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The Death of the 4% Rule:
New Market Conditions and Retiree Needs Require Rethinking Retirement Spending

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KEY FINDINGS

- Withdrawing 4% of the initial portfolio value, adjusted for inflation, heightens the retiree's chance of depleting retirement savings.
- Retirees have the desire for a dynamic retirement strategy, withdrawing varying fund amounts based upon lifestyle requirements at different stages of retirement.
- Two alternative strategies proved preferable: base withdrawals upon an age-based remaining balance strategy using the Internal Revenue Service's Required Minimum Distribution tables; or remove 6% of the remaining portfolio balance, adjusted for inflation, each period.

ABSTRACT

The dilemma of how to effectively decumulate retirement savings poses a critical question: Can retirees withdraw 4% of their retirement wealth each year? This study delves into this inquiry, leveraging Monte Carlo simulation to evaluate six decumulation strategies. Unlike conventional studies that assume normal distributions for stock and bond returns, the authors utilize a machine learning algorithm to identify the most suitable distributions. Their results reveal that a static 4% withdrawal rate adjusting for inflation from the initial portfolio may heighten the risk of retirees depleting their savings prematurely. Instead, the authors advocate a sustainable approach: retirees should align annual withdrawals with the remaining portfolio value, accommodating market changes. Considering the delicate balance of securing long-term financial well-being while preventing early fund depletion, the authors identify two superior strategies: adhering to either age-based withdrawal percentages from the Internal Revenue Service's Required Minimum Distribution tables or to the fixed 6% rate, adjusted for inflation, both tied to the remaining portfolio value.

With the advent of defined contribution retirement plans and the 401(k) plan came the 4% rule: withdraw 4% of your retirement savings each year. However, market conditions today are not the same as they were back in the 1980's when the move from defined benefit to defined contribution retirement plans accelerated. Stock returns have grown, making realizing similar returns going forward more difficult; the Federal Reserve has held down interest rates for the past two decades, negatively impacting the yield on bonds; and stock and bond returns have become more correlated. Does the 4% rule still work?

Historically, retirement plans were defined benefit plans: companies contributed to employee retirement plans, and at retirement, the employee received a constant-dollar amount payout pension. These payouts did not have survivorship benefits, but the employee did not need to decide how the retirement dollars were invested and did not need to worry about outliving the benefit: the payout continued throughout the employee's life.

With the shift to defined contribution plans, employees had to decide how much to save, how to invest the savings, and upon retirement, how much to withdraw from the retirement account (typically a 401(k) account). The advantage to these defined contribution plans purportedly was to put more control into the hands of the employees in terms of how much to save and how to invest these savings; there is also the opportunity to have survivorship benefits, presuming funds remain in the retirement account upon the employee's death.

However, employees having control over retirement savings has not proven to be a good thing. Savings in retirement accounts have not been substantial enough to fund retirement needs, and employees seem unsure how much they can safely withdraw from these accounts without risking running out of funds during their lifetimes. During times of financial stress, retirees tend to tap retirement accounts, despite the substantial penalty in doing so for those under 59.5 years old. A recent study found that since March 2020 (the beginning of the Coronavirus pandemic), 83% of those not currently saving for retirement were saving prior to COVID (Dunaway-Seale 2022). Many believe the purported benefits of defined contribution plans are proving to be false.

While a great deal of research has been done on retirement plans, most of this research has been done on defined benefits plans, the predominant plans until the Revenue Act of 1978 enabled the creation of defined contribution plans. These employees, the early investors in 401(k) plans, have begun retiring in large numbers during the past twenty years. Given the decline in defined benefit plans, more work needs to be done on the defined contribution plans to see if employees have adequate funds available to see them through retirement.

In this article we examine six alternative decumulation strategies and compare their performance. Sun and Webb (2013) explored two distinct age-based decumulation strategies. The first approach involved retirees consuming a percentage of their remaining wealth, as specified in the Internal Revenue Service (IRS) Required Minimum Distribution (RMD) tables. The second strategy incorporated not only the amount determined in the first approach but also factored in dividend and interest earnings over a 35-year retirement period. Employing Monte Carlo simulation, we assessed these two strategies proposed by Sun and Webb, along with four alternative approaches.

In two of the alternative strategies, mirroring the methodology of Sun and Webb (2013), the withdrawal amount was contingent on the portfolio's current value, with preset withdrawal rates of 4% and 6%, adjusted for inflation. Conversely, the remaining two strategies relied on a fixed withdrawal rate based on the initial portfolio value, also set at 4% and 6%. Additionally, we scrutinized the portfolio returns under the assumption of a 35-year retirement horizon, ultimately comparing the outcomes of these six decumulation strategies.

To evaluate the effectiveness of decumulation strategies, prior research has commonly employed Monte Carlo simulations. Monte Carlo simulations can be conducted in two primary ways: the historical simulation method and the Parametric (Variance-Covariance) Method. However, the Parametric method often assumes a normal distribution, disregarding the possibility of fat tails in stock or bond returns (e.g., Pfeiffer, Salter, and Evensky 2013). In this article, we employ a machine learning algorithm to optimize the probability distributions of five essential financial metrics:

total stock returns, total bond returns, and the income components of stock and bond returns (including dividend returns for stocks and coupon payments for treasury bonds), in addition to inflation rates. To capture the correlations among these financial metrics, we utilize Copula.

Our findings unveiled two key insights. First, a consistent pattern emerged when the withdrawal strategy was based on the initial portfolio value, showing suboptimal results for investors. Conversely, strategies that adjusted the withdrawal amount according to the remaining portfolio value consistently outperformed their fixed-rate counterparts, providing a more favorable outcome for retirees.

Secondly, among the tested strategies, the one incorporating age-based percentages from the IRS RMD table, along with interest and dividend earnings as per Sun and Webb (2013), performs the best. This strategy strikes a harmonious balance between securing long-term financial stability and preventing premature fund exhaustion. The second strongest strategy involves a 6% withdrawal rate based on the remaining wealth. Notably, this approach offers an annual withdrawal rate similar to the top performer, all while maintaining a relatively low risk of depleting funds before reaching the age of 99. The third-strongest strategy is a 4% withdrawal rate based on the remaining wealth. Although this approach carries a lower risk of depleting funds before age 99, it entails a slightly reduced level of early retirement-year spending compared to the two leading strategies mentioned above, thus potentially limiting quality of life for retirees. These three approaches are potentially effective options for individuals aiming to prudently manage their retirement finances.

The 4% rule has shown diminished effectiveness in recent years. Withdrawing 4% of the initial portfolio value is more likely to lead retirees to outlive their retirement savings. Nevertheless, recalibrating the withdrawal rate to 4% of the remaining balance, and factoring in inflation, continues to stand as a viable and efficient strategy. Yet, for an even more robust approach, we could consider implementing withdrawal percentages aligned with the IRS RMD tables or opt for a fixed, slightly higher withdrawal rate, such as 6%, tethered to the remaining portfolio value. These strategies offer a more astute balance between securing enduring financial stability and averting early fund depletion, making them potentially superior options in today's complex financial environment.

