# Irrational Logarithms in College Algebra

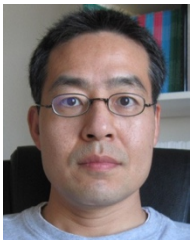
**David Rose, Li Zhou, and Steve Frye,** *Polk State College*

Abstract:

The College Algebra textbook used at our college asks students to solve some exponential equations and approximate the logarithmic solutions if irrational and express exactly if rational. But how can a student decide when a logarithm with rational base and rational argument is irrational? We find an easy answer to this question using the Fundamental Theorem of Arithmetic. Then, using the usual properties of logarithms, we extend our test to apply also to logarithms with bases and arguments which are radicals of rational numbers. Then we investigate irrationality for natural logarithms which are perhaps more prominent in college algebra. Without assuming transcendence of the base e, using only ideas from second-semester calculus, we find a class of irrational natural logarithms. Finally, we observe that all irrational logarithms considered in this note are transcendental in light of the well-known theorems of C. Lindemann and A. O. Gelfond.

**David Rose** has enjoyed teaching mathematics for over 40 years, mostly in four-year colleges and most recently in the two-year college setting. He was blessed to be part of a research team and have about 50 published research papers, primarily in general topology. He has served as a reviewer/editor of the Journal for Advanced Research in Pure Mathematics and has been privileged to present talks at many math conferences, including a brief presentation at an AMATYC conference.

**Li Zhou** (lzhou@polk.edu) teaches mathematics at Polk State College in Florida. He received the Distinguished Teaching Award from the Florida Section of the MAA in 2002. When he is not teaching, he spends his spare time solving challenging problems. He first tasted the pleasure of problem solving in 1999 with the Problem Section of *The AMATYC Review* and has since become an accomplished solver with numerous problems and solutions published in the MAA journals, Cr*ux Mathematicorum*, *The Mathematical Gazette*, and *Mathematical Reflections*.



**Steve Frye** has been a professor of mathematics at Polk Community College since 2004. He received his BS in chemistry from Kansas State University and later attended the University of Wisconsin as a graduate student in chemistry. He attended Officer Candidate School and served as an engineering officer aboard the USS Illusive from 1969–1970. He then received a MA in mathematics from Indiana University in 1973 and began a career at IBM, where he worked as a programmer, analyst, manager, and solution architect from 1974–2000. Follow-

ing his retirement, he roamed much of the US in a motorhome, returning to settle in Florida in 2003. He began teaching at Polk State College in 2003 as an adjunct then joined the faculty as a professor in 2004. Most impor- tantly to him, he is still married and in love with the same woman, with whom he celebrated 49 years of mar- riage in August.