CHAPTER 3: Interpreting the Standards

Instruction and the classroom environment will be characterized by caring faculty who exhibit high expectations for students regardless of race, gender, socioeconomic class, or disability.
The standards for introductory college mathematics described in Chapter 2 provide general guidelines for improving the introductory college mathematics experience for all students. This chapter addresses their implementation in programs for underprepared students, called "the Foundation," as well as in programs for students beyond that level. In particular, the standards imply that

- the mathematical foundation for underprepared students must go beyond the teaching of arithmetic and algebra skills,
- technical programs must prepare students for their chosen occupational fields as well as provide them with flexibility in career and educational choices, and
- mathematics courses for students pursuing bachelor's degrees should not only serve as prerequisites for higher-level mathematics courses but should also provide students with meaningful content that has application to their daily lives and their career choices.

Furthermore, it is assumed that instruction and the classroom environment will be characterized by caring faculty who

- acknowledge and appreciate the diverse experiences of their students;
- exhibit high expectations for students regardless of race, gender, socioeconomic status, or disability;
- are mathematicians who can develop sound instructional strategies and content to meet the needs of all students;
- are available to help students outside the classroom; and
- provide a nonthreatening environment that encourages students to ask questions and take risks.

"The reform of mathematics education in the first two years of college speaks of building mathematical power as a basic goal. In every topic and in every course, students should be discovering the usefulness of mathematics as a means to deal with the world around us. The initial courses available at college can be thought of as general education mathematics which provides sound mathematics instruction and the incorporation of knowledge from other disciplines."

Jack Rotman, Lansing Community College

INTERPRETING THE STANDARDS
On a visit to a nearby four-year college campus, a two-year college mathematics teacher ran into a former student.

"Hi. You probably don't remember me. I was a student in your introductory algebra class four years ago."

The teacher replied, "Why, yes, I do remember you. How are you doing these days?"

"I'm doing really well," the student responded. "I'll be graduating this spring with a degree in business administration. When I first came to college, I didn't think I would amount to anything. I was an adult student returning to college, surrounded by all these kids who knew it all. And to top it all off, I had to take algebra, a course I never took in high school! Plus, whatever math I learned in school was long forgotten."

"Well, you have obviously done very well for someone who didn't think he belonged on a college campus," noted the teacher.

"Yes," replied the student. "Even though I didn't believe in my ability to do the work, you did. You challenged, encouraged, cajoled, and made me realize that I could learn just as well as others, if not better. I hadn't thought of getting a four-year degree or going into anything that might mean more math, like business administration. But you helped me to see the possibilities. Thank you. I just wanted to let you know that I appreciate what you did for me and to let you see how far I have come."

This vignette points out that students come to college with a variety of mathematical backgrounds and career goals. In order to serve this diverse student population better, every institution should make available a common core of mathematics. This common core, called "the Foundation," provides a starting point for well-designed programs that meet the individual needs of the students. The goal of the Foundation is to provide a mathematical basis for pursuing various curricular paths. Students should emerge from foundation courses with the ability and confidence to go on to the study of higher levels of mathematics so that they may use mathematics effectively in their multiple roles as students, workers, citizens, and consumers.

The Foundation should have multiple entry points that depend upon the background of the individual student. Students may master the Foundation by successfully completing high school mathematics based on the NCTM Standards (NCTM, 1989), by earning a GED based on the emerging standards for adult mathematics education, by taking mathematics courses in college, or by some combination of the three.

The Foundation must be solidly based on the standards described in Chapter 2, including an emphasis on developing reasoning ability, using technology appropriately, and solving substantial realistic problems. It should include topics found in high school basic mathematics, algebra, and geometry courses and in college developmental mathematics courses, although the emphasis will be markedly different. In addition, the Foundation will include other topics vital to establishing a solid core of mathematics knowledge and skills, including mathematical modeling, functions, discrete mathematics, probability, and descriptive statistics.
While college foundation courses serve a vital purpose, precollege students should be encouraged to acquire the mathematical knowledge of the Foundation at the secondary level whenever possible. Colleges are encouraged to join with secondary schools at local, state, and regional levels to develop and promote programs alerting students to the importance of studying more mathematics in high school. Such programs already exist in a number of states and have proved to be very successful (Demana, 1990).

The Goals

The Foundation plays a critical part in the revitalization of introductory college mathematics. Its goals are to

- help students develop mathematical intuition along with a relevant base of mathematical knowledge;
- integrate numeric, symbolic, functional, and spatial concepts;
- provide students with experiences that connect classroom learning and real-world applications;
- efficiently, but thoroughly, prepare students for additional college experiences in mathematics;
- prepare students to work in groups and independently;
- enable students to construct their knowledge of mathematics through meaningful applications and explorations as well as techniques of reasoning, regardless of their level of preparation;
- provide multiple entry points to meet the needs of students who enter college mathematics at different levels of mathematical sophistication; and
- challenge students, but at the same time foster positive student attitudes and build confidence in their abilities to learn and use mathematics.

Meeting these goals requires continual assessment of students’ readiness for new material and a system for supplying the prerequisite building blocks of the Foundation when the need arises.

The Students

Higher education will continue to serve older students with faded mathematical backgrounds, younger students with inadequate secondary school preparation, and students with learning disabilities and other special circumstances. In addition, the gradual implementation of the NCTM Standards (NCTM, 1989) in the school mathematics curriculum will add to the diversity of backgrounds of entering students. Furthermore, a disproportionately high number of minorities have entered higher education without the mathematical background to pursue mathematics-intensive coursework (NRC, 1991). The Foundation should play a key role in ensuring equity in and providing access to mathematics-intensive disciplines. The Foundation must expand the educational opportunities and broaden the career options for all students. Faculty must instill positive attitudes in their students about mathematics.

"I am certain that as women and members of the working class and other cultures participate more and more in the established mathematics, our societal conception of mathematics will change and our ways of perceiving our universe will expand. This will be liberating to us all."

David Henderson in Fear of Math, by Claudia Zaslavsky, 1994, p. 25.
Frequently, students enrolled in the Foundation have not been successful in past mathematical experiences; and they do not recognize the roles that mathematics will play in their education, in their careers, and in their personal lives. Faculty must provide meaningful content that will establish the importance of mathematics, use instructional strategies that build confidence in the students' abilities to learn and use mathematics, and provide guidance to ensure that their students use appropriate learning techniques.

**The Content**

The Foundation must be designed for the needs and interests of adult students. While it will include many topics taught in high school, the Foundation should not replicate the high school curriculum. There is a subtle, but critical, difference between building a college curriculum around students' needs and building it around their deficiencies. The greater time constraints, the more focused career interests, and the broader experiences of adult learners, as well as the goals and expectations of postsecondary institutions, necessitate different courses specifically designed for college students.

Traditional developmental courses try to cover every possible skill that students might need in subsequent courses. This coverage is likely to be too shallow to equip students for later study or for applying mathematics outside the mathematics classroom. Instead, faculty should include fewer topics but cover them in greater depth, with greater understanding, and with more flexibility. Such an approach will enable students to adapt to new situations.

