CHAPTER 1: Introduction

The ultimate goals of this document are to improve mathematics education and to encourage more students to study mathematics.
One day, about one-third of the way through the spring semester, an experienced mathematics faculty member walked into her department chair’s office with a worried look. She had volunteered to teach an experimental section of Intermediate Algebra using a textbook that emphasized real problem solving, group projects, and technology. Algebra techniques were introduced as they were needed to solve problems. For example, while there was considerable work that involved linear, quadratic, exponential, and logarithmic functions, as well as systems of linear equations, there was little, if any, work on such traditional algebra topics as factoring, theory of equations, and the solution of rational and radical equations. She explained to the department chair that it was really refreshing to teach the new material using innovative instructional methods. Furthermore, her students, who were hesitant at the beginning of the semester to get involved in group projects, were also beginning to enjoy the new approach to studying mathematics. So, why the worried look?

When she had volunteered to use what she considered an exciting approach to learning algebra, she thought that she understood the underlying principles of the much discussed reform in mathematics education. She wanted to get involved. Now she wasn’t sure that she was doing the right thing. Her course did not include many of the topics that had been the mainstay of intermediate algebra. She was worried that she was not adequately preparing her students for the study of higher levels of mathematics either at her college or at transfer institutions.

If you were the department chair, how would you have responded to your faculty member’s concerns?

THE NEED FOR CROSSROADS IN MATHEMATICS: STANDARDS FOR INTRODUCTORY COLLEGE MATHEMATICS BEFORE CALCULUS

Higher education is situated at the intersection of two major crossroads: A growing societal need exists for a well-educated citizenry and for a workforce adequately prepared in the areas of mathematics, science, engineering, and technology while, at the same time, increasing numbers of academically underprepared students are seeking entrance to postsecondary education.

Mathematics is a vibrant and growing discipline. New mathematics is continually being developed and is being used in more ways by more people than ever before. In fact, the rate of growth in mathematically based occupations is about twice that for all other occupations (NRC, 1990). Yet, an alarming situation now exists in postsecondary mathematics education. More students are entering the mathematics “pipeline” at a point below the level of calculus, but there has been no significant gain in the percentages of college students studying calculus (Albers et al., 1992). The purpose of Crossroads in Mathematics is to address the special circumstances of, establish standards for, and make recommendations about introductory college mathematics. The ultimate goals of this document are to improve mathematics education and encourage more students to study mathematics.

The students addressed in this document are seeking Associate of Arts (AA), Associate of Science (AS), Associate of Applied Science (AAS), and bachelor’s degrees.
degrees. Some are traditional full-time students who are recent high school graduates. Others, particularly those at two-year colleges, are from widely diverse populations and fall into one or more of the following categories. They

- are older,
- work a full- or part-time job while attending college,
- manage a household,
- are returning to college after an interruption in their education of several years,
- intend to enter the work force after obtaining an associate degree,
- intend to work toward a bachelor's degree either at a transfer institution or in the upper division of their present four-year college or university,
- are studying for a degree as a part-time student,
- have English as a second language,
- need formal developmental work in a variety of disciplines and in study skills,
- have no family history in postsecondary education, or
- have disabilities that require special accommodations.

All of these characteristics dramatically affect introductory college mathematics instruction.

**Basic Principles**

The following principles form the philosophical underpinnings of this document:

- **All students should grow in their knowledge of mathematics while attending college.** College students who are not prepared for college-level mathematics upon entering college can obtain the knowledge necessary by studying the Foundation (as described in Chapter 3). These students, along with others who enter college prepared for college-level mathematics, will continue to study mathematics to reach the level of sophistication required for their educational, career, and life goals.

- **The mathematics that students study should be meaningful and relevant.** Basic skills, general principles, algorithms, and problem-solving strategies should be introduced to the students in the context of real, understandable problem-solving situations so that students gain an appreciation for mathematics as a discipline, are able to use it as a base for further study, and can transfer this knowledge to problem-solving situations at work or in everyday life. Intuitive justifications for mathematical principles and procedures should be emphasized.
• *Mathematics must be taught as a laboratory discipline.* Effective mathematics instruction should involve active student participation. In-depth projects employing genuine data should be used to promote student learning through guided hands-on investigations.

• *The use of technology is an essential part of an up-to-date curriculum.* Faculty and students will make effective use of appropriate technology. The technology available to students should include, but not be limited to, that used by practitioners in the field. Faculty should take advantage of software and graphing calculators that are designed specifically as teaching and learning tools. The technology must have graphics, computer algebra, spreadsheet, interactive geometry, and statistical capabilities.

• *Students will acquire mathematics through a carefully balanced educational program that emphasizes the content and instructional strategies recommended in the standards along with the viable components of traditional instruction.* These standards emphasize problem solving, the use of technology, intuitive understanding, and collaborative learning strategies. Skill acquisition and mathematical abstraction and rigor, however, are still critical components of mathematics education. Furthermore, direct whole-class instruction (lecturing, questioning, and discussion) is a viable option when working with highly structured content (Secada, 1992).

• *Introductory college mathematics should significantly increase students' options in educational and career choices.* When students master the content of introductory college mathematics, they will have the problem-solving skills that are required in many disciplines and careers.

• *Increased participation by all students in mathematics and in careers using mathematics is a critical goal in our heterogeneous society.* Mathematics instruction must reach out to all students: women, minorities, and others who have traditionally been underrepresented in the discipline, as well as students with learning difficulties, differing learning styles, disabilities, and language and socialization difficulties. Furthermore, faculty must provide a supportive learning environment and promote appreciation of mathematics.

"We must equip all of our students—regardless of age, sex, ethnic background, educational goal, occupational goal, personal history or capabilities—to think for themselves and to solve their own problems and those of society to the very best of their individual abilities. That's what the right mathematics does."

Michael Davidson,
Cabrillo College

This document makes no attempt to define "college-level mathematics," nor does it address the issue of whether courses at the introductory level should be credit bearing (to meet graduation requirements).

**SUMMARY**

Introductory college mathematics must serve well all college students who are not prepared to study at the calculus level or beyond. This document offers a new paradigm for this level of mathematics education. The standards that follow in Chapter 2 are not a "quick fix" for what is wrong. Rather, they provide a strong and flexible framework for the complete rebuilding of introductory college mathematics.