Our research proceeds with a review of the literature on retirement withdrawal strategies followed by our data and study methodology, results, and, finally, our conclusions.

REVIEW OF LITERATURE

Historically, employers established a pension plan for employees, guaranteeing a set monthly retirement benefit for the life of the employee. Employers bore the risk of investing funds to cover the cost of these retirement obligations, and employees had little control over how employers chose to fund retirement benefits. These defined benefit retirement plans were expensive for employers, and many smaller companies could not afford retirement plans, thus leaving many employees with no employer retirement funds.

Enter Ted Banna with his idea for an employee-funded, tax-exempt defined contribution retirement plan: the 401(k). The 401(k) offered employees the ability to save for retirement on a tax-exempt basis and determine how much to save as well as how to invest the savings. This turned out to be both an advantage and a detriment to retirement savings.

The decisions employees needed to make in managing their retirement accounts relate to the following:

Amount. With defined contribution plans, the belief was that employees could withdraw 4% of their retirement account each year without penalty once reaching the age of 59.5. Bengen (1994) essentially developed the 4% rule and found it to be sustainable through time. To simplify one aspect of defined contribution plans, the rule of thumb became: "retirees can withdraw 4% of the initial value of the portfolio each year." This was based upon the growth rate of the stock market, the relatively low inflation rate, and the belief that stock returns and bond yields move in opposite directions. Still today there are proponents of the 4% rule, and some financial advisors tout a withdrawal at an even higher rate (Staib 2019; Finke, Pfau, and Williams 2012; Guyton and Klinger 2006). A recent Morningstar study of 30-year rolling retirement portfolios from 1930–1990 found retirees could safely withdraw 1.4% to 2.5% with an all-cash portfolio and 3.2% to 6.5% with an all-stock portfolio (Benz, Ptak, and Rekenthaler 2021). Further, with 40%–60% invested in stocks, a safe withdrawal rate was 3.3%. Most of these studies presume the withdrawals are of the same amount in equivalent intervals.

Others propose a dynamic withdrawal strategy, noting that retirees' cash needs are different at different stages of retirement (Sevy 2021; Toland, Nikolic, and Habib 2020; Mendes and Lavrado 2017). Sevy (2021) tests a series of withdrawal schemes and concludes retirees should reduce spending when there is a bear market and increase spending when the markets are strong.

Another study finds that in the early years of retirement, retirees spend more freely, traveling and making upgrades to their homes. In the middle stage of retirement, spending declines as retirees age, and in the later years of retirement, medical expenses and assisted living facilities can cause cash needs to spike for some retirees (Sheikh, Roy, and Lester 2014). Retirees' cash needs are not steady throughout retirement.

In fact, Pfau (2014) finds retiree satisfaction increases by creating a dynamic portfolio. He notes the value in retirees paying attention to market returns and adjusting spending. Scruggs (2019) takes this idea of a dynamic portfolio further by noting the three levers that will reduce longevity risk: adjusting asset allocation; changing initial withdrawal rate; and increasing withdrawal flexibility. Blanchett (2023) finds adjusting withdrawals throughout retirement based upon the retiree's expected utility modeled by prospect theory leads to virtually all needs of a retiree being met and most of the wants being funded.

Finke, Pfau, and Williams (2012) find withdrawal rates may be higher for retirees with higher risk tolerances, particularly for those that have a minimum guaranteed income. Scott, Sharpe, and Watson (2008) also note the introduction of other income sources and retiree preferences in the creation of a dynamic withdrawal strategy. Clare et al. (2022) find that the addition of gold to a portfolio serves to balance portfolio returns during down stock years. Clare et al. (2020) note the advantage of purchasing a deferred annuity to manage income needs in later retirement years thus eliminating longevity risk. However, Kieren and Weber (2022) found in their survey that investors do not like annuities.

A strict 4% withdrawal rate rule may be highly ineffective for how retirees actually spend. Researchers note the first few years of retirement withdrawals are critical and prefer a dynamic withdrawal strategy (Rook 2015; Blanchett, Kowara, and Chen 2012). The current economy may make constant-rate withdrawal challenging given the difficulty for the markets to capture the same level of returns as we saw in the 1980s and 1990s, plus there is increased volatility in the markets (Staib 2019). The 4% rule was created in a different era: one of high growth in the market and lower volatility; this may suggest a dynamic withdrawal strategy will be more successful (Sheik, Roy, and Lester 2014). Sun and Webb (2013) looked at alternative divestment strategies and used RMDs, finding that investors withdraw too little in the early years of retirement. They also found that spending RMD-based percentages of remaining wealth *plus* interest and dividends performed better than other alternative decumulation strategies.

Researchers are finding a more customized retirement withdrawal plan is the safest, given longer lifespans, higher inflation rates, and higher healthcare costs. A dynamic withdrawal strategy also allows for shifts in withdrawal percentages as lifestyle needs shift (Kapadia 2021). One study found that steady state spending, together with inflation adjustments, may be overstating the spending needs by 26% or more by age 95 (Roy 2019).

Invest how? A question also exists about how retirement funds should be invested. Some analysts believe that a balanced (50% stocks and 50% bonds) investment approach is best. Others note a dynamic strategy, with the percentage of the portfolio invested in stocks declining later in retirement to reduce the volatility of the portfolio returns. Still others believe that stock investment is critical to reducing longevity risk, the risk of running out of funds during the retiree's lifetime. Kitces and Pfau (2014) find that an investment strategy that has high stock allocations reduces longevity risk, but this finding does come with the caveat that stock investing leads to more volatility in portfolio value. Vora and McGinnis (2000) find retirees who remove dollars from their portfolio on a regularly scheduled basis see superior portfolio performance if the portfolio is invested in stocks as compared to a portfolio of bonds. The dominance of the stock returns over the bond returns increases as the investment's horizon increases. These results call into question the concept that bonds are a safer investment as consumers age.

Time horizon. The proper time horizon for retirees is also subject to debate. Some researchers presume retirement begins at age 62 and lasts for 30 years. Others presume the retirement window is 35 years. Still others use the actuarial tables to randomly select the retirement horizon (Sevy 2021). As medical advances have extended life spans, reducing longevity risk has become a large concern for many retirees. A 65-year-old woman in 2015 had a 34% chance of living to 90 and beyond (The Hamilton Project 2022). Studies need to include realistic time horizons to reduce longevity risk.