The Foundation must fully embrace the use of technology and prepare students for a revised mathematics curriculum at the next level. Course content should emphasize how and when to use technology in balance with paper-and-pencil work and manipulatives.

The content for foundation courses is built around the content standards introduced in Chapter 2.

**NUMBER SENSE.** Number sense involves the intuitive understanding of the properties of numbers and the ability to solve realistic arithmetic problems using appropriate mathematical tools. The latter includes the use of mental arithmetic and calculators, with a significantly reduced emphasis on paper-and-pencil algorithms. Number sense is developed through concrete experiences. It includes knowledge of basic arithmetic facts and equivalent numerical representations and the ability to estimate answers. For instance, a person with number sense will recognize immediately that the sum of one-half and one-third is slightly less than one, or that $\sqrt{10}$ is close to 3. Likewise, numerically literate people recognize that 25%, 0.25, and $\frac{1}{4}$ are equivalent. Such intuition is based not on being able to perform an algorithm, but rather on meaningful experiences with numbers.

Number sense includes a conceptual understanding of numerical relationships and operations. Students should be able to use numbers to express mathematical relationships that occur in everyday situations. In particular, they should know how to use percent and proportionality relationships. They should also understand concepts on which arithmetic algorithms are based and be comfortable devising their own methods for performing mental arithmetic. Students with a well-developed number sense will have a basis for building an understanding of algebra and the properties of real numbers.

**SYMBOLISM AND ALGEBRA.** The study of algebra in the Foundation must focus on modeling real phenomena via mathematical relationships. Students
should explore the relationship between abstract variables and concrete applications and develop an intuitive sense of mathematical functions. Within this context, students should develop an understanding of the abstract versions of basic number properties (which assumes they have acquired a reasonably sophisticated level of number sense) and learn how to apply these properties. Students should develop reasonable facility in simplifying the most common and useful types of algebraic expressions, recognizing equivalent expressions and equations, and understanding and applying principles for solving simple equations.

Rote algebraic manipulations and step-by-step algorithms, which have received central attention in traditional algebra courses, are not the main focus of the Foundation curriculum. Topics such as specialized factoring techniques and complicated operations with rational and radical expressions should be eliminated from foundation courses. The inclusion of such topics has been justified on the basis that they would be needed later in calculus. This argument lacks validity in view of the reforms taking place in calculus and the mathematics being used in the workplace. Extensive recommendations for needed reform in algebra instruction are given in *Algebra for the Twenty-First Century* (Burrill, Choate, Phillips, & Westegaard, 1993).

GEOMETRY AND MEASUREMENT. These topics have received scant attention in the typical introductory college mathematics program (see Albers et. al., 1992, p. 85). However, the ability to visualize and mentally manipulate objects is an essential component of the Foundation. The study of measurement in the Foundation will enable students to use both the U.S. Customary System and the International (Metric) System of measurements in problems and in everyday situations. In addition, students should be able to do unit conversions and apply principles of accuracy and precision.

Geometry will include the study of basic properties of angles, polygons, and circles and the concepts of perimeter, area, and volume for basic plane and solid figures. Dynamic geometry software can enliven and deepen this study. Students should use coordinate geometry to make connections between algebra and geometry. Geometry may also be used as a vehicle to acquaint students with the study of logic and to provide an awareness of valid and invalid forms of argument. Although geometry should not be presented as a series of formal proofs, students should be able to construct simple fundamental proofs.

Right triangle trigonometry should be included with the study of geometry in the Foundation. The topic provides a context to connect arithmetic operations, algebraic formulas, and geometric properties. Furthermore, it acts as a basis for the study of analytic trigonometry by mathematics-intensive majors and for the more advanced trigonometry applications studied by many technical majors.

FUNCTIONS. Through the study of functions, students will be able to compute numerical values for, plot, and interpret the graphs of a variety of basic functions. They should be able to create and identify a variety of functions based on patterns in collected data. Students will analyze functions for periodicity, maximum and minimum values, increasing or decreasing behavior, domain and range, and average rate of change. The study of functions will include the use of their multiple representations in order to solve problems. Students should be able to make connections between the parameters of a function and the behavior of the function. Finally, students should be able to use functions to model real-world relationships.

DISCRETE MATHEMATICS. Discrete mathematics can enliven and enrich the Foundation by presenting some traditional topics from a different perspective.

"Geometry is a vehicle that provides so much of the basic core of knowledge that the student of mathematics should possess–basic geometric facts, structure of the system, applications, a study of two- and three-dimensional spaces (including some dimensional analysis), introduction to forms of argument, introduction to forms of proof, the deductive skills that reach far away places (an author writing in a lucid style or a lawyer preparing a strong court case), building the foundation for the coursework of trigonometry, analytic geometry, or calculus. And the list continues in an era that reflects ultra-modern applications of geometry."

Daniel Alexander, Parkland College

**INTERPRETING THE STANDARDS**
At the most basic level, students can use procedures such as tree diagrams, Venn diagrams, and permutation and combination formulas as aids to solving counting and probability problems. Students can use the ideas of recursion and difference equations to model phenomena from areas of human endeavor much as differential equations are used at a more sophisticated level. In addition, students can learn how to use matrices to store data and to solve problems involving the data (see Problem 10 in the Appendix). At a more advanced level, students can then proceed to use matrices to solve systems of equations.

Discrete procedures offer faculty the opportunity to integrate and connect topics in mathematics. Choppin (1994) describes a hands-on activity, for example, that connects topics in algebra and geometry and involves the students in pattern discovery, series, estimation, and recursion at an elementary level.

PROBABILITY AND STATISTICS. Foundation courses will help students develop an understanding of concepts from probability and statistics. Students will collect, summarize, and display data in such a way that reasonable conclusions may be drawn. In addition, students will be able to determine basic measures of central tendency and dispersion and be able to solve problems involving random events using basic theoretical probability and simulations. Descriptive statistics offers a particularly meaningful context for arithmetic and algebraic problem solving.

DEDUCTIVE PROOF. Formal deductive proof will not be a major emphasis of foundation courses. However, as pointed out in the discussion on geometry, students should be aware of valid and invalid forms of mathematical arguments. Informal deductive proofs can provide this awareness and at the same time give meaning to the content under discussion.

The Pedagogy

The pedagogy used in presenting material in the Foundation should mirror the standards in Chapter 2. Of particular importance in the Foundation is teaching with technology. Mathematics faculty who teach foundation material should make effective use of appropriate technology. Technology should be a routine part of instruction. Paper-and-pencil algorithms, however, should be applied to basic computations that are as easily done with paper and pencil as with a calculator. Graphing calculators and computer software should be used when beneficial or advantageous. Their use is especially helpful for geometry, functions, discrete mathematics, probability, and statistics.

The use of cooperative learning strategies is also critical to providing positive learning experiences. Many students at this level have low self-esteem. Faculty must avoid reinforcing student perceptions that the teacher is the sole authority and that the student cannot learn except through the teacher. As faculty take on the role of a coach, rather than that of an authority figure, and as students learn to work together, they will begin to realize the mathematical power they possess.

Faculty teaching the Foundation will have to walk a finer line than those in the other areas. They will have to be compassionate enough to help students work through their frustrations but show enough "tough love" to encourage them to become independent thinkers and help them realize that sustained effort will be required to truly master the material.

