Standards for Introductory College Mathematics before Calculus

Prepared by the Writing Team and Task Force of the Standards for Introductory College Mathematics Project,
Don Cohen, Editor
September 1995
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