Strategy for investing. There has been much research into the proper way to invest in the stock market: lump sum or dollar cost averaging. Research shows that dollar cost averaging leads to better wealth accumulation (Leggio and Lien 2001) and further, an augmented dollar cost averaging strategy is superior (Kapalcyzinski and Lien 2021). The same question exists for decumulation of wealth: is it best to set a withdrawal rate at the beginning of retirement and maintain that withdrawal rate through time or should the withdrawal rate vary with the size of the portfolio? Reverse dollar cost averaging (RDCA) is effective, particularly in good market conditions. It reduces the market risk of the portfolio and leaves money in the market, thus providing for growth opportunities (Sevy 2021).

Investment risk. Initially defined contribution plans were an add-on to defined benefit plans; over time, 401(k) plans have replaced traditional employer-funded defined benefit plans, thus transferring the risk of retirement savings to the individual (Merton 2014). Analysts note the goal with retirement savings is to minimize the probability of ruin (i.e., running out of retirement savings during your lifetime) as opposed to maximizing the probability of accumulating wealth (Rook 2015). In fact, most studies presume retirees spend their last dollar or retirement savings just prior to death. There are few studies that consider maximizing terminal wealth.

Recent research notes that retirees with a defined benefit plan draw down retirement wealth slower which raises some concern for baby boomers and those with defined contribution accounts as the risk of outliving savings may be greater for these individuals (Siliciano and Wettstein 2021). Templin (2022) finds that there is a 65%–70% chance that the 4% rule works for retirees, meaning that retirees do not exhaust retirement wealth prior to death. This means the 4% rule fails 30–35% of the time and retirees outlive their savings.

Conversely, Templin (2021) notes that a recent study found that 75% of 62–75 year-olds have seen their retirement wealth remain steady or grow (EBRI 2021). However, a recent study found that retirees who began with retirement wealth of $200,000 had $86,000 less in their retirement account if their retirement account was a 401(k) plan instead of a traditional pension (Nova 2022).

Inflation. Inflation is a consideration for retirement portfolios. Some studies use a flat rate, typically 3% annually (Staib 2019). Other studies adjust the inflation rate based upon actual inflation (Rook 2015).

DATA AND METHODOLOGY

The empirical study data comes from *Ibbotson Associates Valuation Edition 2022 Yearbook.* The annual returns for 1926–2022 are used. The risky assets considered are large company stocks (S&P 500 composite), and the bonds are long term government bonds.

Decumulation Strategies

We propose six decumulation strategies (or methods) as below. The objective is to maximize funds available during the 35-year retirement period while minimizing terminal wealth.

Method 1: The RMD Strategy. This approach, adapted from Sun and Webb (2013), involves retirees consuming a percentage of their remaining wealth based on their age, as specified in the IRS Required Minimum Distribution (RMD) tables. Calculations for RMD are included in Appendix A.

Method 2: The RMD PLUS Dividend & Interest Strategy. Also adapted from Sun and Webb (2013), this strategy suggests retirees consume the age-related percentage of remaining wealth as per IRS RMD tables. Additionally, retirees include interest and dividends in their withdrawal calculations. This method has been identified as optimal by Sun and Webb (2013).

Method 3: The 4% of Initial Value Strategy. Retirees employing this method withdraw a fixed 4% of their initial portfolio value each year, with this amount being adjusted for inflation at the beginning of each year. Importantly, this withdrawal amount remains constant before inflation adjustment, unaffected by market fluctuations.

Method 4: The 4% of Remaining Balance Strategy. Initially, this strategy mirrors Method 3 in that the first withdrawal is based on 4% of the initial portfolio value. However, at the conclusion of each period, the portfolio's value is recalculated, and subsequent withdrawals are based on 4% of the remaining total portfolio value. This approach helps manage longevity risk but requires retirees to adapt to changing withdrawal amounts, considering both market performance and inflation.

Method 5: The 6% of Initial Value Strategy. Similar to Method 3, this strategy involves retirees withdrawing 6% of their initial portfolio value each year, with adjustments for inflation considered.

Method 6: The 6% of Remaining Balance Strategy. This strategy closely resembles Method 4, but with one key distinction: the withdrawal rate is adjusted from 4% to 6% in our baseline scenario.

In all six decumulation strategies, the terminal value is set to zero if the account balance is exhausted either before reaching or at the age of 99.

Monte Carlo Simulation

We presume the investor retires at age 65 and has a remaining life span of 35 years. This is more conservative than previous research which presumed a 30-year retirement time horizon. We estimate annual withdrawals and portfolio balances for retirees over a 35-year retirement period when evaluating the effectiveness of these decumulation strategies. This estimation hinges on the accurate prediction of stock and bond returns. Many studies note the decline in the US equity premium (Jagannathan et al. 2000). Others note the anticipated future decline of stock market returns (Diamond 2000). Given the constraints posed by the limited availability of 35-year investment periods within historical data and the need to account for possible serial correlation arising from overlapping returns, we employ Monte Carlo simulation to evaluate these decumulation strategies. Our approach involves an initial step where we leverage a machine learning algorithm to optimize the distributions of five crucial financial metrics: total stock returns, total bond returns, and the income components of stock and bond returns (comprising dividend returns for stocks and coupon payments for treasury bonds), as well as inflation rates. We also consider the correlations among these financial metrics. Subsequently, we utilize the optimized parameters to conduct a Monte Carlo simulation, generating simulations that closely align with the best-fit scenarios. This meticulous process enables us to gain a comprehensive understanding of the performance and potential outcomes associated with each decumulation strategy, while considering the intricacies of financial markets and economic factors.

Our analysis of the auto-correlation function (ACF) and the augmented Dickey-Fuller test (ADF) on the observed data yields the finding that the return series of stocks and bonds do not exhibit significant auto-correlation. As a result, we proceed by directly selecting the most appropriate probability distributions, drawing insights from historical annual data spanning 1926 to 2022. Our methodology initiates with an initial pool of 109 distributions using the Python "fitter" package.[1] The best-fitting five distributions for the observed data are determined through a combination of goodness-of-fit statistics, including AIC, BIC, and Kolmogorov-Smirnov (K-S) (Massey 1951). It's worth noting that among the five financial metrics, four can be fitted by the varied asymmetric distributions, whereas inflation rates are best described by a symmetric non-central t-distribution. This rigorous approach allows us to select distributions that closely align with the empirical data, thereby enhancing the reliability and accuracy of our analytical framework.

To model the interdependence among the distributions of the five financial metrics, we adopt the Frank Copula methodology (Hofert, Mächler, and McNeil 2012) using Frank Copula API from the Python "Statsmodels" module.[2] This entails scrutinizing a total of 243 (three raised to the power of five) combinations involving the top three marginal distributions of the five financial metrics. The best combination of five marginal distributions is determined by minimizing the sum of K-S statistics from five K-S tests conducted between 97 observed yearly data and the 2,000 random samples drawn from the best-fit marginal distribution with a default theta (θ) of 0.066 directly from observed data.