Students entering foundation courses bring with them some knowledge of mathematics. Faculty should help students build on this knowledge and recognize its value. Faculty should use manipulatives and other concrete models of

"Restructure remedial courses for success. Incorporate cooperative learning, peer tutoring and computer assisted instruction as supplements to traditional teaching methods. Have experienced, well motivated, and talented teachers work with students in small classes, and promote group study inside and outside of the classroom setting."

Beverly Anderson,
The AMATYC Review,
mathematics phenomena to help students make the transition from concrete to abstract thinking.

The Foundation need not be tied to traditional course structures. Goldblatt (1994) suggests that traditional remedial courses be replaced by courses that introduce students “to new areas of mathematics that do not require a highly developed skill in either algebra or arithmetic. Topics such as elementary probability and statistics, game theory, linear programming, and symbolic logic introduce new fields of study from the world of mathematics in ways that do not require the students to retrace the steps of their previous failures” (pp. 7, 9). Skills can be introduced and practiced as they are needed.

Another model for a foundation program centers on students who have at one time gained a necessary level of mathematical sophistication but need to review previously learned skills and techniques. This model calls for placing the students in courses beyond the Foundation level (e.g., precalculus, technical mathematics, and statistics). Then, as the need arises, underprepared students are given special instruction and assignments either by faculty, or as part of a program organized in an academic support center. This approach provides students with the mathematical prerequisites they lack, while involving them in mathematics more immediately relevant to their career goals.

The strategies that are used to implement the Foundation must accommodate students with disabilities and other special needs. For example, flexible scheduling is an important consideration. Students with family or work responsibilities frequently need to attend class in the evenings or on weekends. Students with disabilities or students who have been away from coursework for several years may need an extended time frame in order to complete course requirements. Haney and Testone (1990) describe an after-semester workshop program that allows students to complete course requirements between college semesters.

Increased and Decreased Attention

This vision of the Foundation mandates changes in emphasis as well as in content and pedagogy. Traditionally, mathematics at the Foundation level has emphasized the teaching of arithmetic and algebra skills and the solving of “textbook” problems. This document calls for a more balanced approach to skill and concept development. Areas that should receive increased attention include

- the active involvement of students in solving real multi-step mathematics problems;
- the introduction of needed skills in the context of real applications;
- mental arithmetic, estimation, geometric properties, and the translation of problem situations into algebraic models;
- the integration of mathematical topics so that students may use a wide range of mathematical content and techniques to solve problems;
- the conceptual understanding of mathematical ideas and the ability to use valid arguments; and
- the appropriate use of technology throughout the curriculum for computational work, graphing, geometry, probability, and statistics.

"One of the aspects of the class that I have particularly enjoyed has been working in groups. When I came to the class and heard this idea discussed, I was very wary. I was older than most of the people in the class and felt that it might be an isolating experience. After the newness of the concept wore off, I found that working with my classmates was very rewarding. We argued, we laughed, and we griped, but we all learned from sharing one another’s ideas and methods. I felt very much accepted, and even sought out by the group, which only increased my positive feeling about the experience."

A student’s comment

INTERPRETING THE STANDARDS
Areas that should receive decreased attention include

- paper-and-pencil drill with arithmetic algorithms; longhand simplification of polynomial, rational, exponential, and radical expressions; and factoring;

- solving contrived word problems, equations, and inequalities;

- the isolated topic approach to teaching and learning; and


Summary

The Foundation described in this document is radically different from the traditional secondary and developmental curricula. Students who successfully complete a study of this Foundation will have acquired a basic knowledge of mathematics that will give them the ability and confidence to go on to higher levels of mathematics that are needed in their particular areas of study and to become effective citizens in a modern society. This goal can be reached if the faculty of foundational courses engage their students in activities designed to enhance their intuitive understanding of mathematics and their belief in their own ability to do mathematics. Changing the existing curriculum to conform to the guidelines outlined here will be a formidable task. Such a change is needed, however, so that the educational experiences of underprepared students will be more relevant and valuable.
Lynn walked into her office on Monday morning to find a memo marked "URGENT." As she set her briefcase down, she glanced at the contents and was relieved to see that it was the request she had been anticipating. The company that employs Lynn had begun to make a new line of components to be shipped to other companies for use in manufacturing. It was Lynn's job, as a time study analyst, to determine the standard for the amount of time required to package these components.

The memo requested that Lynn provide as soon as possible a standard for placing hardware components in a bag. The components vary in size and weight, and different size bags are used to package the individual orders. It went on to say that the company was already receiving calls for the items and needed to establish standards to price them properly. The standards would be difficult to establish since the three factors all affect the time required to package each order. Lynn was prepared; she had already been on the production line and taken data (a sample of the data appears in Table 1) from which she could prepare the standard. Now she had to analyze the data to develop a formula or system of curves to serve as a model for predicting the time required to bag the components. She would then have to test the model and, if it proved to be satisfactory, write a report.

"Workers are less and less expected to carry out mindless, repetitive chores. Instead they are engaged actively in team problem-solving, talking with their co-workers and seeking mutually acceptable solutions."


<table>
<thead>
<tr>
<th>Study No.</th>
<th>Time (min.)</th>
<th>Weight of Components (lb)</th>
<th>Bag Size</th>
<th>No. of Components</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.264</td>
<td>6.62</td>
<td>4</td>
<td>11</td>
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<tr>
<td>2</td>
<td>0.130</td>
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</tr>
<tr>
<td>4</td>
<td>0.169</td>
<td>2.91</td>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 1. Time Required to Place Hardware in a Bag.

Students in technical programs should be prepared to function effectively in an ever-changing workplace. As the vignette shows, the mathematical tools workers use have changed drastically in the last few years. As a result of implementing computing and networking technology, companies have restructured, pushing decision making down the organizational structure. Technicians often work with their peers in teams and are required to acquire and process information and make decisions based on data formerly available only to engineers and managers. In this data-driven, technologically advanced environment, technicians must be proficient in the application of technology, in mathematics at the level of the Foundation and beyond, and in critical thinking.

The mathematical preparation of technical students should focus on applications. The effectiveness of their education will be very limited, however, if they do not become proficient in performing basic mathematical skills and have...
"Students' dispositions for change between the ages of 15 and 20 virtually guarantee that no inflexible system of early tracking can be personally or educationally sound."


The mathematics studied by students as part of their technical programs must support them if their careers change, or if they decide to study additional more sophisticated mathematics.

Collins, Gentry, and Crawley (1993) recommend that two- and four-year colleges and universities should "enhance upward academic mobility (e.g., from Associate of Applied Science to Bachelor of Engineering Technology programs) through articulated curricula with emphasis on applied content and skills. Bridging or transitional programs should be available where necessary" (p. 7). Although the focus of a technical student's mathematical interest may be applications to his or her chosen field, these students should also learn to appreciate the usefulness of mathematics for solving problems from a variety of fields. The development of the ability to reason and to analyze problems from other disciplines will assist technical students in becoming better workers and better informed citizens.

