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Piecewise Functions
Underappreciated, but Oh So Important
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Mathematics for AAS Programs Committee
Lee Wayand, Columbus State Community College
lwayand@cscc.edu
Stefan Baratto, Clackamas Community College
sbaratto@clackamas.edu
**Piecewise Functions: Drug Profile ABCIXIMAB**

- Hemorrhagic stroke and intracranial bleeding
- Thrombocytopenia
- Other adverse effects (incidence greater than 1 percent):
  - Cardiovascular - Hypotension, Bradycardia, atrial fibrillation/flutter pulmonary edema
  - Central nervous system - abnormal thinking, dizziness
  - Genitourinary - urinary tract infection

**Notes on Administration:**
- Weight-based dosing of both abciximab and concomitant heparin is essential to decrease the incidence of major and minor bleeding episodes. Patients should be managed following an accepted, literature-based standard of practice.
- Abciximab infusions must be administered through a low protein binding 0.2 or 0.22 micron in line filter.
- Infusion pump is required in management of abciximab infusions.

**Incompatibilities/Drug Interactions:**
- Other medications that effect hemostasis: thrombolytics, oral anticoagulants, nonsteroidal anti-inflammatory agents, dipyridamole, ticlopidine, clopidogrel.
- IV dextran in combination with abciximab results a high incidence of bleeding.

**Adult Dosage:**
Based on the EPILOG (NEJM. 1997; 336: 1689) and CAPTURE (Lancet. 1997; 349: 1429) studies:
- **Loading Dose:** 0.25mg/kg IV over 5 minutes slow IV push
- **Infusion:** 0.125mcg/kg/min (0.09mg/kg) if patient less than 80kg
  - 10mcg/min (7.2mg) if patient equal to or greater than 80kg
  - in 250ml D5W or NS at 21ml/hr for 12 hours

**Pediatric Dosage:**
Safety and efficacy in children have not been established.

**Routes of Administration:**
- Intravenous bolus followed by infusion

**Onset of Action:**
- A few minutes

**Peak Effects:**
- In less than 30 minutes
# Piecewise Functions: Electricity Bill

## PGE: Residential Meter

<table>
<thead>
<tr>
<th>Service Period</th>
<th>Meter Reading</th>
</tr>
</thead>
<tbody>
<tr>
<td>08/03/15</td>
<td>66,826</td>
</tr>
<tr>
<td>07/02/15</td>
<td>65,391</td>
</tr>
<tr>
<td>32 days</td>
<td>1,435 kWh</td>
</tr>
</tbody>
</table>

## Details

- **Basic Charge**  
  - 1 Residence × $10.00
- **Energy Use Charge**  
  - 800 kWh × 6.500¢
  - 635 kWh × 7.222¢
- **Transmission Charge**  
  - 1,435 kWh × 0.246¢
- **Distribution Charge**  
  - 1,435 kWh × 3.926¢

## Adjustments

- **RPA Exchange Credit**  
  - 800 kWh × (−0.855)¢
  - 635 kWh × 0.000¢
- **Regulatory Adjustments**  
  - 1,435 kWh × (−0.066)¢
- **Energy Efficiency Funding Adj**  
  - 1,435 kWh × 0.318¢
- **Energy Efficiency Customer Svc**  
  - 1,435 kWh × 0.007¢
- **Renewable Resource Adjustment**  
  - 800 kWh × 0.018¢
  - 635 kWh × 0.026¢
- **Decoupling Adjustment**  
  - 800 kWh × 0.034¢
  - 635 kWh × 0.047¢
- **Solar Payment Option Cost Recovery**  
  - 1,435 kWh × 0.047¢
- **Spent Fuel Adjustment**  
  - 1,435 kWh × (−0.127)¢
- **Capital Projects Adjustment**  
  - 800 kWh × 0.161¢
  - 635 kWh × 0.047¢
- **Boardman Decommissioning Adj**  
  - 1,435 kWh × 0.037¢

