To Pre-Algebra and Beyond without and with Technology

Sample Activities

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Pre-Lecture Exercises

**Pre-Algebra – HOMEWORK 4 – Page 4**  
Chapter 4: Fractions – Chapter 5: Algebra

When we want to talk about any number in a set, we use a letter, and we call it a variable. For example, we said, “A fraction with equal (and non-zero) numerator and denominator is equal to one,” and we wrote: \( \frac{N}{N} = 1 \). \( N \) was the variable used, and it could take any non-zero value.

That is, for \( N = 3 \), \( \frac{N}{3} = \frac{3}{3} = 1 \); for \( N = 4 \), \( \frac{N}{4} = \frac{4}{4} = 1 \); for \( N = -9 \), \( \frac{N}{9} = \frac{-9}{9} = 1 \); and so on.

6. Write a fraction equal to 1 for \( N = 8 \):

7. Write mathematical expressions for the following:
   - Add three to five:________
   - Add three to ten:________
   - Add three to fourteen:_______

8. Use a variable to write a mathematical expression for: Add three to a number:______________

**Pre-Algebra – HOMEWORK 7 – Page 4**  
Chapter 7: Geometry – Chapter 8: Percents

Given the grid to the right, calculate:

10. The area of the grid

11. The total shaded area

12. The fraction of the grid area that has been shaded

13. The equivalent fraction, with denominator 100, of the shaded area in the grid.

**Pre-Algebra – HOMEWORK 8 – Page 4**  
Chapter 8: Percents – Chapter 9: Probability & Statistics

The spinner shown is separated in 5 equal sectors (parts).

7. What fraction of the spinner is grey?
   - What fraction of the spinner is white?

8. What percent of the spinner is grey?
   - What percent of the spinner is white?

9. If the spinner is “fair”, and you are asked to guess the color of the sector the pointer will most likely land on the next time it is spun, what would your guess be? Explain.
Pre-Lecture Notes (Videos)

Pre-Algebra – PREPARATION FOR CHAPTER 2

Chapter 2: Integers

Read section 2.1 (page 41) and complete the notes below; then, answer exercises 1 and 2.

2.1. THE NUMBER LINE

Remember that the set of **Integers** is the set of positive and negative whole numbers: \( \{0, \pm 1, \pm 2, \pm 3, \pm 4, \ldots \} \). We use the **number line** as a way of graphing (picturing/drawing) numbers. Every point on the line represents a number, called the **coordinate of that point**. The point with coordinate zero (i.e., the point corresponding to the number zero) is called the **origin**. The origin and the location of 1 are sufficient to define any number line. All positive numbers are positioned to the **right of the origin** and all negatives to the left of the origin. The positive half line is symmetric to the negative half line. The relationship between numbers on the number line is **< (less than) from left to right**, which means that smaller numbers are on the left of larger numbers, and larger numbers are on the right of smaller numbers.

![Number Line Diagram]

1. Place the given numbers on the number line: \(-7, -8, 0, -4, 3, -5, 2, -2, -1, -3, 3^2, 7^0\).

2. Use < or > and the number line above to compare the numbers in each pair:

   (a) \(-7 \text{ _____ } -8\)  
   (b) \(-4 \text{ _____ } 0\)  
   (c) \(3 \text{ _____ } -5\)  
   (d) \(-2 \text{ _____ } 2\)  
   (e) \(-1 \text{ _____ } -3\)  
   (f) \(7^1 \text{ _____ } 3^2\)
Readings (Flip)  

Chapter 4: Fractions – Chapter 9: Probability & Statistics

4.R.4. READING ACTIVITY 4 – Page 1  Name: ______________________

Probability

The probability of an event to occur is a fraction of the number of positive outcomes (desired outcomes) over (out of) the number of possible outcomes (total outcomes):

\[ P(E) = \frac{\text{Number of outcomes in } E}{\text{Number of total outcomes}} \]