Technicians need to be flexible workers who can adapt to the changing needs of their jobs. These changes often necessitate the acquisition of additional skills, especially in the application of mathematics. Therefore, the educational environment should encourage students to develop strategies for learning additional mathematics independently in order to change from one job to another and to meet their evolving educational and career objectives.

To accomplish these goals, the content and structure of the mathematics curriculum for technical students must be both rigorous and relevant. Druckman and Bjork (1994), in a summary of research on transfer of learning, point out:

> Although concrete experience is very important, the teaching of abstract principles plays a role in acquiring skills over a broad domain of tasks. ... A training program that provides learners with varied contexts and general procedures allows them to adapt to new situations not encountered during training. (p. 11)

Mathematics courses must be designed around the concepts and applications that connect topics and make mathematics meaningful. Courses organized around the manipulation of algebraic symbols and routine exercises do little to promote transfer of learning. To prepare students for the world of work, the mathematics they study must be broad based. It must provide the necessary skills and conceptual understanding that will allow for the study of more advanced concepts, as well as the appropriate problem-solving strategies for solving real problems in a variety of contexts and interpreting their results.

**The Content**

"The key issue for mathematics education is not whether to teach fundamentals but which fundamentals to teach and how to teach them," writes Lynn Steen (1990, p. 2). If mathematics courses for technical students are to include realistic problem solving, extended projects, collaborative work, and portfolios, faculty must reexamine the structure and content of the curriculum. In conjunction with professionals from other disciplines and representatives from business and industry, they must decide what mathematics is most important for technical students to learn.

The curriculum should provide substantive mathematical challenges, building upon the Foundation to include an understanding of numeric, algebraic, and geometric topics. Some programs might also include trigonometry, statistics, or an introduction to calculus (Collins et al., 1993). Blending applications and skills, the curriculum should develop intuitive understanding through a combination of
paper-and-pencil and technology-oriented activities. For example, once students understand simple exponential functions and can evaluate them with paper and pencil, a computer or graphing calculator may be used to evaluate and graph more complicated examples. Time can then be spent giving meaning to the solutions of problems involving exponential functions and determining the effect changes in the parameters have on the solutions. Enough attention to proof and formal derivations should be given to provide students with an appreciation of the supporting mathematical theory; however, the focus of the technical curriculum should be on applications.

Central to the mathematics education of technical students is the development of the ability to design and use algorithmic procedures for solving problems. Traditional programs have emphasized continuous mathematics. Technical students should also be introduced to discrete algorithms in such areas as counting and graph theory. Technical applications often involve determining the optimal way to perform certain procedures, and discrete algorithms can be used to determine the solution. Gardiner (1991) warns, however, that using algorithms should not degenerate "into a succession of meaningless routines" (p. 12). At the introductory level, students should be introduced to a small number of central techniques that can be applied to realistic problems in a meaningful manner.

The content of courses designed to provide the mathematics needed by technical students in such areas as health-related services, business, and engineering technologies will vary. All courses should build on the Foundation to meet the needs of students in individual programs. Course proliferation due to excessive customization, however, should be minimized. Programs with similar mathematical needs should enroll students in common mathematics courses in which applications and student projects may be program-specific.

The curriculum should also include applications from the natural sciences. For example, experimenting with various weights on a spring can motivate the study of linear functions, and examining the change in the temperature of a liquid as it cools can motivate the study of exponential functions. Mathematics courses should not only be designed to meet the immediate needs of technical students. Rather, they should also be broad-based and rich in content in order to meet the students' employment and personal needs now and in the future.

Some of the factors that impact the mathematics curriculum for technical students are accrediting agency guidelines, programs at other colleges, recommendations of professional organizations, the views of the mathematics faculty, and experts from other fields. Two-year colleges should also attempt to secure articulation agreements with four-year colleges and universities for students who wish to transfer after obtaining a two-year technical degree. Within these constraints, colleges should design the mathematics curriculum so that technical students will be able to change from one technical program to another, or from a two-year associate in an applied science degree program to a bachelor's degree program, with a minimum of backtracking.

Mathematical content should be introduced in the context of real problem-solving situations. Specific courses should integrate mathematical themes, with less regard for traditional classifications such as algebra, trigonometry, and geometry. In addition, technology should prompt faculty to rethink the presentation of certain topics. For example, the idea of linear function is one topic that deserves emphasis. Rather than teach that topic as an isolated segment in a chapter on functions, it might be motivated by an application in which students have to find a line that best fits some real data. With the aid of technology the discussion could be continued with different data sets, and the models generated could then be used for making predictions. The application would give real meaning to $x$- and $y$-intercepts and provide a reason for finding them. Slope would acquire meaning...
with particular emphasis given to the units describing the change in one variable in terms of another. The problems could be extended by evaluating residuals. Changing the parameters of the model would then demonstrate the effect of the parameters on the residuals.

Curricular content should undergo continuous evaluation and updating. In industry, technology has led to new expectations of employees. In education, it has led to drastic changes in what can be taught and how. In addition to adjusting to meet the demands of technology, industries are being restructured due to economic trends, government regulations, and political pressure. All these developments will dictate the direction of change in curricular content in courses for technical students.

**The Pedagogy**

In mathematics courses for technical students, instructional strategies should include

- interactive learning through writing, reading, and collaborative activities;
- projects and apprenticeship opportunities that encourage independent thinking and require sustained effort;
- use of multiple approaches (numerical, graphical, symbolic, and verbal) to solve meaningful problems; and
- the use of interactive and multimedia technology.

While the instructional methodology associated with technical programs will not differ significantly from the pedagogy for more general courses, faculty should adapt instructional strategies for particular technical programs where such adaptation will enhance the learning environment. For example, a faculty member teaching mathematics to electronics students can design a laboratory experience that explores sine waves of voltage using an oscilloscope. Mathematics classroom experiences with equipment specific to a technology area may be team-taught with a technology faculty member or prepared in consultation with a practitioner.

As indicated in *What Work Requires of Schools: A SCANS Report for America 2000* (Secretary's Commission on Achieving Necessary Skills [SCANS], 1991), mathematics should be taught in context. That is, students should learn content while solving realistic problems. A contextual approach will make liberal use of technology and focus as much on what solutions mean as how they were obtained. Using interactive and multimedia tools, the classroom can become an open gateway to the workplace. Classrooms will no longer be bounded by walls, local resources, or a single faculty member’s knowledge. Students will be challenged to develop solutions to real problems in a virtual workplace.

Individuals working in business or industry who are qualified teachers can be brought in as adjunct faculty. Such individuals can enhance mathematical instruction for technical students by bringing to the classroom valuable expertise as practitioners in their fields. Care must be taken, however, in the use of part-time faculty, as outlined in Chapter 4. In addition, all faculty should regularly consult with practitioners in the field in order to remain current in the applications of mathematics.
Increased and Decreased Attention

Mathematics courses for technical students should develop mathematical intuition through an understanding of the content and how it may be applied to solving problems. According to the 1993 SCANS report (SCANS, 1993), programs for technical students should place increased attention on

- organizing and processing information,
- estimating,
- working in groups,
- reading technical charts and graphs,
- reading and learning from other technical materials,
- working with formulas computationally and algebraically,
- solving problems from real applications,
- making regular use of appropriate and field-specific technology, and
- communicating results.

