

In Memoriam Kenneth J. Arrow (1921-2017)



Kenneth J. Arrow, the renowned economist who received the Nobel prize¹ in 1972 for “pioneering contributions to general economic equilibrium theory and welfare theory,” passed away on February 17, 2017. Arrow made path-breaking contributions in many other areas of economics and related fields, notably including social choice theory, health economics, and finance.

Arrow’s most important and enduring contributions to financial economics, by far, are in his 1953 paper, “The Role of Securities in the Optimal Allocation of Risk Bearing,” which introduces three closely related concepts that remain central to asset pricing theory: (a) the notion of state-contingent consumption streams, (b) the definition of a security as a claim to future state-contingent wealth, whose eventual payoff in each particular state of the world can be spent on the consumption of goods and services in that state, and (c) the concept of a “state price,” meaning the price of a claim to a unit of wealth in some particular future state of the world. A claim to a unit of state-contingent wealth is now commonly known as an “Arrow security.”

Arrow made other important contributions to financial economics. A prominent example is his work on risk aversion (see Arrow, 1965), including what is now called the Arrow-Pratt measure of risk aversion, among other

¹ This formal name of this award is the “The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel.” [Arrow shared this award with John R. Hicks.](#)

theoretical properties of preferences for uncertain consumption. However, the passage of years has clarified the much greater importance of Arrow's seminal 1953 paper, which essentially founded the modern theory of asset pricing.

Before Arrow's 1953 paper, uncertain consumption had generally been modeled by economists in terms of the properties of a probability distribution of future consumption levels. Instead, Arrow conceived the notion of a state-contingent consumption choice, represented by a list of bundles of goods, one such bundle for each possible future state of the world. Arrow's model foretells the modern definition of a stochastic consumption process, which maps a set of times and states of the world to the amounts of consumption in each state-time pair.

In a 2006 [interview of Arrow for the American Finance Association](#), I asked him how he came up with the notion of state-contingent consumption. In his response, Arrow provided a rich history of influences, including Daniel Bernoulli, Stanley Jevons, Jacob Marschak, John von Neumann and Oscar Morgenstern, Irving Fisher, Frank Knight, and Leonard (Jimmy) Savage. As a particular stimulus, Arrow cited John Hicks' (1946) *Value and Capital*, in which Hicks had treated a stream of consumption across time as a bundle of period-by-period consumption claims. Arrow saw how to extend Hicks' idea to handle uncertainty in a general-equilibrium setting.²

For his model of a financial security, Arrow avoided Gérard Debreu's later concept of markets for claims to future state-contingent consumption bundles. Instead, Arrow defined a security as a claim to future state-contingent units of account, for example "dollars," whose payoffs could then be used to purchase consumption bundles in each state. Not only was this a more realistic approach, it was the basis of another important new idea,³ "dynamic spanning."

² A common misperception is that the famous Arrow-Debreu (1954) paper, explicitly covers markets for state-contingent consumption. Strangely, the case of markets for uncertain consumption is not actually mentioned in the Arrow-Debreu paper, despite the fact that this paper was completed after Arrow's 1953 paper, on which Arrow had been working as early as the fall of 1951. In his AFA interview, Arrow basically ascribed this oversight to the manner which he and Debreu had collaborated, by postal mail while Arrow was traveling in Europe, and also to the primacy of their objective of proving the existence of market-clearing prices in a general equilibrium setting.

³ In his 2006 AFA interview, Arrow remarked: "Well, I guess I knew enough about the world to know that securities usually didn't pay commodities. There were futures contracts. So there was

The idea of economizing on the use of financial markets by using dynamic trade of wealth claims to span state-and-time contingent consumption is now a mainstay of asset pricing theory and also of financial industry practice. Although it appeared in only a primitive way in Arrow's 1953 paper, this idea was eventually developed and redeveloped into the approach that Bob Merton suggested for deriving the Black-Scholes option-pricing formula,⁴ and which is now the standard model for dynamic hedging and contingent claims pricing. That is, suppose one wants to create some future stream of state-contingent funds, which sits in a high-dimensional space of stochastic processes. Using only a small set of securities, one can then calculate an initializing portfolio of securities and how to re-trade the securities over time so as to replicate that target stream of contingent funding. By arbitrage reasoning, the initial cost of this replicating trading strategy must be the value of the target funding stream.⁵

From the perspective of the current development of the field of asset pricing, the ideas in Arrow's 1953 paper may now seem obvious. At the midpoint of the 20th century, however, Arrow's conceptions of state-contingent consumption, state pricing, and the role of financial markets and securities in achieving efficient consumption allocations, were revolutionary, and provided strong foundations that still support the basic paradigms of the field today.

For me and so many others across the broad field of economics, as witnessed at the [Academic Tribute to Arrow held at Stanford University](#) in October, 2017, Kenneth Arrow was a huge research influence and a tremendously important role model as both an academic and a human being.⁶

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some set of markets where you actually delivered goods. But I knew that most securities did not. It struck me that this reduced the number of markets. So there was an efficiency, due to the fact that securities are paid in money, which you then translate [into consumption goods]. It's a question of stating it. Anything you say there is obvious once stated. The problem is to state it."

⁴ Fischer Black and Myron Scholes (1973) cite Merton for his approach.

⁵ For a standard and nearly definitive treatment, see J. Michael Harrison and David Kreps (1979).

⁶ Arrow was a member of my Ph.D. dissertation committee. I took his doctoral course at Stanford University on general equilibrium theory. He was a close colleague for the duration of my career at Stanford until his death in February 2017.

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