For example, during our trip to New Zealand in the 1980s, we wanted to visit Lake Taupo. We wrote the days of the week on 7 pieces of paper, folded them, and put them in a shoe box. Katerina shook the shoe box and picked one paper at random. The probability of picking a weekend day to take the trip to Lake Taupo is \( \frac{2}{7} \) (2 out of 7) since there are two weekend days (Saturday and Sunday) out of the 7 days in the week. That is, the number of outcomes for the event ‘Weekend’ is 2, and the number of total outcomes for ‘Days of the Week’ is 7:

\[ P(\text{Weekend}) = \frac{\text{Number of days in Weekend}}{\text{Number of total days in Week}} = \frac{2}{7} \]

A number cube (die) is a cube with the numbers 1 through 6 on each face of the cube. When we roll the die, the number of total possible outcomes is 6: for the outcomes 1, 2, 3, 4, 5, 6. The probability to roll a 3, \( P(3) \), is \( \frac{1}{6} \) because there is only one outcome equal to 3 out of the 6 outcomes altogether:

\[ P(\text{Rolling 3}) = \frac{\text{Number of ways we can get a 3}}{\text{Number of total outcomes}} = \frac{1}{6} \]

Write the fraction that describes the probabilities when a number cube is rolled:

\[ P(\text{roll a 4}) \quad \text{P(roll a 5)} \]

\[ P(\text{get an even number}) \quad \text{P(get an odd number)} \]

\[ P(\text{get a prime number}) \quad \text{P(get a positive number)} \]

\[ P(\text{roll at least a 2}) \quad \text{P(roll at most a 2)} \]

Probability

When we write a probability as a fraction, it is preferred that we do not simplify the fraction to lowest terms. This way, the denominator shows the total possible outcomes, and the numerator shows the total desired outcomes.

A bag contains 3 grey, 4 black, and 5 white balls. I reach inside and pick a ball without looking. How many balls are there altogether in the bag?

Determine the probabilities:
P(pick a grey ball) P(pick a black ball)
P(pick a white ball) P(pick a red ball)

Luisito’s piggy bank contains 4 pennies, 2 nickels, 10 dimes, and 9 quarters. One day he was shaking his piggy bank to hear the loud noise of the coins, and one coin, at random, came out of the piggy bank. Find the probabilities below in regards to coin that randomly fell out:
P(coin was a penny) P(coin was a nickel)
P(coin was a dime) P(coin was a quarter)
P(coin was worth at least 10 cents)
P(coin was worth at most 10 cents)
P(the value of the coin was a single-digit number)
Readings (Flip)

4.R.5. READING ACTIVITY 5 – Page 1  Name: ____________________

Percent Fractions

The percent fraction of a given fraction is a fraction with denominator 100 that is equivalent to the original fraction:

\[
\frac{a}{b} = \frac{p}{100}
\]

The fraction \( \frac{p}{100} \) is the percent-fraction for \( \frac{a}{b} \)

For example, to find the percent fraction for \( \frac{3}{20} \), we need to find an equivalent fraction with denominator 100; since \( 20 \times 5 = 100 \), we can multiply numerator and denominator by 5 to get the percent equivalent fraction:

\[
\frac{3}{20} = \frac{3 \times 5}{20 \times 5} = \frac{15}{100}
\]

A percent fraction \( \frac{p}{100} \) can be written in the form of a percent as \( p\% \)

That is, \( \frac{15}{100} \) written as a percent is 15%. This means that \( \frac{3}{20} = \frac{15}{100} = 15\% \)

Find the percent fraction for each fraction below; then, write as a percent:

\[
\begin{array}{ll}
\frac{8}{50} & \frac{13}{50} \\
\frac{9}{25} & \frac{6}{25} \\
\frac{9}{20} & \frac{17}{20} \\
\frac{3}{5} & \frac{7}{10}
\end{array}
\]
Percent Fractions

Inversely, given a percent, we can write the percent fraction and simplify to find the equivalent, simplest fraction: \(15\% = \frac{15}{100} = \frac{3 \times 5}{2 \times 2 \times 5} = \frac{3}{20}\).