To be precise, the combination of marginal distributions for five financial metrics is a generalized Gamma distribution for total stock returns, an asymmetric Laplace distribution for total bond returns, a non-central t-distribution for inflation rates, a Burr distribution for stock income returns, and an asymmetric Laplace distribution for bond income returns. Subsequently, the theta parameter of Frank Copula is fine-tuned

[1] https://pypi.org/project/fitter/.
[2] https://www.statsmodels.org/stable/generated/statsmodels.distributions.copula.api.FrankCopula.html.

by minimizing the sum of pair-wise absolute differences of correlation coefficients between observed data and 2,000 random samples from a best-fit marginal distribution. This process identifies the optimized theta value, which best aligns the observed data with the selected copula structure. In our specific case, the optimized theta is 0.26. Finally, we generate a simulated dataset comprising four million random samples drawn from the joint distributions resulting from the copula model. This dataset forms the foundation for our analytical investigation.

The annual withdrawals and portfolio balances of the retiree over a 35-year retirement horizon (from age 65 to 100) are then computed by employing the random samples from the simulated five financial metrics (returns and inflation rates). This process involves generating a total of 10,000 iterations, allowing us to compute both the value at risk (VaR) and the conditional value at risk (CVaR).

Performance Measures

The VaR measures the amount of the maximum potential loss within a specified period with a degree of confidence: it is a one-sided risk measure. Variance is a two-sided risk measure, implying that the individual dislikes any deviation from the mean regardless of the direction of the deviation. This is hardly the notion of risk perceived by an individual. Performance criteria for disinvestment is a bit tricky. Herein the goal is to be able to have funds available throughout the retiree's life without leaving too much unspent. Thus, we are looking at the VaR and CVaR.

To interpret the results with VaR, let's look to an example. If VaR 95 is equal to $10,000, this means that there is a 5% chance that the terminal value of the portfolio will be less than $10,000. If CVaR95 = $8,000, this means that if the terminal value is less than $10,000 as defined by VaR95, then the expected terminal value is $8,000 (Sarykalin, Serraino, and Uryasev 2008).

Traditionally, for the negative side the focus is on the probability of default. We, however, are looking at the probability of funds surviving. Therefore, VaR is measuring the potential for surviving funds. CVaR looks at the extreme outcomes in the tail of the distribution of possible returns, beyond the (VaR) cutoff point.

To calculate VaR, we look at the simulated 10,000 iterations, order them from worst losses to greatest gains and get the k-th percentiles.

Let X be a random variable with the cumulative distribution function $F_X(z) = P(X \leq z)$. In our case, X relates to the amount of unspent wealth at the end of the retiree's life (Sarykalin, Serraino, and Uryasev 2008). The VaR of X with confidence level a $\in$ (0,1) is

$$VaR_\alpha(X) = \min(z | F_X(z) \geq \alpha).$$

The CVaR of X with confidence level $a \in (0,1)$ is the mean of the generalized α-tail distribution (Sarykalin, Serraino, and Uryasev 2008):

$$CVaR_\alpha(X) = \int_{-\infty}^{VaR_\alpha(X)} \left(\frac{1}{\alpha}\right) z dF_X(z).$$

For the purpose of this article, investors are presumed to have chosen an asset allocation for their portfolio. The standard presumption is 60% of the assets are invested in government bonds, and 40% are invested in large company common stock (a conservative asset allocation). We assume the investors have a fixed sum of money available to invest at time T_0. The portfolio value will grow (shrink) based upon the proportional gains (losses) in the stocks and bonds in the investment portfolio. The portfolio value will be adjusted by the amount of the withdrawal each period. We calculate VaR and CVaR of the terminal value at the 90%, 95%, and 99% confidence levels.

SIMULATION RESULTS

Base Analysis

According to the Federal Reserve, the median household has $65,000 in retirement savings, and this will grow to $255,200 by retirement age (Federal Reserve 2021). We therefore use $255,200 as the presumed retirement wealth. We presume the portfolio wealth is invested 60% in government bonds and 40% in large company stocks.

Our analysis commences with an examination of the annual withdrawal patterns over the course of retirement for each strategy. Specifically, we scrutinize the median annual withdrawals from age 65 to 99. The simulation results, as presented in Exhibit 1, highlight that only Method five exhausts retirement funds before reaching

EXHIBIT 1

Median Annual Withdrawal with 60/40 Asset Allocation (60% in bonds 40% in stocks)

Age	Method 1 (RMD)	Method 2 (RMD plus int & dividend)	Method 3 (4% initial)	Method 4 (4% of remaining balance)	Method 5 (6% initial)	Method 6 (6% remaining balance)
65	7,528	17,911	10,208	10,208	15,312	15,312
66	8,108	17,272	10,472	10,884	15,708	15,986
67	8,735	17,644	10,769	11,569	16,154	16,628
68	9,360	17,924	11,081	12,303	16,621	17,292
69	10,031	18,159	11,423	13,040	17,134	17,916
70	10,759	18,326	11,772	13,820	17,657	18,528
71	11,563	18,460	12,136	14,614	18,202	19,093
72	12,438	18,831	12,501	15,452	18,750	19,696
73	13,305	18,926	12,872	16,277	19,301	20,197
74	14,365	19,184	13,288	17,155	19,918	20,695
75	15,443	19,348	13,711	18,081	20,540	21,175
76	16,524	19,548	14,128	18,987	21,141	21,561
77	17,682	19,666	14,547	19,868	21,723	21,840
78	18,897	19,807	14,978	20,768	22,267	22,109
79	20,294	19,874	15,448	21,580	22,849	22,247
80	21,667	19,966	15,953	22,458	23,368	22,357
81	23,079	20,023	16,408	23,282	23,779	22,404
82	24,772	20,162	16,899	24,110	24,158	22,342
83	26,293	20,116	17,427	24,873	24,481	22,264
84	28,096	19,987	17,949	25,584	24,716	21,975
85	29,836	20,116	18,430	26,330	24,930	21,737
86	31,890	20,025	18,931	27,010	24,947	21,420
87	33,678	19,868	19,434	27,599	24,828	20,911
88	35,472	19,737	19,988	28,171	24,642	20,481
89	37,471	19,583	20,482	28,509	23,891	19,845
90	39,723	19,258	20,994	28,774	22,576	19,184
91	41,882	19,000	21,603	29,226	18,878	18,502
92	43,582	18,619	22,150	29,698	1,460	17,774
93	45,702	18,175	22,659	29,882	0	16,987
94	47,070	17,726	23,243	29,703	0	16,024
95	48,467	17,017	23,705	29,674	0	15,140
96	49,106	16,083	24,264	29,423	0	14,147
97	50,025	15,531	24,796	29,042	0	13,268
98	50,490	14,583	25,356	28,582	0	12,271
99	50,175	13,538	25,932	28,101	0	11,251

age 99; more precisely, exhausted funds occur at age 93. In contrast, Methods one, three, and four depict a progressive increase in median withdrawals. For instance, Method one initiates annual withdrawals of less than $10,000 for the first four retirement years, eventually exceeding $50,000 at age 99. This observation prompts us to consider whether these approaches align with typical retiree spending patterns, which often involve higher expenses during the initial years of retirement. It raises the question of whether retirees might encounter unnecessary financial constraints during their early retirement years, despite possessing ample resources that could enhance their quality of life. Conversely, Models two and six consistently sustain median funds without depletion and permit relatively generous withdrawals in the early stages of retirement.

