CROSSROADS IN MATHEMATICS

Standards for Introductory College Mathematics Before Calculus

EXECUTIVE SUMMARY

AMERICAN MATHEMATICAL ASSOCIATION OF TWO-YEAR COLLEGES

SEPTEMBER 1995

Dear Colleague,

Several efforts at reform in mathematics education have been made in recent years. Possibly the two most noteworthy of these are the NCTM Curriculum and Evaluation Standards, which represent recommendations for grades K-12, and the calculus reform movement. Until now, however, no group has attempted to establish standards for the mathematics programs that bridge the gap between high school mathematics and college calculus and that satisfy the needs of students whose educational plans do not include calculus.

This document, Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus , was developed to address these issues. The preparation of these standards was directed by the American Mathematical Association of Two-Year Colleges (AMATYC), the only national organization exclusively devoted to improving mathematics in the first two years of college, and was made possible by funding from the National Science Foundation and Exxon Education foundation. Participating in the effort were representatives from the American Mathematical Society, the Mathematical Association of America, the National Association of Developmental Education, and the National Council of Teachers of Mathematics. Many outstanding educators from all levels of postsecondary education contributed to the development of this document. Their efforts are gratefully acknowledged

The major goals of Crossroads in Mathematics are to improve mathematics education in two-year colleges and at the lower division of four-year colleges and universities and to encourage more students to study more mathematics.

A copy of the full document was sent to AMATYC members, to interested members of those organizations participating in the development of the standards, and to others in the mathematics community. In addition, copies were sent to Mathematics departments at colleges throughout the United States and Canada. Instructions for ordering copies of the document appear at the end of the summary.

AMATYC is proud of its effort to reform mathematics education. We invite you to read the following summary and join us in supporting mathematics reform.

Sincerely,
Don Cohen, Editor

Marilyn Mays, Project Director
President AMATYC 1993-95

Karen Sharp, Co-Principal Investigator
President AMATYC 1991-93

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CROSSROADS IN MATHEMATICS: *Standards for Introductory College Mathematics Before Calculus*

Higher education is situated at the intersection of two major crossroads: A growing societal need exists for a well-educated citizenry and for a workforce adequately prepared in the areas of mathematics, science, engineering, and technology while, at the same time, increasing numbers of academically underprepared students are seeking entrance to postsecondary education.

Mathematics programs at two- and four-year colleges as well as at many universities serve students from diverse personal and academic backgrounds. These students begin their postsecondary education with a wide variety of educational goals and personal aspirations. Some enter college with solid mathematical backgrounds, ready to study calculus. Many others intend to study calculus but lack the preparation to do so. A third group can prepare for their personal and career goals through the study of mathematics below the level of calculus. The students in the last two groups are the ones whose needs are addressed in *Crossroads in Mathematics.* These students make up the majority of the nation's college students who are studying mathematics; they represent over 75 percent of the mathematics students at two-year colleges and 49 percent at four-year colleges and universities *(Statistical Abstract of the Undergraduate Programs in the Mathematical Sciences and Computer Science in the United States, 1990-91 CBMS Survey).*

Mathematics is a vibrant and growing discipline and is being used in more ways by more people than ever before. An alarming situation now exists, however, in the nation's postsecondary institutions. Each year greater numbers of students enter the mathematics "pipeline" at a point below the level of calculus, yet, there is no significant gain in the numbers of students studying at higher levels. The failure of many of these students to persist in mathematics not only prevents them from pursuing their chosen careers, but it also has a negative impact on our nation's economy as fewer members of the workforce are prepared for jobs in technical fields.

The purpose of *Crossroads in Mathematics* is to address the special circumstances of, establish standards for, and make recommendations about introductory college mathematics. The recommendations are based upon research evidence and the best judgment of the educators who contributed to the document.

