecture Notes**

Objectives  
1. Apply note taking strategies, such as writing problems as well as explanations.  
2. Use appropriate abbreviations in notes.

Study the set of sample math notes in this section, and read the comments about them. Then try to incorporate the techniques into your own math note taking in class.

- The **date and title** of the day’s lecture topic are always at the top of every page. **Always begin a new day with a new page.**
- Note the **definition** of expression is written in and examples are included.
- Notice the **labels** on all parts.
- **Skipping lines** makes the notes easier to read.
- See the **box** which reminds you about an important idea. **Underlining** is used for emphasis.
- See how the direction word (**evaluating**) is emphasized and explained.
- Notice the two columns, which allow for the example and its explanation to be close together. **Whenever you know you’ll be given a series of steps to follow, try the two-column method.**
- Also, notice the reminder comment about how the final answer should look.
- **See the warning—Careful!** This alerts you to a common point of confusion.
Why are these notes brain friendly?

The notes are easy to look at, and you know that the brain responds to things that are visually pleasing. Other techniques that are visually memorable are the use of spacing (the two columns), stars, underlining, and circling. All of these methods allow your brain to take note of important concepts and steps.

The notes are also systematic, which means that they use certain techniques regularly. This way, your brain easily recognizes the topic of the day, the signals that show an important point, and steps to follow for procedures. When you develop a system that you always use in your notes, your notes are easy to understand later when you are reviewing for a test.

NOW TRY THIS:

Find one or two people in your math class to work with. Compare each other’s lecture notes over a period of a week or so. Ask yourself the following questions as you examine the notes.

1. What are you doing in your notes to show the main points or larger concepts? (Such as underlining, boxing, using stars, capital letters, etc)

2. In what ways do you set off the explanations for worked problems, examples, or smaller ideas (subpoints)? (Such as indenting, using arrows, circling or boxing)

3. What does your instructor do to show that he/she is moving from one idea to the next? (Such as saying “Next” or “any questions,” “now,” or erasing the board, etc.)

4. How do you mark that in your notes? (Such as skipping lines, using dashes or numbers, etc.)

5. What explanations (in words) do you give yourself in your notes, so when those new dendrites you grew in lecture are fading, you can read your notes and still remember the new concepts later when you try to do your homework?

6. What new ideas did you learn by examining your classmates’ notes?

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7. What will you try in your own note taking? List four techniques that you will use next time you take notes in math.

Study Skills

**Using Study Cards**

Objectives
1. Create study cards for all new terms.
2. Create study cards for new procedures.

You may have used “flash cards” in other classes before. In math, study cards can be helpful too. However, they are different because the main things to remember in math are not necessarily terms and definitions, they are *sets of steps to follow* to solve problems (and how to know which set of steps to follow), and *concepts about how math works* (principles). So, the cards will look different, but be just as useful. Look carefully at what kinds of information to put on them and where to put it. Then use them the way you would any flash card:
   o to quickly review when you have a few minutes,
   o to do daily reviews,
   o to review before a test.

**But remember, the most helpful thing about study cards is making them.** It is in the making of them that you have to do the kind of thinking that is most brain friendly and will improve your neural network of dendrites. After each card description you will find an assignment to try. It is marked “NOW TRY THIS.”

**New Vocabulary Cards**

Put the word (spelled correctly) on the front of the card. On the back, write:
   o the definition (in your own words if possible),
   o an example, an exception (if there are any),
   o any related words, and
   o a sample problem (if appropriate).

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Front of Card

variable

p, 84
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Back of Card

Def: A letter representing a number that varies (changes) depending on the situation.

Example: \[ c + 5 \] where \( c \) represents the class limit which varies for different classes. \\
\[ \text{variable} \quad \uparrow \] \\
\[ \text{constant} \quad \downarrow \text{varies} \leftrightarrow \text{variable} \]

- Use any letter you like: \( a, b, c, d, h, k, m, n, p, x, y \) etc.
- Related words: constant, coefficient

NOW TRY THIS: List 4 new vocabulary words/concepts you need to learn right now. Make a card for each one.

______________________________  ______________________________

Procedure ("Steps") cards

Write the name of the procedure at the top on the front of the card. Then write each step in words. If you need to know abbreviations for some words, include them along with the whole words written out. On the back, put an example of the procedure, showing each step you need to take. You can review by looking at the front and practicing a new worked example, or by looking at the back, and remembering what the procedure is called and what the steps are.

Front of Card

**Evaluating an expression**

**Step 1:** Replace each variable with the value you are given.

**Step 2:** Do the calculations, following the order of operations.
NOW TRY THIS: What procedure are you learning right now? Write below the steps that you will put on your study card.

Procedure: ________________________________________________

Step 1. ___________________________________________________

Step 2. ___________________________________________________

Step 3. ___________________________________________________

Step 4. ___________________________________________________

Step 5. ___________________________________________________

Example: Evaluate $3c - 5$ when $c$ is 20.