Next, we assess the likelihood of retirees depleting their retirement funds at various ages throughout the retirement period. Exhibit 2 reveals that Methods three and five, which involve the traditional approach of withdrawing a constant percentage (either 4% or 6%) of the initial balance, exhibit the least favorable performance. These methods carry a high risk of the retiree running out of funds before the 35-year retirement draw period concludes. Specifically, Method three presents a 5.6% chance of depletion at age 88 and an 18.9% chance at age 99, while Method five presents a 38.9% chance at age 88 and a 66.8% chance at age 99.

In contrast, for the Sun and Webb (2013) methods (Methods one and two), the retiree never exhausts the funds because the withdrawal amount is always determined based on the remaining balance. Similarly, Methods four and six almost never deplete investment funds, except during periods of exceptionally high inflation.

Exhibit 3 provides a comprehensive overview of VaR and CVaR metrics, evaluated at confidence levels of 99%, 95%, and 90%, for the terminal wealth of retirees across the six withdrawal strategies. Additionally, we include the VaR at the 50% confidence level, which essentially represents the median portfolio balance at age 99.

Decumulation strategies Three and Five are clearly not preferred choices, as even at the 90% confidence level they lead to retirement account depletion before reaching age 99, as shown in both Exhibits 2 and 3.

Our favored VaR and CVaR outcomes aim for a balanced approach. Excessive VaR and CVaR values may indicate that retirees accumulate excessive terminal wealth, potentially at the expense of their quality of life during retirement. Therefore, retirees should aim for a harmonious equilibrium between annual spending and preserving terminal wealth.

Method one exhibits very high VaR and CVaR values at the 95% and 90% confidence levels, suggesting a low risk of running out of retirement funds. However, these methods result in very low median withdrawal amounts, such as $7,528 for the first year of retirement and less than $10,000 in early retirement years (as shown in Exhibit 1). This strategy leaves retirees with an unnecessarily high median terminal value of $290,919.

Although lower than Method one, Method four also shows a high median terminal value at VaR95 and VaR90. The high terminal value comes at the expense of low early-year retirement withdrawals. For example, the median withdrawal amount for the first year of retirement is $10,208.

In contrast, Methods two and six stand out with relatively modest median terminal values, positive, albeit low, VaR and CVaR for terminal value, and a history of maintaining stable withdrawals throughout retirement. These two methods strike a better balance, offering a reasonable withdrawal amount that doesn’t compromise retirees’ quality of life while maintaining a low risk of depleting funds by age 99. In essence, these withdrawal methods enable retirees to savor a relatively high and consistent standard of living during retirement while safeguarding against the depletion of their retirement savings. Method two, for instance, yields a median terminal

EXHIBIT 2

Frequency of Portfolio Depletion across Ages (out of 10,000 iterations) with 60/40 Asset Allocation

Age	Method 1 (RMD)	Method 2 (RMD plus int & dividend)	Method 3 (4% initial balance)	Method 4 (4% of remaining balance)	Method 5 (6% initial balance)	Method 6 (6% remaining balance)
66	0	0	0	0	0	0
67	0	0	0	0	0	0
68	0	0	0	0	0	0
69	0	0	0	0	2	0
70	0	0	1	0	3	0
71	0	0	1	0	6	0
72	0	0	3	0	15	1
73	0	0	4	0	23	1
74	0	0	4	0	38	1
75	0	0	12	0	69	1
76	0	0	20	0	119	1
77	0	0	29	0	205	2
78	0	0	38	0	346	2
79	0	0	50	0	548	2
80	0	0	67	0	819	2
81	0	0	102	1	1,174	3
82	0	0	126	1	1,534	4
83	0	0	230	1	2,330	4
84	0	0	230	1	2,330	4
85	0	0	284	1	2,735	4
86	0	0	358	1	3,136	6
87	0	0	455	2	3,532	6
88	0	0	563	2	3,889	6
89	0	0	666	2	4,261	6
90	0	0	782	2	4,586	7
91	0	0	887	4	4,919	11
92	0	0	1,024	5	5,196	12
93	0	0	1,139	6	5,457	15
94	0	0	1,263	7	5,728	17
95	0	0	1,389	8	5,940	18
96	0	0	1,514	8	6,149	18
97	0	0	1,623	8	6,336	21
98	0	0	1,740	11	6,514	25
99	0	0	1,879	13	6,679	29

NOTES: The instances of balance depletion in methods Four and Six are attributed to simulated periods of exceptionally high inflation. These instances of inflation surpass the portfolio's rate of return multiplied by the withdrawal rate, leading to the depletion of account balances.

value of $58,641, slightly surpassing Method six's $57,810. Nevertheless, Method two exhibits a more predictable withdrawal pattern during retirement, with median withdrawals ranging from $13,358 to $20,162, compared to Method six, which shows a range of $11,251 to $22,404 for median withdrawals.

In summary, the most favorable withdrawal strategy is Method two, as it keeps the terminal value low, carries a minimal risk of depleting retirement funds, and provides consistent withdrawals. The next best option is Method six, which offers a comparable terminal value and withdrawal pattern. As for the third-best strategy, it is Method four, characterized by a low risk of fund depletion but relatively modest early retirement withdrawals.

EXHIBIT 3

VaR and CVaR of Terminal Value with 60/40 Asset Allocation

Probability	Method 1 (RMD)	Method 2 (RMD plus int & dividend)	Method 3 (4% of initial)	Method 4 (4% of remaining balance)	Method 5 (6% of initial)	Method 6 (6% remaining balance)
Panel A: VAR with 60/40 Asset Allocation						
99%	76,506	14,405	0	1,870	0	13
95%	114,174	21,968	0	36,980	0	3,692
90%	139,749	27,351	0	69,944	0	11,021
50%	290,919	58,641	742,079	226,894	0	57,810
Panel B: Conditional Value at Risk (CvaR) with 60/40 Asset Allocation						
99%	63,854	11,788	0	426	0	2
95%	91,173	17,404	0	16,274	0	1,203
90%	109,555	21,133	0	35,704	0	4,290

NOTE: The value "0" indicates that the retiree has depleted his/her retirement funds by age 99.