Basic Principles

The following principles form the philosophical underpinnings of the document:

1. *All college students should grow in their knowledge of mathematics while attending college.* Students who are not prepared for college-level mathematics upon entering college will obtain the necessary knowledge by studying the Foundation.
2. *The mathematics that students study should be meaningful and relevant.* Mathematics should be introduced in the context of real, understandable problem-solving situations.
3. *Mathematics must be taught as a laboratory discipline*. Effective mathematics instruction should involve active student participation using in-depth projects.
4. *The use of technology is an essential part of an up-to-date curriculum*. Faculty and students will make effective use of appropriate technology.
5. *Students will acquire mathematics through a carefully balanced educational program that emphasizes the content and instructional strategies recommended in the standards along with the viable components of traditional instruction.* These standards emphasize problem solving, technology, intuitive understanding, and collaborative learning strategies. Skill acquisition, mathematical abstraction and rigor, and whole-class instruction, however, are still critical components of mathematics education.
6. *Increased participation in mathematics and in careers using mathematics is a critical goal in our heterogeneous society*. Mathematics education must reach out to all students.

The Standards

The standards provide goals for introductory college mathematics and guidelines for selecting content and instructional strategies for accomplishing the goals.

The Standards for Intellectual Development address desired modes of student thinking and represent goals for student outcomes.

Standard 1-1: Problem Solving

Students will engage in substantial mathematical problem solving.

Standard 1-2: Modeling

Students will learn mathematics through modeling real-world situations.

Standard 1-3: Reasoning

Students will expand their mathematical reasoning skills as they develop convincing mathematical arguments.

Standard 1-4: Connecting with Other Disciplines

Students will develop the view that mathematics is a growing discipline, interrelated with human culture, and understand its connections to other disciplines.

Standard 1-5: Communicating

Students will acquire the ability to read, write, listen to, and speak mathematics.

Standard 1-6: Using Technology

Students will use appropriate technology to enhance their mathematical thinking and understanding and to solve mathematical problems and judge the reasonableness of their results.

Standard 1-7: Developing Mathematical Power

Students will engage in rich experiences that encourage independent, nontrivial exploration in mathematics, develop and reinforce tenacity and confidence in their abilities to use mathematics, and inspire them to pursue the study of mathematics and related disciplines.

The *Standards for Content* provide guidelines for the selection of content that will be taught at the introductory level.

Standard C-1: Number Sense

Students will perform arithmetic operations, as well as reason and draw conclusions from numerical information.

Standard C-2: Symbolism and Algebra

Students will translate problem situations into their symbolic representations and use those representations to solve problems.

Standard C-3: Geometry

Students will develop a spatial and measurement sense.

Standard C-4: Function

Students will demonstrate understanding of the concept of function by several means (verbally, numerically, graphically, and symbolically) and incorporate it as a central theme into their use of mathematics.

Standard C-5: Discrete Mathematics

Students will use discrete mathematical algorithms and develop combinatorial abilities in order to solve problems of finite character and enumerate sets without direct counting.

Standard C-6: Probability and Statistics

Students will analyze data and use probability and statistical models to make inferences about real-world situations.

Standard C-7: Deductive Proof

Students will appreciate the deductive nature of mathematics as an identifying characteristic of the discipline, recognize the roles of definitions, axioms, and theorems, and identify and construct valid deductive arguments.

The *Standards for Pedagogy* recommend the use of instructional strategies that provide for student activity and interaction and for student-constructed knowledge.

Standard P-1: Teaching with Technology

Mathematics faculty will model the use of appropriate technology in the teaching of mathematics so that students can benefit from the opportunities it presents as a medium of instruction.

Standard P-2: Interactive and Collaborative Learning

Mathematics faculty will foster interactive learning through student writing, reading, speaking, and collaborative activities so that students can learn to work effectively in groups and communicate about mathematics both orally and in writing.

Standard P-3: Connecting with Other Experiences

Mathematics faculty will actively involve students in meaningful mathematics problems that build upon their experiences, focus on broad mathematical themes, and build connections within branches of mathematics and between mathematics and other disciplines so that students will view mathematics as a connected whole relevant to their lives.

Standard P-4: Multiple Approaches

Mathematics faculty will model the use of multiple approaches-numerical, graphical, symbolic, and verbal-to help students learn a variety of techniques for solving problems.