Other topics that deserve increased attention in technical programs include statistics; probability; rate change; conversions; difference equations; matrix methods; evaluating the results of numerical computations and graphical displays obtained from computers and graphing calculators; data collection, manipulation, transfer, and analysis; exponential and logarithmic functions; and discrete algorithmic problem-solving strategies. All programs need to ensure that graduates can understand and apply basic principles of statistics and probability.

Topics deserving less attention include determinants and Cramer's rule; trigonometric identities; complicated factoring; graphing functions with paper-and-pencil; Descartes' rule of signs; formulas for finding roots; secant, cosecant, and cotangent functions; radical equations with more than one radical; complex rational expressions; and complex expressions involving exponents.

Summary

Beyond the Foundation, the mathematical needs of technical students may vary according to their field of study. The courses that provide students with the required mathematics, whether they are taught in an integrated approach or not, must provide a broad base of mathematical knowledge. They must also contain the appropriate rigor and depth to allow students to study additional mathematics that their careers may require and to ease the switch from one technical area to another or the transfer from an associate's degree to a bachelor's degree program.

All students need to have experience using mathematics, combined with technology, to solve real-world problems. "Plug-and-chug" drill should not be translated to a computer screen; students must be given substantive exercises that develop mathematical understanding as well as facility with technology and solid understanding of its power and limitations. In addition to being able to arrive at results, technical students must be able to interpret and use them. These goals require faculty to think deeply about what they teach technical students and what teaching strategies are most effective.

"To me, any tracking that keeps young people out of the mathematics pipeline should be discouraged throughout the students' formative years. But after the student makes a decision about his or her career and after the student has completed a common core of mathematics, I see no problem with classes geared toward the aspirations of the students. The statement is made, of course, provided that students have entry to any other tracks at the appropriate level if they change their minds. Tracking at this level comes from the student, and not from anyone who may have low expectations for the student."

Jack Price,
NCTM News Bulletin,
December 1994, p. 3.
Mathematics-intensive programs include mathematics, science, engineering, computer science, economics, and business. Included with the mathematics majors are those students who are preparing to be secondary school or college mathematics teachers. Because students in these programs are required to study calculus, they need mathematics preparation, often termed “precalculus,” that goes beyond the Foundation. This section describes the special needs of such students and the role that introductory college mathematics courses can play in helping them to attain their educational goals.

The mathematicians, scientists, engineers, and economists of the future emerge from our mathematics-intensive majors. Faculty must demand quality performance of these students at this level of their education. Excellence does not materialize suddenly in calculus or upper-division mathematics, or when these students begin designing bridges, telescopes, or business strategies; it emerges from sustained work in challenging courses that offer a rich variety of mathematical experiences.

The job market for mathematics-intensive majors has become increasingly competitive. Two major factors in career success are flexibility of outlook and approach and the ability to work in teams. Introductory college mathematics programs must provide multiple problem-solving methods, promote teamwork, and emphasize meaningful problems that require extensive thought and develop insight.

The content

The precalculus curriculum must prepare students to be successful in a wide variety of calculus programs. The topics outlined below are basic to the modeling and problem-solving standards that should form the heart of precalculus education. While not departing from concerns about mathematical processes and techniques, more emphasis should be placed on developing student understanding of concepts, helping them make connections among concepts, and building their reasoning skills in preparation for higher-level courses in mathematics and related fields.

FUNCTIONS. “Having a sense for number and having a sense for functions are among the most important facets of mathematical thinking” (Eisenberg & Dreyfus, 1994, p. 45). Just as a sense for number allows students to reason efficiently with numerical information, a sense for functions allows students to
gain insights into the relationships among variables in problem-solving situations. While students begin their study of functions before reaching the precalculus level of mathematics, Demana (1994) points out that building a strong understanding of functions is a common theme in college precalculus reform projects. At this level, students learn to treat functions as objects and to reason formally about operations on sets of functions (Thompson, 1994).

Mathematics-intensive programs should include the study of linear, power, polynomial, rational, algebraic, exponential, logarithmic, trigonometric, and inverse trigonometric functions. Students should also develop a general understanding of the relationship of a function to its inverse, if that inverse exists. Although the rectangular form for functions should be emphasized, parametric and polar representations should also be studied.

Students should be able to categorize and organize functions into families and explore their properties. In addition, students should be able to use the algebra of functions and analyze functions graphically and numerically: find zeros, locate intervals where the function is increasing or decreasing, describe the concavity of the function, and approximate extreme values.

DISCRETE MATHEMATICS. Recognition and use of patterns, including those dealing with aspects of the very large and the very small, are essential to problem solving in mathematics. Numerical techniques that have always played a role in estimation and measurement take on new significance with the increasing use of technology. Spreadsheets and graphing packages allow students to use iterative processes to approximate solutions and guide investigations. The ability to build templates and then change the values of the parameters enables students to investigate the effects of these changes on the model. Recursion is also an important technique in building models for many applications. At the precalculus level, students can use difference equations to model phenomena that are studied with differential equations in calculus, including population growth, Newton's laws of heating and cooling, and simple harmonic motion.

Matrices are very powerful mathematical tools that are often overlooked in introductory college mathematics. Students can use matrices to store data, represent graphs, represent geometric transformations, or solve systems of linear equations. Examples of the use of matrices in modeling situations include representations of production strategies and probability distributions.

STATISTICS. The standards emphasize using real data and probabilistic concepts. Data analysis is especially important for students in mathematics-intensive programs. Students should work with real data, transform them to linearize them, undo the transformation to produce a function that models the original data, and make inferences based on the results. Students should gain experience with probabilistic models, including normal and binomial distribution models, and use Monte Carlo simulations to provide information on processes that cannot be assessed deterministically. These topics cannot all be integrated into a single calculus preparation course. In particular, statistical inference requires separate attention. Students who have not studied introductory statistics prior to studying calculus should do so before they graduate.

"The calculus reform movement is demonstrating that we should come to expect greater fragmentation in the mathematics curriculum. It is possible for different schools to offer quite different calculus courses that are successful locally and transferable (in several senses) globally—transferable to other institutions for credit, and transferable to other disciplines in terms of students being able to apply the mathematics they have learned. We will likely see this trend extending to other parts of the curriculum in the future."

Sheidon Gordon, Suffolk Community College

The Pedagogy

The standards for pedagogy in Chapter 2 provide appropriate instructional guidelines for mathematics-intensive majors. In particular, these standards
advocate building connections with other fields and approaching problem solving with a variety of strategies. Such pedagogical techniques prepare students to use mathematics effectively in their own fields of study. For mathematics majors, these kinds of experiences foster a broad outlook on the field and provide opportunities for developing deeper insight. For prospective secondary school mathematics teachers, the pedagogical strategies model those that the students will use when they become classroom teachers.