Results of 70/30 Asset Allocation

We have also explored a more conservative investment approach, where 70% of the retiree's wealth is allocated to the relatively secure bond market, with the remaining 30% invested in stocks. In this scenario, our analysis reveals that retirees employing Method five deplete their retirement savings earlier, precisely by age 90, as indicated by the median annual withdrawal presented in Panel A of Exhibit 4.

When examining the VaR and CVaR results (as displayed in Panels B and C of Exhibit 4), we observe a consistent trend, mirroring the simulations conducted with a 60/40 asset allocation. Methods three and five continue to expose retirees to the risk of outliving their investments, while Methods one and four may potentially leave retirees with more terminal wealth than is optimally utilized.

Under the 70/30 asset allocation, the preferred decumulation strategy remains Method two, with Method six finding similar results. Method two adeptly balances the provision of a reasonable withdrawal amount without jeopardizing retirees' quality of life while effectively managing the risk of depleting their funds, making it a good choice for investors. Method six remains the second-best option as it exhibits a slightly lower initial withdrawal pattern, higher mid-range withdrawals, and a slightly lower median terminal value compared to Method two. Method four is less desirable since it has an unnecessarily large terminal value at the expense of low consumption during the early retirement years.

State Retirement Results

World Population Review (2022) created an analysis of how much retirees need by state. The breakdown is the total dollar amount needed in savings as well as the amount that will be withdrawn annually. The percentage needed to be withdrawn is not necessarily 4%; in fact, in most cases, it is more than that. We run the analysis on the state with the lowest required retirement account balance at retirement (Mississippi, at $617,671 and a need to withdraw $44,758 annually, or 7.25%, Exhibit 5); and on the state with an average retirement represented by New Hampshire with a need for $948,850 in a retirement account with an annual withdrawal of $55,960, or 5.90%, (Exhibit 6).

In Panel A of Exhibit 5, we present data on the median withdrawals by age throughout retirement, considering various decumulation strategies for Mississippi,

EXHIBIT 4

70/30 Asset Allocation

Panel A: Median Annual Withdrawal by Ages

Age	Method 1 (RMD)	Method 2 (RMD plus int & dividend)	Method 3 (4% of initial)	Method 4 (4% of remaining balance)	Method 5 (6% of initial)	Method 6 (6% remaining balance)
65	7,528	17,854	10,208	10,208	15,312	15,312
66	8,022	17,143	10,472	10,777	15,708	15,828
67	8,595	17,425	10,769	11,386	16,154	16,364
68	9,160	17,519	11,081	12,039	16,621	16,928
69	9,787	17,720	11,423	12,740	17,134	17,488
70	10,456	17,758	11,772	13,420	17,657	17,990
71	11,120	17,807	12,136	14,084	18,202	18,415
72	11,912	17,989	12,501	14,828	18,750	18,879
73	12,688	18,045	12,872	15,545	19,302	19,299
74	13,646	18,168	13,288	16,311	19,917	19,650
75	14,611	18,243	13,710	17,092	20,541	20,017
76	15,510	18,295	14,129	17,847	21,138	20,282
77	16,486	18,238	14,548	18,629	21,716	20,480
78	17,603	18,285	14,978	19,322	22,256	20,604
79	18,823	18,261	15,449	20,002	22,816	20,633
80	19,986	18,233	15,950	20,690	23,276	20,565
81	21,175	18,125	16,406	21,379	23,632	20,455
82	22,516	18,117	16,893	21,988	23,918	20,430
83	23,788	17,852	17,408	22,565	24,121	20,143
84	25,321	17,729	17,934	23,081	24,221	19,882
85	26,735	17,704	18,411	23,592	24,152	19,440
86	28,369	17,447	18,896	24,145	23,824	19,144
87	29,932	17,310	19,375	24,466	23,211	18,602
88	31,279	16,992	19,860	24,912	21,542	18,143
89	33,007	16,756	20,358	25,052	15,024	17,462
90	34,579	16,355	20,850	25,254	0	16,720
91	36,267	16,045	21,406	25,418	0	16,056
92	37,612	15,553	21,908	25,474	0	15,303
93	39,286	15,150	22,415	25,585	0	14,602
94	40,208	14,636	22,893	25,382	0	13,720
95	41,158	13,940	23,305	25,201	0	12,917
96	41,543	13,143	23,709	24,854	0	12,026
97	42,115	12,571	24,153	24,528	0	11,209
98	42,058	11,710	24,596	23,958	0	10,338
99	41,650	10,816	24,950	23,559	0	9,435

Panel B: Value at Risk (VaR)

VAR	Method 1 (RMD)	Method 2 (RMD plus int & dividend)	Method 3 (4% of initial)	Method 4 (4% of remaining balance)	Method 5 (6% of initial)	Method 6 (6% remaining balance)
99%	71,219	13,127	0	1,712	0	11
95%	102,024	18,793	0	32,379	0	3,159
90%	124,324	22,968	0	61,030	0	9,385
50%	241,488	46,363	477,546	189,048	0	48,263

Panel C: Conditional Value at Risk (CVaR)

CVAR	Method 1 (RMD)	Method 2 (RMD plus int & dividend)	Method 3 (4% of initial)	Method 4 (4% of remaining balance)	Method 5 (6% of initial)	Method 6 (6% remaining balance)
99%	60,764	10,743	0	356	0	2
95%	83,923	15,243	0	13,967	0	1,017
90%	98,921	18,107	0	30,946	0	3,676

EXHIBIT 5

Mississippi—Required Balance of $617,661 and Spend of $44,758 (7.25% of initial)

Panel A: Median Annual Withdrawal by Ages with 60/40 Asset Allocation—Mississippi