## Taxes and Fees

<table>
<thead>
<tr>
<th>Tax</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>City of Portland Tax</td>
<td>1.5%</td>
</tr>
<tr>
<td>Multnomah County Tax</td>
<td>0.073%</td>
</tr>
<tr>
<td>Low Income Assistance (fixed)</td>
<td>$0.84</td>
</tr>
<tr>
<td>Public Purpose Charge</td>
<td>3.0%</td>
</tr>
</tbody>
</table>
Details Section
1. What is the Basic Charge according to the Details section?
2. Find the total per kWh charge given in the Details section for the first 800 kWh used.
3. Find the total per kWh charge given in the Details section for the next 635 kWh used.
4. Construct a function showing the total cost (from Details) of the first 800 kWh used. Your input should be kWh, and your output should be cost; do not include the Basic Charge.
5. Graph the function constructed in Exercise 4.
6. Construct a function showing the total cost (from Details) of the next 635 kWh used. Your input should be kWh, and your output should be cost; do not include the Basic Charge.
7. Combine Exercises 4 and 6 to give a total cost function (from Details) for the energy used, including the Basic Charge.
8. Graph the function constructed in Exercise 7.

Adjustments
9. Compute the cost/credit per kWh for the first 800 kWh used from the Adjustments section.
10. Compute the total cost/credit for the first 800 kWh used from the Adjustments section.
11. Compute the cost/credit per kWh for the next 635 kWh used from the Adjustments section.
12. Compute the total cost/credit for the next 635 kWh used from the Adjustments section.
13. Give a total for the Adjustments section.
14. Modify the function constructed in the Details Exercise 4 to account for Adjustments.
15. Modify the function constructed in Details Exercise 7 to account for Adjustments.
16. Graph the function constructed in Exercise 15 of this section.

Total Cost of Electricity Use
17. Modify the function constructed in Adjustments Exercise 15 to account for Taxes and Fees.
18. Graph this function.
19. Determine the total Current Charges for energy use this month.
20. Use your function from Exercise 17 to determine the charges if a residence used 800 kWh in a month.
21. Use your function from Exercise 17 to determine the charges if a residence used 1,200 kWh in a month.
22. Find the Average kWh per day used this month.
23. Find the Average Cost per day this month.
24. Construct a function to determine the Average Cost per day.
Piecewise Functions: Grades

Instructor’s Notes

Student’s familiarity with these functions readily allow for several “teachable moments.” With spreadsheets, we immediately demonstrate the “need” for algebra. The function that translates a student’s raw score to a later grade gives an example of a function in which the range is naturally discrete and not a set of numbers, but is not “fake” either. It is an excellent example, in the sense that students readily understand it, of why we allow two different inputs map to the same output, but not allow a single input map to two different outputs. It provides an example where the “set of ordered pairs” might have a repeated element, but we still have a function because it consistently maps to the same output. We can use the grade function to demonstrate to students why we need people as well as technology. The technology can assign a letter grade to each student’s raw score, but the instructor decides whether to boost a student’s grade (e.g. a student whose raw score is 89.6). You can better demonstrate such events and even more in-depth piecewise defined functions using conditional formatting in the spreadsheet.

A student’s course grade is determined by adding up the total number of points the student earned that term and dividing by the total number of points available. This number is then multiplied by 100 to put the student’s raw score on a 100-point scale (assume there is no extra credit available).

A computer program assigns a letter grade to the student based on their raw score. It assigns the grade “A” for raw scores between 90 and 100; “B” for scores greater than or equal to 80, but less than 90; “C” for scores greater than or equal to 70, but less than 80; “D” for scores greater than or equal to 60, but less than 70; and “F” for scores less than 60.

a. What letter grade would be assigned to a student whose raw score is 92?
b. What letter grade would be assigned to a student whose raw score is 96?
c. Consider a set of students with raw scores 74, 86, 92, 86, and 96. What grades would be assigned to these five students?
d. Write the raw scores and letter grades from (c) (above) as ordered pairs.
e. In this context, are letter grades a function of raw scores?
f. Are raw scores a function of letter grades?
g. Use the definition of a function to justify your answers to (e) and (f) (above).