The pedagogy standards also place emphasis on cooperative learning and the use of technology to encourage student investigation, discovery, and insight. The use of technology is especially critical for students in mathematics-intensive programs, who will enter a workplace where expertise in technology will be assumed. Students must become sufficiently comfortable with graphing calculators and computers so that they automatically reach for them when, and only when, these mathematical tools offer a better or quicker way to solve a problem. To attain this technological confidence, students must use these tools in class, at home, and on examinations other than those specifically designed to test for knowledge of basic skills. Students will never have a job in which they will be restricted from using these “tools of the trade.” Faculty who restrict calculator or computer use on examinations should reexamine their policies and ask what purpose the restrictions serve. If calculator use on an examination yields an automatic “A” grade, faculty are asking the wrong questions, or teaching the wrong material, or both. For example, instead of asking students to graph a polynomial function, a task easily done with a graphing calculator, faculty could provide a graph or a set of data points and ask students to write an equation that would generate that type of graph. This approach requires deeper understanding of graphing and also assesses understanding of modeling.

"Calculus reform has come to focus more on how calculus is taught and less on what is actually taught. . . . The overall focus on raising students’ conceptual understanding, problem solving skills, analytic ability, and transferability of calculus skills to work in other disciplines has led to general changes not in the list of topics and techniques covered, but in how these topics are developed."


**Increased and Decreased Attention**

Increased attention should be given to providing students with a global view of the concept of a function. Students should be able to

- distinguish between classes of functions;
- understand periodic behavior and properties that cut across classes of functions, such as transformations;
- use functions in modeling situations;
- use exponential and logarithmic functions in problem solving in a variety of applications;
- use decomposition of functions to analyze the behavior of complicated functions; and
- interpret the behavior of graphs of functions near asymptotes and for very large and very small values of the variable.

Increased attention should be given to determining the real roots of any
equation by a combination of graphical and numerical methods. Similarly, graphing calculator or computer features should be used to solve systems of equations.

Faculty should introduce these concepts and techniques in the context of solving real problems. Such problems should lend themselves to solutions by a variety of strategies and should include student-generated data and data from outside sources. The properties of plane and solid figures offer a rich source of meaningful applications.

Decreased attention should be given to such traditional topics as

- graphing functions with paper and pencil;
- the cotangent, secant, and cosecant functions;
- reduction formulas and the proofs of complicated trigonometric identities;
- conic sections (especially complex algebraic manipulations);
- linear interpolation and other table manipulations;
- partial fractions and factoring beyond the level of the Foundation;
- equation-solving strategies such as the upper- and lower-bounds theorem and Descartes' rule of signs; and
- drill and practice on routine exercises and contrived applications.

Brief descriptions of several precalculus reform projects are given in *Preparing for a New Calculus* (Solow, 1994).

"Traditionally, college students taking precalculus have previously been exposed to basic ideas from an algebraic point of view. The students seem to become overconfident and/or bored and do not work consistently. The graphical approach adds new light to known concepts, thereby allowing the students to gain depth while maintaining their interest. The hands-on approach and the ability to check their answers that the graphing calculator provides, seem to have further enhanced students' motivation."

Antonio R. Quesada and Mary E. Maxwell,
*Educational Studies in Mathematics*,
Vol. 27, 1994 p. 213.

**Summary**

Faculty who teach students pursuing mathematics-intensive majors face several challenges. They must adapt the curriculum to prepare students with diverse mathematical backgrounds and learning styles for reformed calculus courses. They need to vary their teaching strategies in response to research on student learning. Even classroom layouts must change. Classrooms must be designed to accommodate cooperative learning, laboratory investigations, and computers. Faculty must face these challenges with renewed enthusiasm to increase the effectiveness of precalculus mathematics and subsequent calculus programs. The entire mathematics community must cooperate to transform the mathematics "filter" into a "pump."
Liberal Arts Programs

Curricula in the humanities and social sciences are grouped under the term “liberal arts programs.” Typically, these programs have been planned with little emphasis on mathematics. Yet as mathematics and its applications become increasingly pervasive in society, the need for all citizens to understand and use mathematical ideas increases as well. All college educated individuals should be competent and confident in the Foundation presented in this chapter. As indicated in the following description from National Goals for Education (U.S. Department of Education, 1990), however, additional mathematics will be needed by all college graduates:

Our people must be as knowledgeable, as well trained, as competent, and as inventive as those in any other nation. All our people, not just a few, must be able to think for a living, adapt to changing environments, and understand the world around them.

(p. 1)

The following vignette illustrates the kind of mathematics liberal arts programs can use to prepare students to “think for a living” and “adapt to changing environments.”

A mathematics faculty member invited the Director of Supply Management of a large television manufacturer to speak to his class. The director described his own educational background in mathematics as being below calculus and stated that he had to use a considerable amount of mathematics to solve a variety of work-related problems. He then presented a spreadsheet of industry data on the yearly sales of televisions and asked the class to predict what would happen to the sales of television sets the next year; this problem was important to his position and the company. He described briefly some of the company decisions influenced by the prediction. Upturns in sales might indicate the need for a new factory, while downturns would result in a backlog of finished sets which would have to be stored. Following questions, the class divided into groups with the assignment to analyze the data, make observations and hypotheses, propose a prediction model, and write group reports. Motivation and interest were high.

The group made oral reports to the director in a later class period. They observed that downturns in sales occurred at about the same time as the oil embargo and the Gulf War. They built several linear prediction models after making scatter diagrams. Some tried polynomial and exponential curve fitting. All used statistical software in a variety of ways. The students and faculty member were very pleased with the director’s feedback on the oral presentations. ❑

The students in the vignette used substantial mathematics including the graphical presentation of data as well as linear and nonlinear regression. Students were able to see that mathematical results need to be interpreted in contexts that involve history, economics, and other disciplines. It was also helpful for students to tackle a problem posed by someone who did not have extensive background in mathematics. Such an experience enables students to develop a broader view of the nature of mathematics and the role it plays in the world.

The Content

The traditional college algebra or precalculus courses, which are primarily
designed to prepare students for calculus, do not provide the breadth of mathematics needed by liberal arts students (Sons, 1995). Haver and Turbeville (1995) describe the goals of a capstone course that they developed for nonscience majors as follows:

The goals of the course are to develop, as fully as possible, the mathematical and quantitative capabilities of the students; to enable them to understand a variety of applications of mathematics; to prepare them to think logically in subsequent courses and situations in which mathematics occurs; and to increase their confidence in their ability to reason mathematically. (p. 46)

The Haver and Turbeville proposal fits the guidelines for a "foundation" course recommended by the CUPM Subcommittee on Quantitative Literacy (Sons, 1995). Note that the term "foundation" in the CUPM report is being used to denote a portion of a broad quantitative literacy program aimed at developing "capabilities in thought, analysis, and perspective" (p. 12). The mathematical content of that "foundation" is intended to be beyond what is normally studied in high school.

It is not appropriate or possible to be prescriptive about the specific mathematical content needed in a liberal arts program. For example, a history major is not necessarily preparing to be a historian, nor is every psychology major preparing to be a psychologist. Some students will seek jobs immediately after earning associate's or bachelor's degrees, while others will enter graduate school. Nevertheless, liberal arts students are likely to encounter formulas, graphs, tables, and schematics and to be asked to draw conclusions from them in the course of work or study.