Age	Method 1 (RMD)	Method 2 (RMD plus int & dividend)	Method 3 (4% of initial)	Method 4 (4% of remaining balance)	Method 5m (7.25% of initial)	Method 6m (7.25% remaining balance)
65	18,221	43,349	24,706	24,706	44,780	44,780
66	19,624	41,803	25,346	26,342	45,940	46,128
67	21,140	42,704	26,065	28,000	47,243	47,320
68	22,654	43,383	26,818	29,777	48,608	48,527
69	24,278	43,949	27,647	31,561	50,110	49,516
70	26,041	44,355	28,491	33,448	51,639	50,407
71	27,985	44,678	29,372	35,371	53,226	51,205
72	30,104	45,576	30,257	37,398	54,825	51,856
73	32,201	45,808	31,155	39,394	56,413	52,284
74	34,769	46,430	32,162	41,519	58,162	52,659
75	37,378	46,827	33,184	43,762	59,824	52,942
76	39,993	47,313	34,194	45,953	61,242	52,716
77	42,795	47,597	35,208	48,087	62,504	52,506
78	45,736	47,940	36,252	50,266	63,354	51,922
79	49,117	48,101	37,388	52,230	64,018	51,167
80	52,441	48,324	38,611	54,356	64,251	50,424
81	55,857	48,461	39,712	56,349	63,938	49,347
82	59,955	48,799	40,901	58,354	62,537	48,159
83	63,638	48,688	42,179	60,200	59,336	46,719
84	68,000	48,374	43,441	61,921	43,887	45,121
85	72,212	48,686	44,607	63,727	0	43,468
86	77,184	48,467	45,818	65,372	0	41,734
87	81,510	48,088	47,037	66,798	0	39,602
88	85,852	47,771	48,377	68,181	0	37,853
89	90,690	47,398	49,572	69,000	0	35,514
90	96,141	46,610	50,811	69,641	0	33,295
91	101,368	45,987	52,286	70,737	0	31,007
92	105,481	45,064	53,611	71,877	0	28,835
93	110,613	43,988	54,841	72,323	0	26,703
94	113,924	42,903	56,254	71,889	0	24,340
95	117,304	41,187	57,373	71,821	0	22,209
96	118,851	38,926	58,725	71,212	0	20,070
97	121,075	37,591	60,014	70,290	0	18,013
98	122,201	35,296	61,368	69,177	0	15,990
99	121,439	32,765	62,763	68,012	0	14,237

Panel B: VAR—Mississippi

VAR	Method 1 (RMD)	Method 2 (RMD plus int & dividend)	Method 3 (4% of initial)	Method 4 (4% of remaining balance)	Method 5m (7.25% initial or $44,758)	Method 6m (7.25% remaining balance)
99%	185,168	34,864	0	4,526	0	1
95%	276,336	53,168	0	89,502	0	1,848
90%	338,235	66,198	0	169,286	0	7,620
50%	704,112	141,929	1,796,054	549,151	0	57,495

Panel C: CVAR—Mississippi

CVAR	Method 1 (RMD)	Method 2 (RMD plus int & dividend)	Method 3 (4% of initial)	Method 4 (4% of remaining balance)	Method 5m (7.25% initial or $44,758)	Method 6m (7.25% remaining balance)
99%	154,546	28,531	0	1,031	0	0
95%	220,667	42,122	0	39,388	0	495
90%	265,156	51,148	0	86,414	0	2,520

EXHIBIT 6

New Hampshire—Required Balance of $946,850 and Spend of $55,960 Annually (5.9% of initial)

Panel A: Median Annual Withdrawal by Ages—New Hampshire

Age	Method 1 (RMD)	Method 2 (RMD plus int & dividend)	Method 3 (4% of initial)	Method 4 (4% of remaining balance)	Method 5nh (5.90% initial or $55,960)	Method 6nh (5.90% remaining balance)
65	27,932	66,452	37,874	37,874	55,959	55,959
66	30,083	64,083	38,855	40,381	57,408	58,476
67	32,407	65,464	39,957	42,923	59,036	60,886
68	34,728	66,504	41,112	45,647	60,742	63,388
69	37,217	67,372	42,381	48,382	62,618	65,739
70	39,919	67,994	43,676	51,274	64,530	68,055
71	42,900	68,490	45,027	54,222	66,521	70,211
72	46,149	69,866	46,382	57,330	68,526	72,512
73	49,363	70,221	47,760	60,390	70,540	74,460
74	53,299	71,175	49,303	63,647	72,791	76,395
75	57,299	71,784	50,871	67,086	75,079	78,300
76	61,308	72,528	52,418	70,445	77,272	79,810
77	65,603	72,965	53,972	73,716	79,425	80,988
78	70,111	73,490	55,572	77,055	81,476	82,066
79	75,294	73,737	57,315	80,067	83,652	82,686
80	80,390	74,079	59,189	83,325	85,579	83,216
81	85,627	74,289	60,876	86,381	87,189	83,480
82	91,909	74,807	62,699	89,455	88,585	83,502
83	97,554	74,636	64,659	92,284	89,991	83,401
84	104,241	74,155	66,593	94,922	90,940	82,396
85	110,698	74,633	68,381	97,691	91,829	81,529
86	118,320	74,298	70,238	100,213	92,435	80,587
87	124,951	73,716	72,106	102,398	92,702	78,828
88	131,607	73,230	74,161	104,519	92,065	77,405
89	139,024	72,659	75,992	105,774	90,426	75,116
90	147,381	71,451	77,892	106,757	87,457	72,700
91	155,393	70,496	80,152	108,437	81,109	70,348
92	161,698	69,081	82,183	110,185	60,798	67,631
93	169,565	67,432	84,069	110,868	0	64,889
94	174,640	65,769	86,236	110,204	0	61,410
95	179,822	63,138	87,951	110,099	0	58,057
96	182,193	59,672	90,024	109,165	0	54,438
97	185,604	57,625	91,999	107,752	0	51,138
98	187,329	54,108	94,075	106,045	0	47,501
99	186,160	50,228	96,213	104,260	0	43,706

Panel B: VAR—New Hampshire

VAR	Method 1 (RMD)	Method 2 (RMD plus int & dividend)	Method 3 (4% of initial)	Method 4 (4% of remaining balance)	Method 5nh (5.90% initial or $55,960)	Method 6nh (5.90% remaining balance)
99%	283,855	53,444	0	6,939	0	62
95%	423,613	81,505	0	137,203	0	15,253
90%	518,501	101,479	0	259,508	0	44,595
50%	1,079,376	217,571	2,753,281	841,827	0	228,665

Panel C: CVAR—New Hampshire

CVAR	Method 1 (RMD)	Method 2 (RMD plus int & dividend)	Method 3 (4% of initial)	Method 4 (4% of remaining balance)	Method 5nh (5.90% initial or $55,960)	Method 6nh (5.90% remaining balance)
99%	236,913	43,736	0	1,581	0	10
95%	338,274	64,572	0	60,381	0	5,047
90%	406,473	78,408	0	132,470	0	17,575

a state with the lowest required retirement funds. This data illustrate that retirees could deplete their retirement funds by age 85 if they employ Method five (5m), which involves withdrawing $44,758 annually, equivalent to 7.25% of their initial balance of $617,671, adjusted for inflation. On the other hand, Method one (based on the Sun and Webb (2013) RMD method) results in low median withdrawals at the start of retirement, for instance, $18,211 at age 65, but significantly higher withdrawals toward the end of retirement, reaching as high as $121,439 at age 99. Method Six (6m in Exhibit 5), involving a consistent 7.25% withdrawal of the remaining balance each year, exhibits notably higher median withdrawals in the early retirement years compared to the later years.