- $3c - 5$ Replace $c$ with 20.
- $3(20) - 5$ Do the calculations.
- $60 - 5$ Follow order of operations; multiply first, then subtract.
- $55$ No variables in final answer.
Interpreting Different Types of Numerical Information

Textbooks across the curriculum provide information in numerical form. Unlike math, students are sometimes not computing, solving, or manipulating the numbers. In the examples below, students are supposed to gain meaning about data, interpret the significance of numerical information, and often, to remember numerical information for exams. Practice in moving from percent to fractional reporting and understanding very large numbers written in decimal form (or words) is helpful.

Students should become comfortable interpreting each statement below. Can they restate in their own words? Do they understand what the numbers mean?

1. In selective colleges, fifty-five percent of freshman come from families in the top 25% of income.
   - About what fraction of students come from the top level of income?
     a) 1/4;
     b) 1/2;
     c) or 2/3

2. Four out of five immigrants in the 1980s came from Asia, Latin America, and the Caribbean.
   - What percent of immigrants in the 1980s came from (etc)? (Open ended, no choices given.)

3. California remained the leading state of residence of the illegal immigrant population in 2015, with 2.9 million, nearly 25 percent of the total number.
   - Write 2.9 million in digits.
   - What was the total number of illegal immigrants in 2015?

4. Absolute poverty is defined by the World Bank (2014a) as when someone lives on less than $1.25 a day. According to the most recent estimates, in 2011, about 17 percent of people in the developing world lived at or below $1.25 a day, a decrease of 26 percent compared to ten years ago, and an overall decrease of 35 percent compared to twenty years ago.
   - 17% is about
     - One half
     - One quarter
     - One fifth
   - Is the absolute poverty situation getting better or worse, according to this statement? [Why is this difficult to understand?]
## Disciplinary Literacy: Comparison of Four Disciplines

<table>
<thead>
<tr>
<th>Discipline</th>
<th>Purpose</th>
<th>Belief</th>
</tr>
</thead>
<tbody>
<tr>
<td>History</td>
<td>To create a <strong>plausible</strong>, complete accounts of the past, based on evidence from the historical record.</td>
<td>There is no one true account of the past.</td>
</tr>
<tr>
<td>Science</td>
<td>To create <strong>replicable, reliable</strong> findings about scientific phenomena that can be used to <strong>predict</strong> what will happen under similar conditions.</td>
<td>We constantly strive to get closer to the truth.</td>
</tr>
<tr>
<td>Math</td>
<td>To find the <strong>answer</strong> to a problem; the abstract <strong>truth</strong>.</td>
<td>Logical, accurate solutions produce true answers.</td>
</tr>
<tr>
<td>Literature</td>
<td>To create artificial worlds that provide <strong>insight</strong> into the human condition.</td>
<td>Truth is irrelevant.</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Disciplines</th>
<th>Methods</th>
<th>Evidence</th>
</tr>
</thead>
<tbody>
<tr>
<td>History</td>
<td>Analysis of already existing artifacts and writings of other historians to answer questions about cause/effect, significance, goals/motivations, etc.</td>
<td>Is the analysis plausible, corroborated, complete. Is there bias in perspective?</td>
</tr>
<tr>
<td>Science</td>
<td>Controlled experiments and systematic observations to discover features, processes, interactions, etc. of phenomena under study.</td>
<td>Is the evidence experimental, replicable. Is process predictable?</td>
</tr>
<tr>
<td>Math</td>
<td>Logical thought to solve problems.</td>
<td>Is the logic accurate?</td>
</tr>
<tr>
<td>Literature</td>
<td>Use of story and poetry to interpret the human condition.</td>
<td>Is an interpretation of meaning based on textual clues?</td>
</tr>
</tbody>
</table>

<table>
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<th>Discipline</th>
<th>Texts</th>
<th>Approaches to Reading</th>
</tr>
</thead>
<tbody>
<tr>
<td>History</td>
<td>Recounts, explanations, arguments</td>
<td>Everything should be read critically, with an eye to the perspective of the author and the credibility of the information.</td>
</tr>
<tr>
<td>Science</td>
<td>Explanations, arguments about features, processes, interactions</td>
<td>Experiments/findings read critically for adherence to scientific methods; replicated, reliable information read in learning mode (as with science textbooks).</td>
</tr>
<tr>
<td>Math</td>
<td>Proofs (of hypotheses), solutions to problems, explanations of mathematical processes</td>
<td>Every word and symbol should be read carefully, and texts should be reread. Need to look for errors.</td>
</tr>
<tr>
<td>Literature</td>
<td>Poems, short stories, novels, plays; critiques; most emphasizing character, plot, rising action, climax, theme, literary devices;</td>
<td>Read in accordance with a particular literary tradition (e.g. close reading, reader response, scholarly reading, using a particular interpretive lens/poststructural).</td>
</tr>
</tbody>
</table>
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Somatosensory Cortex video
https://www.youtube.com/watch?time_continue=22&v=jziqfFp9Rq0


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