Find the fraction in simplest form equivalent to each percent:

- 50%  
- 25%

- 10%  
- 20%

- 40%  
- 60%

The same way we calculate \(\frac{3}{20}\) of 60 \(\left(\frac{3}{20} \times 60 = \frac{180}{20} = 9\right)\), we can calculate 15% of 60; we just need to use the simplest-form, fraction equivalent to 15% in place of the percent. That is, 15% of 60 = \(\frac{3}{20}\) of 60 = \(\left(\frac{3}{20} \times 60 = \frac{180}{20} = 9\right)\).

Find 50% of 24  
Find 25% of 32

Find 10% of 70  
Find 20% of 30

Find 40% of 45  
Find 60% of 55
Readings (Practice)  

Chapter 4: Fractions

4.R.6. READING ACTIVITY 6 – Page 1  Name: ____________________

Fraction Estimation

Estimation of the value of fractions usually occurs in relation to zero, one, the nearest integer, the nearest half, or the nearest quarter of a unit. That is, we approximate at 0, ¼, ½, 1, integer value, or the nearest quarter/half mark between integers.

- A fraction is equal to zero if its numerator is zero. A fraction is approximately equal to zero if the numerator is much smaller compared to the denominator.
  For example, \( \frac{0}{11} = 0 \), \( \frac{0}{3} = 0 \); and \( \frac{\frac{1}{200}}{} \approx 0 \), \( \frac{\frac{5}{3,000}}{} \approx 0 \)

- A fraction is equal to one if its numerator is equal to the denominator. A fraction is approximately equal to one if the numerator is approximately equal to the denominator.
  For example, \( \frac{1}{11} = 1 \), \( \frac{122}{122} = 1 \); and \( \frac{\frac{12}{13}}{} \approx 1 \), \( \frac{\frac{41}{40}}{} \approx 1 \)

- A fraction is equal to one half if its numerator is half of its denominator. A fraction is approximately equal to one half if the numerator is approximately half of the denominator.
  For example, \( \frac{\frac{30}{60}}{} = \frac{1}{2} \), \( \frac{\frac{150}{300}}{} = \frac{1}{2} \); and \( \frac{\frac{11}{20}}{} \approx \frac{1}{2} \), \( \frac{\frac{7}{15}}{} \approx \frac{1}{2} \)

- A fraction is equal to one quarter if its numerator is a quarter of its denominator (or the denominator is four times the numerator). A fraction is approximately equal to one quarter if the numerator is approximately a quarter of the denominator (or the denominator is approximately four times the numerator).
  For example, \( \frac{\frac{100}{400}}{} = \frac{1}{4} \), \( \frac{\frac{50}{200}}{} = \frac{1}{4} \); and \( \frac{\frac{11}{45}}{} \approx \frac{1}{4} \), \( \frac{\frac{7}{30}}{} \approx \frac{1}{4} \)

Find the exact value or closer approximation for each of the fractions below. Use the appropriate symbol, = (equal to) or \( \approx \) (approximately equal to), to indicate if the value you found is exact or an estimate.

\[
\begin{array}{ccc}
\frac{9}{36} & \frac{40}{75} & \frac{2}{105} \\
\frac{21}{19} & \frac{115}{230} & \frac{15}{28} \\
\frac{-3}{1000} & \frac{4 + (-4)}{16} & \frac{15}{16}
\end{array}
\]
Fraction Estimation

We are also able to estimate whether the value of a fraction is **over** (greater than) or **under** (less than) the estimate. We may do so by comparing the original fraction to a fraction equivalent to the exact value of the estimate. Easiest comparisons are those between fractions with equal numerators or denominators; we use this property to create the appropriate equivalent fraction to compare with the original fraction.

For example, \( \frac{1}{11} \approx \frac{1}{4} \) because \( 11 \approx 4 \times 3 \).