Standard P-5: Experiencing Mathematics

Mathematics faculty will provide learning activities, including projects and apprenticeships, that promote independent thinking and require sustained effort and time so that students will have the confidence to access and use needed mathematics and other technical information independently, to form conjectures from an array of specific examples, and to draw conclusions from general principles.

Interpreting the Standards

The standards reflect many of the same principles found in school reform and calculus reform. They focus, however, on the needs and experiences of college students studying introductory mathematics in various instructional programs. These standards place emphasis on using technology as a tool and as an aid to instruction, developing general strategies for solving real-world problems, and actively involving students in the learning process.

In particular,

- The *Foundation* includes topics traditionally taught in "developmental mathematics" but also brings in additional topics that all students must understand and be able to use. Courses at this level should not simply be repeats of those offered in high school. Their goal is to prepare college students to study additional mathematics, thus expanding their educational and career options.
- *Technical programs* place strong emphasis on mathematics in the context of real applications. They should prepare students for the immediate needs of employment. At the same time, students should learn to appreciate the usefulness of mathematics and to use mathematics to solve problems in a variety of fields. The content and structure of the mathematics curriculum for technical students must be both rigorous and relevant. The mathematics that technical students study must broaden their options both in careers and in formal education.
- *Mathematics-intensive programs* include the study of calculus and beyond. Consequently, the mathematics that these students study must prepare them to be successful in a wide variety of calculus programs. The study of functions is the heart of precalculus education. While not departing from concerns about mathematical processes and techniques, more emphasis should be placed on developing student understanding of concepts, helping them make connections among concepts, and building their reasoning skills.
- *Liberal arts programs* are designed for bachelor degree-intending students majoring in the humanities and social sciences. These students should gain an appreciation for the roles that mathematics will play in their education, in their careers, and in their personal lives. Each institution has the responsibility of evaluating local needs and resources to determine how best to educate liberal arts majors beyond the Foundation. Options include interdisciplinary modules and introductory statistics.

- *Prospective teacher programs* should shift the emphasis from teaching isolated mathematical knowledge and skills to helping students to apply knowledge and develop in-depth understanding of the subject that goes beyond what they will be expected to teach. Courses for prospective teachers must provide an awareness of what research reveals about how children learn mathematics, models for effective pedagogy, and an understanding of the power and limitations of the use of technology in the classroom.

Implications

Although the standards focus on curriculum and pedagogy, they have wide implications for institutions and their mathematics programs.

- Professional development opportunities must be made available to all faculty members so that they may experience these reform recommendations as learners.
- Colleges must have laboratory and learning center facilities and provide adequate support personnel.
- Appropriate technology must be available for faculty and student use.
- Assessment instruments must measure the full range of what students are expected to know.
- Ongoing program evaluation must be used to make recommendations for improvement while retaining the effective aspects of the program.
- Articulation with high schools, with other colleges and universities, and with employers enables faculty at all levels to work in concert to improve mathematics education.

Implementation

Crossroads in Mathematics provides a framework for the development of improved curriculum and pedagogy. Adoption and implementation of the standards will require a systemic nationwide effort which must be supported by postsecondary institutions, business and industry, and public and private funding agencies. Faculty, with the support of administrators, must lead the way. National professional organizations and their affiliated groups must promote reform through their conferences and by providing more extensive faculty development opportunities.

Looking to the Future

Introductory college mathematics holds the promise of opening new paths to future learning and fulfilling careers to an often neglected segment of the student population. Mathematics education at this level plays such a critical role in people's lives that its improvement is essential to our nation's vitality. *Crossroads in Mathematics* outlines a standards-based reform effort that will provide all students with a more engaging and valuable learning experience.

Crossroads in Mathematics was prepared by the Task Force and Writing Team of the Standards for Introductory College Mathematics Project with Don Cohen as Editor, Marilyn Mays as Project Director, and Karen Sharp and Dale Ewen as Project Co-Directors.

Individual copies of this document may be obtained by completing a copy of the form at the right and mailing it to AMATYC, State Technical Institute at Memphis, 5983 Macon Cove, Memphis, TN 38134. A limited number are available free. When the supply is exhausted, copies will be made available at a moderate charge.

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Address	