Instructor’s Note: On the left, HW 3 was due the day these were distributed, so if student’s had completed HW 3, it’s included, if not, then it’s omitted.
# Piecewise Functions: Tuition

This term, at Clackamont College, tuition for in-state students is calculated as shown.

<table>
<thead>
<tr>
<th>Credits</th>
<th>Tuition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-14</td>
<td>$50/credit</td>
</tr>
<tr>
<td>15-18</td>
<td>$750</td>
</tr>
<tr>
<td>Above 18</td>
<td>$50/credit</td>
</tr>
</tbody>
</table>

a. Find the tuition for a student taking 12 credit hours.

b. Fill in the table.

<table>
<thead>
<tr>
<th>Credits</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tuition</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

c. Graph the data from the table in (b).

d. Suppose a new student asked you how the tuition structure worked. Use complete sentences and write an explanation.

e. Now express the tuition as a function of the number of credit-hours taken (construct a piecewise function, expressed algebraically).

Instructor’s Note: To flesh out part (e) when using as a small-group activity in class.

\[
C(x) = \begin{cases} 
\text{Multiply the number of credit hours by $50} & \text{When the number of credit hours is between 1 and 15} \\
\text{Pay $750} & \text{When the number of credit hours is between 16 and 18} \\
\text{Multiply the number of credit hours by $50} & \text{When the number of credit hours is more than 18}
\end{cases}
\]

Now write these expressions in the language of algebra.
Fred is driving a car along a straight road. The graph shows the number of miles Fred is from home after \( x \) hours.

**a.** Use the graph to evaluate \( f (1.5) \).
   Interpret your result in the context of the application.

**b.** Use the graph to evaluate \( f (5) \).
   Interpret your result in the context of the application.

**c.** Find all values of \( x \) such that \( f (x) = 125 \).
   Interpret your result in the context of the application.

**d.** Find all values of \( x \) such that \( f (x) = 50 \).
   Interpret your result in the context of the application.

**e.** State the domain and range of this function.

**f.** Use complete sentences to describe the motion of the car.

**g.** Construct a formula for this function.

### Piecewise Functions: Additional Graphs

**School schedule**

<table>
<thead>
<tr>
<th>Subject</th>
<th>Room</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math</td>
<td>DH 310</td>
<td>9:00am - 10:20am</td>
</tr>
<tr>
<td>English</td>
<td>NH 212</td>
<td>10:30am - 12:50pm</td>
</tr>
<tr>
<td>History</td>
<td>WD 407</td>
<td>1:00pm - 1:50pm</td>
</tr>
<tr>
<td>French</td>
<td>WD 301</td>
<td>2:00pm - 3:20pm</td>
</tr>
<tr>
<td>Chemistry</td>
<td>NH 311</td>
<td>3:30pm - 4:20pm</td>
</tr>
</tbody>
</table>

**Graph**

- \( y = \text{Elevation}(t) \)
- Time:
  - 9:00 to 10:00
  - 11:00 to 12:00
  - 1:00 to 2:00
  - 3:00 to 4:00
  - 5:00

Graph of \( y = \text{Elevation}(t) \) from 9:00 to 5:00.
Discharge

\[ \text{Discharge (psi)} = \text{gallons/min} \]

Select a curve based on parameters (hose diameter, friction coefficients, etc)

Leakage rates:

\[ \text{Leakage (feet)} = \frac{C_{O_2}}{\text{min} \cdot \text{ft}^2} \]

The appropriate curve is determined by the CO$_2$ concentration.
**Piecewise Functions: Additional Exercises**

A private museum charges $40 for a group of 10 or less. More than 10 people must pay $2 per person for each person beyond the tenth, plus a $40 flat fee. The maximum group size is 50.

a. Sketch a graph of this situation (the cost to the group should be a function of the number of people in the group).

b. How much does a group of 16 pay?

c. Find a formula for the cost function.