But is \( \frac{1}{11} > \frac{1}{4} \) or \( \frac{1}{11} < \frac{1}{4} \)? To find out, we need to create a fraction equivalent to \( \frac{1}{4} \) that can be easily compared to \( \frac{1}{11} \). Such a fraction must have either numerator equal to 3 or denominator equal to 11. It is easier to create an equivalent fraction with numerator equal to 3: \( \frac{1}{4} = \frac{3}{12} \).

Since \( \frac{3}{11} > \frac{3}{12} \), we can conclude that \( \frac{1}{11} > \frac{1}{4} \).

Find the closest estimate to each fraction, and compare the actual values of the fraction and the estimate. Show the equivalent fraction used.

\[
\begin{array}{cc}
\frac{4}{7} & \frac{5}{22} \\
\frac{19}{20} & \frac{5}{12}
\end{array}
\]

We can use estimation when we perform operations with fractions. For example, \( \frac{4}{9} \) of 20 should be a number a less than \( \frac{1}{2} \) of 20 (since \( \frac{4}{9} < \frac{1}{2} \)). Indeed, \( \frac{4}{9} \times 20 = \frac{80}{9} = 8.888 < 10 \).

Estimate the value of “\( \frac{1}{10} \) of 24,” and predict whether the value of the exact answer is over or under your estimate; then, find the exact answer, and verify your prediction.

**Estimate:**

**Exact Answer Should Be**

Over / Under

**Exact Answer:**
Readings (Practice)  

**Chapter 7: Geometry**

**7.R.2. READING ACTIVITY 2 – Page 1**  
Name: ____________________

Right Triangle Trigonometry

Given a right triangle and one of its acute angles $t$, the **trigonometric functions** sine, cosine, and tangent of $t$ are defined as the ratios of the lengths of the sides of the right triangle:

\[
\text{Sine of } t: \quad \sin t = \frac{\text{opposite}}{\text{hypotenuse}}
\]

\[
\text{Cosine of } t: \quad \cos t = \frac{\text{adjacent}}{\text{hypotenuse}}
\]

\[
\text{Tangent of } t: \quad \tan t = \frac{\text{opposite}}{\text{adjacent}}
\]

To find the trigonometric functions of angle $t$ in the right triangle below, we need to find all sides of the right triangle. We can find $c$ using the Pythagorean Theorem: $a^2 + b^2 = c^2$

\[
c^2 = 3^2 + 4^2 = 9 + 16 = 25 \quad \Rightarrow \quad c = \sqrt{25} \Rightarrow c = 5
\]

Therefore,

\[
\text{Sine of } t: \quad \sin t = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a}{c} = \frac{3}{5}
\]

\[
\text{Cosine of } t: \quad \cos t = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{b}{c} = \frac{4}{5}
\]

\[
\text{Tangent of } t: \quad \tan t = \frac{\text{opposite}}{\text{adjacent}} = \frac{a}{b} = \frac{3}{4}
\]
Right Triangle Trigonometry

Find the trigonometric functions of angle $t$:

- **Sine of $t$:** $\sin t = \frac{a}{c} = \frac{8}{13}$
- **Cosine of $t$:** $\cos t = \frac{b}{c} = \frac{6}{13}$
- **Tangent of $t$:** $\tan t = \frac{a}{b} = \frac{8}{6}$

Find the trigonometric functions of angle $t$:

- **Sine of $t$:** $\sin t = \frac{b}{c} = \frac{12}{13}$
- **Cosine of $t$:** $\cos t = \frac{c}{c} = 1$
- **Tangent of $t$:** $\tan t = \frac{b}{b} = 1$
Complete Lecture Notes

Ch 1 - Whole Numbers

Reading: 1.5.4 p. 24-50 (Shis) bring to next
1.5.5 p. 27-28 (Shis via email)

Homework: p. 33-35 (10 pks)

Use @ 1200 noon Rm 218 Math Club

Sets

\[
\begin{align*}
\text{Natural} & \{1, 2, 3, 4, \ldots\} & \text{Whole} & \{0, 1, 2, 3, 4, \ldots\} \\
\text{Integers} & \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\} & \text{Rational} & \{\frac{p}{q}, p, q \in \text{Integers}\} \\
\text{Irrational} & \{\text{not rational}\} & \text{Real Numbers} & \mathbb{R} = \{0, 1\}
\end{align*}
\]