Panels B and C of Exhibit 5 provide VaR and CVaR results. Methods one and four may leave retirees with more terminal wealth than is optimal, while Methods three and five (involving 4% and 7.25% withdrawals of the initial value) expose retirees to a higher risk of depletion, as indicated by VaR90, VaR95, and VaR99. Overall, Methods two and Method six (6m) strike an effective balance between maintaining stable withdrawals to ensure reasonable consumption throughout retirement and mitigating the risk of outliving their funds, making them the most favorable choices for retirees.

These findings align with our earlier results, highlighting that basing withdrawals on the total remaining portfolio value rather than the initial portfolio value consistently offers superior outcomes for retirees.

In Panel A of Exhibit 6, we present data on the median withdrawals by age throughout retirement, considering various decumulation strategies for New Hampshire, a state with retirement needs closest to the national average. The data illustrate that retirees could deplete their retirement funds by age 93 if they employ Method five (5nh), which involves withdrawing $55,960 annually, equivalent to 5.9% of their initial balance of $946,850, adjusted for inflation. On the other hand, Method one results in low median withdrawals at the start of retirement, for instance, $27,932 at age 65, but significantly higher withdrawals toward the end of retirement, reaching as high as $186,160 at age 99.

Panels B and C of Exhibit 6 provide VaR and CVaR results. Methods one and four may leave retirees with more terminal wealth than is optimal, while Methods Three and Five (involving 4% and 5.9% withdrawals of the initial value) expose retirees to a higher risk of depletion, as indicated by VaR90, VaR95, and VaR99. Consistent with previous results for Mississippi, Methods two and Method six (6nh) strike an effective balance between maintaining stable withdrawals to ensure reasonable consumption throughout retirement and mitigating the risk of outliving their funds, making them the most favorable choices for retirees.

CONCLUSION

Is the 4% rule dead? Maybe not dead, but severely wounded. We have more volatile markets, returns on stocks and bonds more highly correlated, and lower asset returns than in the past; inadequate retirement savings since the creation of the defined contribution model for retirement savings; and data that demonstrate that retirees do not have the desire for fixed withdrawals: rather, the withdrawal needs to evolve throughout different life stages. For all of these reasons, it appears a dynamic retirement withdrawal strategy makes sense.

Retirees should adjust decumulations based upon the portfolio value and the retiree's needs, and not set a 4% withdrawal rate based upon the initial retirement wealth. Sticking to the original concept of withdrawing 4% of the investor's initial wealth when retirement begins has an increased chance of leading to the retiree outliving his funds; however, withdrawing 4% or even a higher rate of 6% of the portfolio value will preserve funds throughout the retiree's life, though the amount available to withdraw each year will be variable, making planning more difficult. Further, it most frequently makes sense to withdraw funds not based upon a 4% rule but rather based on a higher withdrawal rate of 6% or withdrawal based upon RMD. Especially when retirees spend interest income and dividends (Method two), the tradeoff between zero survivorship risk and lower terminal wealth is strong.

Future work can evaluate portfolios that include small company stocks given recent studies indicating this asset may lead to superior portfolio performance (Johnston et al. 2019). Others recommend a dynamic decumulation of wealth with guardrails in place to determine the withdrawal amount each period (Sevy 2021). Additionally, future research on a dynamic withdrawal strategy that takes into consideration a lifetime spending curve has merit (Roy 2019).

This is not the definitive death of the 4% rule or the end of discussions on retirement funding. Given the inadequate retirement savings of many nearing age 65, the limits of Social Security in funding retirement, and the number of retirees with defined contribution accounts, we have much more to explore on this topic.

APPENDIX A

REQUIRED MINIMUM DISTRIBUTION TABLE[3]

Following Sun and Webb (2013), we calculate Required Minimum Distribution using the life expectancy specified in Internal Revenue Service (IRS) Publication 590-B (2022). Specifically, our calculations are based on Uniform Lifetime (Table III) and Joint Life and Last Survivor Expectancy (Table II) in Appendix B.

Age	RMD	Age	RMD
65	2.95%	83	5.65%
66	3.03%	84	5.95%
67	3.13%	85	6.25%
68	3.22%	86	6.58%
69	3.32%	87	6.94%
70	3.42%	88	7.30%
71	3.53%	89	7.75%
72	3.65%	90	8.20%
73	3.77%	91	8.70%
74	3.92%	92	9.26%
75	4.07%	93	9.90%
76	4.22%	94	10.53%
77	4.37%	95	11.24%
78	4.55%	96	11.90%
79	4.74%	97	12.82%
80	4.95%	98	13.70%
81	5.15%	99	14.71%
82	5.41%	100	15.63%

[3] https://www.irs.gov/pub/irs-pdf/p590b.pdf

APPENDIX B

DISTRIBUTION FUNCTIONS FOR FIVE FINANCIAL METRICS

Financial Metrics	Distribution Parameters	References
Stock total returns (generalized Gamma distribution)	a = 0.6367718067262966 c = 6.16408498490866 loc = –0.6603238123668429 scale = 0.9443431645292288	Stacy, E. W. 1962. "A Generalization of the Gamma Distribution." *Annals of Mathematical Statistics* 33 (3): 1187–1192.
Bond total returns (asymmetric Laplace distribution)	kappa = 0.7122967970189089 loc = 0.009328090401905449 scale = 0.0680593761589974	Kozubowski, T. J., and K. Podgórski. 2000. "Multivariate and Asymmetric Generalization of Laplace Distribution." *Computational Statistics* 15: 531–540. DOI:10.1007/PL00022717
Inflation rates (non-central *t*-distribution)	df = 1.634810887887318 nc = 0.4969406898747222 loc = 0.0162491457053442 scale = 0.016409760249153325	https://en.wikipedia.org/wiki/Noncentral_t-distribution
Stock dividend returns (Burr distribution)	c = 9.318562184665453 d = 0.11565827165986413 loc = 0.011005456579740667 scale = 0.051361008989308315	Burr, I. W. 1942. "Cumulative Frequency Functions." *Annals of Mathematical Statistics* 13 (2): 215–232.
Bond income returns (asymmetric Laplace distribution)	kappa = 0.31288177862513294 loc = 0.022976034479012586 scale = 0.00885650468421982	Kozubowski, T. J., and K. Podgórski. 2000. "Multivariate and Asymmetric Generalization of Laplace Distribution." *Computational Statistics* 15: 531–540. DOI:10.1007/PL00022717

Frank Copula Formula

$$C(\boldsymbol{u}) = -\frac{1}{\theta}\ln\left(1 + \frac{\prod_j (e^{-\theta u_j} - 1)}{e^{-\theta} - 1}\right) \text{ with } \theta \in (-\infty, \infty)\backslash\{0\},\ \boldsymbol{u} \in [0,1]^d$$

In our case, $d = 5$ and we choose $\theta = 0.26$ which provides the best fitting.

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