A manufacturer charges $24 per toaster for orders of 25 or fewer toasters. Orders over 25 toasters receive a $5 per toaster discount.

a. Model the manufacturer’s price structure with a piecewise-defined function.

b. Determine the bill for an order of 15 toasters.

c. Determine the bill for an order of 30 toasters.

d. Determine the number of toasters at which an order should jump to 26 toasters (that is, the point at which it costs more to order 25 or fewer toasters than to order 26 toasters).

A certain printed circuit board fabricator makes small batches of boards. For orders of 25 boards or fewer, the price is $8.35 per board. For orders larger than 25 boards, the price is $5.80 per board. Regardless of the order size, there is a one-time set-up fee of $25 per order.

a. Express the cost of an order as a function of the order size.

b. What is the cost of a 60-board order?

c. What is the cost of a 15-board order?

d. At what point is it less expensive to order 26 boards rather than the number needed?

A company believes there is a linear relationship between the consumer demand for its products and the price charged. When the price was $3 per unit, the quantity demanded was 500 units per week. When the price was raised to $4 per unit, the quantity demanded dropped to 300 units per week. Let \( D(p) \) represent the quantity demanded per week at price \( p \).

a. Find a formula for \( D(p) \).

b. Evaluate and interpret \( D(5) \).

c. Evaluate and interpret \( D^{-1}(5) \).

d. Currently, the company can produce 400 units per week. What should the price be if they want to sell all 400 units (and maximize their revenue)?
A gas company charges an $11 per month service charge plus a usage fee (per therm). The first 40 therms cost $64.551¢ per therm. Gas used beyond the first 40 therms costs $72.481¢ per therm.

a. Give a function that determines a family’s gas bill.

b. What is the charge for using 30 therms in one month?

c. What is the charge for using 130 therms in a month?

A state uses a piecewise-defined function to determine the amount of income tax \( T(x) \) owed as a function of gross income \( x \) (for jointly filed returns). Assume both \( x \) and \( T(x) \) are in dollars.

\[
T(x) = \begin{cases} 
0.061x & \text{if } 0 \leq x \leq 15,000 \\
0.078x & \text{if } 15,000 < x \leq 50,000 \\
0.09(x - 50,000) + 4,038 & \text{if } 50,000 < x \leq 250,000 \\
0.099(x - 250,000) + 22,038 & \text{if } x > 250,000 
\end{cases}
\]

a. Determine the income tax owed for a household with a gross income of $35,396.

b. Determine the income tax owed for a household with a gross income of $125,682.

c. Determine the income tax owed for a household with a gross income of $347,060.

d. Proposition Flat Tax seeks to change the state’s current tax function so that all income is taxed at a constant rate of 8.8%. Write a formula for this Flat Tax function.

e. Determine the income tax that would be owed by households with gross incomes $35,396, $125,682, and $347,060 if Proposition Flat Tax passes.

f. Based on your calculations, do you believe these households will support Proposition Flat Tax? Explain.

Instructor’s Notes: If students live in a state with an income tax, they can be asked to research their state’s rates and construct their own model. If not, then use federal tax rates.

Give two other examples of “real-life” situations that can be modeled with a piecewise function.

Some ideas

Commissions for salespersons (graduated commission rates)
Taxi cab rates (over 17 mph, by distance; under 17 mph, by time)
Postage rates
Hourly wages (including overtime)
Insurance and deductibles
Natural gas rates (given in units, average temperature determines conversion factor for therms)

Not exactly, there are some other factors that determine the conversion factor

Example which is no longer relevant to our students

Multivariable function whose inputs are time (on phone), day of the week, and time of day.

Output is the charge, based on a ceiling function applied to the time, with piecewise conditions depending on day of the week and time of day.