Place Value

<table>
<thead>
<tr>
<th>Expanded Form</th>
<th>346, 879, 507</th>
</tr>
</thead>
<tbody>
<tr>
<td>300,000,000</td>
<td>+ 50,000,000</td>
</tr>
<tr>
<td>400,000,000</td>
<td>+ 8,000,000</td>
</tr>
<tr>
<td>2,000,000</td>
<td>+ 400,000</td>
</tr>
<tr>
<td>500,000</td>
<td>+ 50,000</td>
</tr>
<tr>
<td>20,000</td>
<td>+ 100</td>
</tr>
<tr>
<td>9,000</td>
<td>+ 300</td>
</tr>
<tr>
<td>700</td>
<td>+ 20</td>
</tr>
<tr>
<td>20</td>
<td>+ 1</td>
</tr>
</tbody>
</table>

Write In Expanded Form

\[
\begin{align*}
1,750,943 & = 1,000,000 + 700,000 + 50,000 + 40,000 + 10,000 + 900 + 40 + 3 + 1 \\
8,997 & = 8,000 + 900 + 90 + 7
\end{align*}
\]

Rounding - to a place value

<table>
<thead>
<tr>
<th>313,247,893</th>
<th>to the nearest million</th>
</tr>
</thead>
<tbody>
<tr>
<td>313,000,000</td>
<td>313,000,000</td>
</tr>
</tbody>
</table>

Operations \(+,-,\times,\div\)

(1) Add

\[
\begin{align*}
9,423 & + 4,586 \\
9,919 & + 4,987 \\
8,420 & + 5,890 \\
4,920 & + 3,987 \\
\end{align*}
\]

(2) Subtract

\[
\begin{align*}
8,967 - 41 & = 8,926 \\
8,967 - 91 & = 8,876 \\
8,967 - 79 & = 8,888 \\
8,967 - 1 & = 8,966
\end{align*}
\]

(3) Multiply

\[
\begin{align*}
3 \times 10 &= 30 \\
24 \times 10 &= 240 \\
17 \times 10 &= 170 \\
11 \times 10 &= 110 \\
8 \times 11 &= 88 \\
9 \times 11 &= 99
\end{align*}
\]
Page 9: Estimation and Magnitude (not included in this packet)
# Writing / Journal Entries

## Pre-Algebra – HOMEWORK 4 – Page 4

### Chapter 4: Fractions

Journal Entry – Chapter 4 – OPERATIONS WITH FRACTIONS: Explain how we add, subtract, multiply, and divide two fractions with different denominators. Give two examples in each case.

<table>
<thead>
<tr>
<th>Operation</th>
<th>What You Know (WYK)</th>
<th>Two Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subtraction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multiplication</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Division</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

## Pre-Algebra – HOMEWORK 5 – Page 6

### Chapter 5: Algebra

Journal Entry – Chapter 5 – ALGEBRAIC EXPRESSIONS: Explain each term or quantity below and give two examples for each.

<table>
<thead>
<tr>
<th>Process</th>
<th>What You Know (WYK)</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adding Like Terms</td>
<td>To add like terms,</td>
<td></td>
</tr>
<tr>
<td>Distribution</td>
<td>To distribute,</td>
<td></td>
</tr>
<tr>
<td>Simplifying Algebraic Expressions</td>
<td>To simplify an algebraic expression,</td>
<td></td>
</tr>
<tr>
<td>Simplifying Algebraic Fractions</td>
<td>To simplify an algebraic fraction,</td>
<td></td>
</tr>
</tbody>
</table>
## Reinforcing Multiplication

### Chapter 1: Whole Nos
- Multiply 4×7
- Divide 28÷7
- Find the quotient and remainder of the division 30÷7
- Estimate 389×72,425