On a personal level, these students may need to decide whether it is safe to swim in or eat fish from a local waterway or to evaluate different financing options when making major purchases involving amortization. On the job, liberal arts graduates may be asked to predict whether the need for services will increase and necessitate the hiring of additional personnel; or, as in the vignette, they may be asked to translate raw mathematical information into symbolic, visual, or verbal forms. Students going into research-oriented fields or on to graduate school will need background on how to interpret and conduct research. And virtually all workers must deal with computers and calculators at some level. Mathematics courses provide the natural and appropriate place for such learning to occur. As citizens, workers, and students, liberal arts majors will be called upon to use technology to solve problems that require mathematical methods.

Each institution has the responsibility of evaluating local needs and resources to determine how best to educate liberal arts majors in mathematics beyond the Foundation. This additional study might take the form of one or two courses or a series of modules in an interdisciplinary sequence. Small institutions may have to design a curriculum to serve all majors in the same courses. Faculty must question how well the mathematics requirements in liberal arts programs at their institutions prepare students to function in today's world.

"Mathematics departments need to begin a dialogue with the nonscience departments concerning the needs of their majors and what they can reasonably expect mathematics courses to teach their majors."

James Woeppe, UME Trends, March 1993, pp. 4-5.

Faculty must question how well the mathematics requirements in liberal arts programs at their institutions prepare students to function in today's world. The ultimate goal for such courses is "to instill in the student an appreciation of mathematics. For this to occur, students must come to understand the historical and contemporary role of mathematics and to place the discipline properly in the context of other human intellectual achievement" (Goldstein, 1989, p. 110). This
goal should not be interpreted to mean that courses for liberal arts students must be broad surveys of mathematical topics, nor that historical topics should dominate. Rather, the emphasis should be on the importance of doing mathematics in order to learn mathematics.

Survey courses have been popular for liberal arts students. Although liberal arts students need a wide variety of content, the curriculum should enable them to see mathematics in a few specific contexts studied in depth. Such topics should highlight the usefulness of mathematics and offer problem-solving opportunities in a variety of liberal arts disciplines. Examples of “big ideas” to be studied in depth include randomness as it relates to uncertainty, probability, and sample selection and social choice and decision making as they relate to voting systems, fair division, and game theory. The topics selected should present fresh mathematics to students rather than a rehash of previously studied topics. Furthermore, the topics should be interesting and challenging for students with a variety of mathematical backgrounds.

Liberal arts students frequently complete their study of mathematics by taking an introductory statistics course. Such a course should go beyond the level of statistical topics covered in the Foundation and include modern techniques for data analysis, estimation, hypothesis testing, and regression. Graduates of such a course should be able to address effectively the problem posed in the introductory vignette.

The Pedagogy

Every college graduate should be able “to analyze, discuss, and use quantitative information; to develop a reasonable level of facility in mathematical problem solving; to understand connections between mathematics and other disciplines; and to use these skills as an adequate base of life-long learning” (MAA, 1993, p. 8). Keith and Leitzel (1994) suggest “more student interaction, problem-solving and understandable applications” (p. 6) as a productive approach for producing quantitatively literate graduates. All of these recommendations support the standards for pedagogy presented in Chapter 2.

For students to become active users of mathematics, the role of the teacher must change from a sage who hands down knowledge to a coach who provides guidance and support. Faculty need to build self-confidence in students. Some liberal arts majors may not see themselves as doers of mathematics; indeed, they may be fearful of mathematics. Initial class activities must be designed to lead to student success. Cooperative learning experiences should be devised to use the differing strengths of students. Frequent praise for finding alternative solutions to problems will break down the belief that there is one right answer and one right way of solving a problem. Students should be expected to use technology to solve problems and to write project reports, just as they would do at work. Students who are shy about speaking can use short journal entries in order to express their ideas. Then faculty can have students read their journal entries, stressing that sharing ideas helps both the one who developed the idea and those who listen to it.

The intellectual development standard of modeling has special applicability for liberal arts majors. Mesterson-Gibbons (1989) points out that mathematical modeling can be approached pedagogically along a continuum. At one end of
the continuum faculty assign open-ended case studies in which students select a problem, do research, collect data, and pursue a variety of paths to create a mathematical model. At the other end of the continuum, students are presented with a problem that illustrates an already developed model. The students may be asked for interpretations or predictions based on the model.

The problem posed in the introductory vignette is near the middle of this continuum. It was not possible to capture in the brief vignette all of the communication and group interaction that developed. With the faculty member’s support, students helped each other understand, interpret, criticize, and appreciate the models as they were created. Analyzing a model, understanding how it represents and misrepresents reality, and exercising caution in interpreting from the model are valuable experiences which students will draw upon after formal schooling.

**Increased and Decreased Attention**

Several areas are to receive increased attention in programs for liberal arts students. Students should be exposed to mathematical ideas that are new to them. The mathematics must be useful, meaningful, and not simply preparation for a higher-level course. Because liberal arts students will encounter mathematics in a variety of settings, the approach taken should involve applications from several disciplines. Students should participate in mathematical modeling, either in developing models or in evaluating how well given models fit reality. Increased attention should also be given to having students interpret real data using statistical techniques.

Traditional mathematics offerings for liberal arts students have included a wide variety of topics covered at a superficial level. It is recommended that a few topics be selected based on the needs and interests of the students. These topics should be studied in sufficient depth so that students gain insight into mathematics as a discipline and learn how to learn mathematics.

**Summary**

All students, including those majoring in liberal arts disciplines, should be confident in their ability to do mathematics. The mathematics that they study should prepare them to contribute in the community, to perform effectively in the workplace, and to function as independent learners in mathematics-related areas. It is particularly important that liberal arts students understand the impact that mathematics has on art, history, literature, and many areas of human endeavor. Mathematics faculty should work closely with colleagues who teach introductory courses in the natural and social sciences, economics, and other disciplines that rely on mathematics to encourage them to reinforce and amplify the mathematics capabilities of students. Just as the “writing across the curriculum movement” addresses the need for students to write frequently in order to improve as verbal thinkers, a “mathematics across the curriculum movement” is needed so that students develop as mathematical thinkers.
"The NCTM Standards describe the mathematics classroom as a mathematical community, where students and the teacher are actively involved in creating their learning experience. This learning community needs the strength of a knowledgeable and compassionate leader who considers the needs and talents of the student-citizens, while providing a vision of where the community is headed and support for getting there. Giving students responsibility for their own learning doesn’t mean abdicating leadership. [It means] giving up some control and creating a new kind of classroom leadership that truly guides, encourages, and enlightens."

"Although I don’t feel that I am afraid of math as much as I am frustrated with it, I must be one of those students who suffers from math anxiety. In seventh grade I was tracked into the lower group of my math class. This was the first time that I realized that I must be having trouble in math or at least wasn’t as skilled in this subject as I was in my other classes. The fact that I was put into this lower class, combined with a truly boring and monotonous teacher, simply turned me off to the subject."

This statement, written by a student during the first week of a content course for prospective elementary school teachers, expresses the feelings of many current and prospective elementary school teachers who approach their mathematics courses with trepidation and a lack of confidence. Teachers are vital to our society’s cultural, technological, and economic vigor; yet, currently, we do little to promote self-confidence in learning mathematics, mathematical thinking, or deep understanding of mathematics. These are the challenges for introductory college mathematics. Our nation cannot afford to continue to have students who feel defeated by mathematics in our schools.