### Chapter 2: Integers
- Multiply (-4)×7
- Divide 28÷(-7) and check your division

### Chapter 3: Decimals
- Multiply 0.04×0.07
- Divide 28÷0.007
- Estimate the integer product 3.89×7.2
- Estimate the integer quotient 29.3÷7.2

### Chapter 4: Fractions
- Add 1/7 + 3/4
- Subtract 5/7 – 1/4
- What is ¾ of 28?
- Multiply (5/7)(3/4)
- Divide (3/4) ÷ 7
- What is 4/5 of 35?
- Change 6/7 to an equivalent fraction with denominator 28
- What is 7/9 of 36?
- Change 4/5 to an equivalent fraction with denominator 35

### Chapter 5: Algebra
- Distribute 4(7x + 5y)
- Solve for x: 7x = 280
- Find the average rate of change from (5,3200) to (12,6000)
- A train travelled 280 miles in 4 hours. What was the average speed of the train?

### Chapter 6: Ratios & Proportions
- Write a ratio and simplify: 4 oranges cost 28 cents
- Solve for x: 7/5 = 28 / x
- The scale of a map is 7cm : 30 miles. What is the actual distance of two cities which are 28cm apart on the map?
- The scale of a map is 28mm : 60 miles. What is the actual distance of two cities which are 7cm apart on the map?
- I read 7 pages in 15 minutes. At this rate, how many pages can I read in one hour?

### Chapter 7: Geometry
- The two triangles are similar.
- Find the perimeter and area of each triangle
  - 3cm
  - 4cm
  - 28ft
- The length of a rectangle is 3 inches longer than the width.
- The width is 4 inches. Find the area of the rectangle.
- The length is 5 inches. Find the volume of a cylinder with radius 2mm and height 7mm.
- Give your answer in terms of π.

### Chapter 8: Percents
- At a 30% blow out sale, a TV sold for $280. What was the original tag price?
- 7 out of 28 cars in the parking lot are grey. What percent of the cars in the lot are grey?
- Registration in Rec Camp costs $70. Of that, $28 is for purchase of uniforms and camp supplies. What percent of the camp registration is for uniforms and supplies?

### Chapter 9: Whole Nos
- I planted bell peppers last summer and over the course of a week, I picked 3,5,6,2,2,3,7 peppers from my garden. Find the average number of peppers per day picked.
- If the probability it will snow in February is ¾, how many days do we expect to have snow in February?
Reinforcing Addition

**Chapter 1:** Add in groups of 10
Use Gauss’ method to add $1 + 2 + 3 + \ldots + 49 + 50 + 51 + \ldots + 97 + 98 + 99$

**Chapter 2:** Add $(-4)+(-5)+3+1+(-6)+7 = [(-4)+(-5)+(-6)]+[3+1+7] = (-15)+11 = -4$
Find an integer factor pair of 24 that adds to –10

**Chapter 3:** Estimate the integer sum for $3.2 + 6.9 + 3.2 + 6.9$

**Chapter 5:** Find all possible pairs of natural numbers $a$ and $b$ such that $a + b = 10$

**Chapter 8:** If I get 40% discount, what is the percent I pay?
If I lost 10% weight, what percent of my original weight have I kept?
If 80% of my students take English, what percent do not take English?

Estimation

**Chapter 1:** Estimate the product $389 \times 72,425$ and provide its magnitude
Estimate the closest whole value for $\sqrt{15}$

**Chapter 2:** Estimate the product $-389 \times 72,425$

**Chapter 3:** Estimate the integer product $3.89 \times 7.2$
Estimate the integer quotient $29.3 \div 7.2$

**Chapter 4:** Fraction Estimation (to 0, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ and 1)
Fractions less than 1 or greater than 1

**Chapter 7:** Estimate the circumference and area of a circle with diameter 10ft; then, pick the correct value from a list of possible values.
Between similar figures, what is the ratio of the perimeters? The ratio of the areas?