As pointed out earlier, prospective secondary school teachers are included among the mathematics-intensive majors. The recommendations made in this section on content and pedagogy, however, apply to the mathematics education of all prospective teachers of precollege mathematics who start their collegiate mathematics education at the introductory level.

Among the recommendations in Moving Beyond Myths (NRC, 1991) the following are particularly relevant to the mathematics education of preservice teachers:

- Engage mathematics faculty in issues of teaching and learning.
- Teach in a way that engages students.
- Ensure sufficient numbers of school and college teachers.
- Link colleges and universities to school mathematics (p. 45).

Faculty need to seek new ways to contribute to the preparation of teachers at all levels in partnership with colleges of education and K–12 teachers of mathematics. Mathematics faculty should participate in professional discussions about mathematical preparation of future teachers. Two-year college faculty and faculty teaching introductory mathematics courses at four-year colleges and universities must become full partners in the vital task of preparing school teachers.

The Content

The mathematics studied by preservice teachers must help them develop an understanding of the subject that goes beyond what they will be expected to teach. Research on teacher education [see Brown and Borko (1992) for a summary]
indicates that the depth and character of teachers' subject matter knowledge influences both the style and substance of instruction. For example, Steinberg, Haymore, and Marks (1985) found that greater knowledge enabled teachers to convey the nature of mathematics, connect mathematical topics, provide conceptual explanations, and see problem solving as central to mathematics instruction.

In addition, classroom teachers must be able to foster the intellectual growth of their students and inculcate mathematical ways of thinking. A Call for Change: Recommendations for the Mathematical Preparation of Teachers of Mathematics (Leitzel, 1991) recommends that prospective teachers should learn to

- view mathematics as a system of interrelated principles;
- communicate mathematics accurately, both orally and in writing;
- understand the elements of mathematical modeling;
- use calculators and computers appropriately in the teaching and learning of mathematics; and
- appreciate the historical and cultural development of mathematics.

Prospective teachers need to recognize the relationship between what they teach and what is taught at other levels of school mathematics. According to A Call for Change,

They need, for example, to understand the close parallel among the development of integer arithmetic in the elementary grades, the algebra of polynomials in the middle and early high school curriculum, and the ideas of number systems explored later in high school. They should see that counting processes and the concepts of functions and relations permeate all aspects of mathematics. They should explore the relationships between geometry and algebra and the use of one to investigate the other. (p. 3)

Special mathematics courses for prospective elementary school teachers should revisit school mathematics topics in ways that develop deeper understanding of these topics and of the relationships among them. Furthermore, all preservice teachers should acquire a broad background in the liberal arts and sciences so that they understand how to apply mathematics in a variety of disciplines. They should also learn about the historical and current contributions of non-European cultures to mathematics and related fields; resources on this topic are available in Zaslavsky (1994) and Van Sertima (1989), as well as through the NCTM and the MAA’s program SUMMA (Strengthening Underrepresented Minority Mathematics Achievement). Future teachers need to be prepared to help students who are members of groups underrepresented in mathematics to see the subject as part of their cultures.

The content standards presented in Chapter 2 provide the essential ingredients for the introductory level mathematics curriculum for K–12 teachers. Specific recommendations for the various levels of mathematical knowledge needed by teachers in grades K–4, 5–8, and 9–12 are in the Professional Standards for Teaching Mathematics (NCTM, 1991, pp. 135–40) and in A Call for Change (Leitzel, 1991). The recommendations are designed to ensure that teachers at all

"All faculty must examine the extent to which their preservice mathematics education courses place subject matter in a context that is meaningful to prospective teachers and model the variety of teaching practices and assessment strategies outlined in this and other reform documents."

Task Force

INTERPRETING THE STANDARDS
grade levels have a thorough understanding of the mathematics they are teaching and a clear vision of where that mathematics is leading. Both documents recommend that

- teachers of grades K–4 study a minimum of 9 semester hours of college mathematics. Such courses assume a prerequisite of three years of high school mathematics for college-intending students or an equivalent preparation.

- teachers of grades 5–8 study a minimum of 15 semester hours of college mathematics. Such courses assume a prerequisite of four years of high school mathematics for college-intending students or an equivalent preparation.

- teachers of grades 9–12 have the equivalent of a major in mathematics. Coursework should include an integration of applications from a variety of disciplines. In addition, emphasis on problem solving and the history of mathematics is essential. Such courses assume a prerequisite of four years of high school mathematics for college-intending students or the equivalent.

Introductory college mathematics courses come at a critical stage in the development of future teachers, offering them an opportunity to move beyond their school experiences with mathematics to take a wider view of the subject. In this way, such courses can make an important contribution to K–12 mathematics education reform.

The Pedagogy

The report *Moving Beyond Myths* (NRC, 1991) states one of the central pedagogical problems in the training of future teachers:

It is rare to find mathematics courses that pay equal attention to strong mathematical content, innovative curricular materials, and awareness of what research reveals about how children learn mathematics. Unless college and university mathematicians model through their teaching effective strategies that engage students in their own learning, school teachers will continue to present mathematics as a dry subject to be learned by imitation and memorization. (pp. 28–29)

If teachers are to make problem solving central to learning mathematics, they must take risks. They need to feel confident in their knowledge of mathematics, be willing to explore new mathematical ideas, and be able to stimulate active discourse in the classroom. Faculty who teach mathematics courses for prospective teachers must nurture this spirit of active inquiry. In addition, since it is common for teachers to teach the way that they were taught, faculty must use in their own classes the instructional techniques that prospective teachers will be expected to use.

*Interpreting the Standards*
Faculty should also

- follow new professional recommendations on teaching strategies;
- gain a better understanding of the mathematical needs of future elementary, middle, and high school teachers;
- rethink their teaching to promote in-depth understanding as well as a broad vision of the “big ideas” of K–12 mathematics; and
- keep abreast of the research on how students learn mathematics and adjust their teaching accordingly.

The tremendous impact of technology on education will continue to grow. Prospective teachers must understand its power and its limitations. They must know how to employ technology to enhance conceptual understanding and that technology itself should not be the main focus of instruction. Appropriate and effective uses of technology must be integrated into mathematics courses for preservice teachers.

**Summary**

Courses for prospective teachers should allow them to build on what they know while developing the habits of mind used by mathematicians and scientists. Students are not passive receivers of information who regurgitate correct answers on demand. The view that “learning occurs not by recording information but by interpreting it” (Resnick, 1989, p. 2) implies the need for a new vision of what it means to teach mathematics. The emphasis should shift from teaching isolated knowledge and skills to helping students apply knowledge and develop in-depth understanding of central ideas. This shift needs to occur not only in school mathematics, but also in introductory college mathematics courses taken by preservice teachers.

"There is no other decision that a teacher makes that has a greater impact on students' opportunity to learn and on their perceptions about what mathematics is than the selection or creation of the tasks with which the teacher engages the students in studying mathematics. Here the teacher is the architect, the designer of the curriculum."

Glenda Lappan and Sarah Theule-Lubienski in *Selected Lectures from the 7th International Congress on Mathematics Education* (Quebec, August 17-23, 1992), p. 250.