**Chapter 8:** Percent estimation (to 0%, 25%, 50%, 75%, 100%)
Using 10% and 1% to calculate 35% of 80

**Chapter 9:** Is it possible for the average of 20 quiz scores ranging from 9.3 to 22 to be 6? 23? 22?
Estimation

8.R.4. READING ACTIVITY 4 – Page 1   Name: ______________________

Estimation with Percents

Percents are fractions applied to a quantity considered a unit.

100% of a quantity is the whole quantity  (100% = 1)
50% of a quantity is half of the quantity  (50% = \( \frac{1}{2} \))
25% of a quantity is a quarter of the quantity  (25% = \( \frac{1}{4} \))
10% of a quantity is one-tenth of the quantity  (10% = \( \frac{1}{10} \))

To estimate, we may approximate
- the percent to a familiar, or easily calculated, percent
- the quantity (unit) to a rounded quantity
- both the percent and the quantity as described above

For example, to estimate 22% of 48, we could think in any of the following ways:

- **Approximate the Percent:**
  22% is a little less than 25%, and 25% of 48 is one-fourth of 48 (12)
  Therefore, 22% of 48 is a little less than 12: \( 22\% \cdot 48 \approx 12 \)

- **Approximate the Percent:**
  22% is a little more than 20%; 10% of 48 is 4.8, and 20% of 48 is 4.8 + 4.8 = 9.6
  Therefore, 22% of 48 is a little more than 9.6: \( 22\% \cdot 48 \approx 9.6 \)

- **Approximate the Quantity:**
  48 is approximately 50, and 22% of 50 is 11
  Therefore, 22% of 48 is a little less than 11: \( 22\% \cdot 48 \approx 11 \)

- **Approximate Both:**
  48 is a little less than 50, and 22% is a little less than 25% (\( \frac{1}{4} \))
  Therefore, 22% of 48 is less than \( \frac{1}{4} \) of 50: \( 22\% \cdot 48 \approx 12.5 \)

- **Approximate Both:**
  48 is a little less than 50, and 22% is a little more than 20%; 20% of 50 is 10
  Therefore, 22% of 48 is approximately 20% of 50: \( 22\% \cdot 48 \approx 10 \)

*Note that, in the last case, we approximated up and down, and we are not sure if the actual answer is larger or smaller than our estimation.*

In the above example, we found
- A **lower bound** for our estimation: \( \text{LB} = 9.6 \)
- An **upper bound** for our estimation: \( \text{UB} = 11 \) (the smallest of all upper bounds)
- An **approximate value** for our answer: \( \text{AV} = 10 \)
Metacognitive Overlay

10.9. PROBABILITY & STATISTICS REVIEW

- **9.A) Event & Sample Space**
  List the sample space of rolling a number cube: ______

- **9.B) Probability, Impossible & Certain Events**
  What is the probability of rolling 1? ___

- **9.C) Expectance of Occurrence**
  How many times do we expect 1 if we roll 60 times? ___

- **9.D) Empirical Probability**
  If 70% of our customers like chocolate chip cookies, how many of the 40 cookies we bake should be chocolate chip? ___

- **9.E) Statistics, Populations, Samples, Random Sampling, Raw Data**
  What is the population for testing performance of microchips? ________

- **9.F) Frequency Distribution**
  Construct a frequency distribution for the raw data:
  3,4,4,5,6,5,5,3,5,4,3,7,5,4,5,4,3,7,6,7,8,3,8,7 ________

- **9.G) Sample Size**
  What is n for the data above? ______

- **9.H) Histogram**
  Draw a histogram for the data above ______

- **9.I) Measures of Central Tendency**
  List three measures of central tendency ________

- **9.J) Arithmetic Mean**
  Find the mean of the data above _______

- **9.K) Median**
  Find the median of the data above _________

- **9.L) Mode**
  Find the mode of the data above _________

- **9.M) Range**
  Find the range of the data above _________

10.R.1. READING ACTIVITY 1 – Page 1  Name: _______________________

Review of Items Missed

In the space below, and in consultation with your instructor, select one of the items you did not answer correctly during the review, write notes, and solve all exercises on the back.

Title:

Notes:

Solved Example: