Towards a Theory-based Measure of Purchase Variation

Els Gijsbrechts
University of Antwerp and the Catholic University of Mons

Katia Campo
Accuris Group

Patricia Nisol
Ghent University

Els Gijsbrechts is Professor of Marketing at the University of Antwerp and the Catholic University of Mons – FUCAM, Prinsstraat 13, 2000 Antwerp, Belgium, + 32 3 220 41 08, els.gijsbrechts@ufsia.ac.be. Katia Campo is Senior Manager Modeling and Analytics at Accuris Group Belgium, and part-time Marketing Professor at the University of Antwerp, Accuris and University of Antwerp – UFSIA, Plantin & Moretuslei 220 b10, 2018 Antwerp, Belgium, + 32 486 89 61 25, katia.campo@accuris.com. Paricia Nisol is reseach assistant at Ghent University – Marketing Department, Hoveniersberg 4, 9000 Ghent, Belgium, +32 9 264 35 28, patricia.nisol@rug.ac.be.

EXECUTIVE SUMMARY

Variety seeking behavior has gained quite some attention in the recent marketing literature (see, e.g., Baumgartner and Steenkamp 1996, Van Trijp et al 1996, and Sethuraman and Chintagunta 1998). Important mechanisms underlying this search for variation are *satiation* and *boredom* (Pessemier 1978, Pessemier and Mc Alister 1982). Following these mechanisms, the stimulation that consumers derive from using a product in a category depends on how different this product is from previously consumed units. Various 'behavioral' measures have been developed to quantify the degree of stimulation present in a consumer's purchase history. Limitations of existing measures, though, are that they are often incomplete, ad hoc, and applicable only to single purchase histories - that is, sequences where one product is bought from the category at a time.

The MOP Measure of Variation in Purchases
This paper presents MOP, a new Measure Of Purchase variation. Inspired by traditional loyalty indices (Guadagni and Little, 1983), the measure adapts and extends these indices to quantify the variation inherent in a consumer's purchase sequence. In line with theories of variety seeking behavior, the measure quantifies this variation based on a 'discounted' comparison of each category purchase with products previously adopted by the household. For a given sequence of purchase occasions n, the level of variation is given by:

$$MOP = \sum_n \frac{MOP_n}{N}$$

where MOP_n represents the degree of variation brought about by purchases on occasion n. This variation results from two possible sources: variation within the set of products bought on occasion n itself, and stimulation caused by differences between currently and previously bought items. Or:

$$MOP_n = \left[\sum_{i=1}^{\infty} DIF_{n\,n-i} * w_{n\,n-i}\right] + DIF_{n\,n} * w_{n\,n}$$

where $DIF_{n,n}$ refers to the average difference between products simultaneously bought in n, and $DIF_{n,n-i}$ represents the average difference between products bought in n and those purchased i shopping trips earlier. Both $DIF_{n,n-i}$ and $DIF_{n,n}$ are attribute-based, that is, expressed as average product differences across attributes k:

$$DIF_{n\ n-i} = \sum_{k=1}^{K} \frac{DIF_{n\ n-i}(k)}{K} \quad \text{and} \quad DIF_{n\ n} = \sum_{k=1}^{K} \frac{DIF_{n\ n}(k)}{K}.$$

These average product differences contribute more or less strongly to the consumers' variation level reached in n, to the extent that they cover more consumption units (larger quantities), and they are 'more closely together' in the purchase sequence. The weights $w_{n,n}$ and $w_{n,n-i}$ are designed in such a way that they capture the effect of both consumption importance and discounted experience.

The *MOP* measure is complete in the sense that it accounts for product differences on all relevant attributes in the category, like brand, flavor, form and package type.

It is widely applicable in the sense that it can accommodate multiple purchase sequences, where different products are purchased from the category simultaneously. An appealing feature of the measure is that it does not just offer summary insights into the consumers' overall degree of purchase variation, but automatically produces more information on the sources of variation. Consumers with a given variety drive may exhibit different purchasing patterns: while some buy different products from the category on a given purchase occasion, others primarily switch among items over time (Walsh 1995). The 'Temporal Decomposition' of MOP sheds light on this issue. It can be used to assess whether variation is primarily caused by the purchase of varied product sets simultaneously or on subsequent occasions. Also, various contributions emphasize that consumer's category perceptions are organized along product attributes, and that the degree of variety sought may differ between attributes (see, e.g. Boatwright and Nunes 1999, Hoch et al, 1998, Lattin and Mc Alister 1985, Lattin 1987). The 'Attribute Decomposition' of MOP allows to analyze along which product dimensions, i. e. brand, flavor, form or package type, variation is primarily sought.

Empirical Validation

Analysis of scanner panel data for four product categories confirms the measure's construct validity. At the same time, comparison with existing measures reveals when and why MOP provides superior results. It also indicates the importance of being able to deal with multiple purchase sequences, which are found to be inherently much more varied than typical single purchase sequences.

CONCLUSION AND DIRECTIONS FOR FUTURE RESEARCH

The MOP measure developed in this paper offers an advantage over existing measures from a theoretical and a managerial perspective. From a theoretical point of view, the measure is complete and developed in a way that is consistent with variety seeking behavior theories. From a managerial perspective, an advantage of MOP is that it can be used for all purchase sequences, even those including multiple purchases. A drawback of MOP is that it is more tedious to calculate than other purchase variation measures. This disadvantage has to be weighed against the additional insights gained by decomposing MOP. Temporal and Attribute decomposition might produce highly relevant managerial insights for promotional and assortment management, an area that we leave for future research.

TOWARDS A THEORY-BASED MEASURE OF PURCHASE VARIATION

Recent research on consumer buying behavior exhibits a great deal of interest in variety seeking behavior (see e.g. Baumgartner and Steenkamp 1996, Van Trijp et al. 1996, Seetharaman and Chintagunta 1998). Given the importance of variety seeking behavior, a number of behavioral measures have been developed to assess the consumer's degree of purchase variation in a product category, using households' purchase histories in the category (see e.g. Van Trijp and Steenkamp, 1990). Yet, existing behavioral measures have important limitations. The purpose of this paper is to present a new measure of variation in behavior, called MOP (Measure Of Purchase variation), that improves existing measures in two respects. *First*, it is more comprehensive and logical than previously developed measures, and applicable in a wider range of situations. *Second*, the measure can be readily split up into variation generated by simultaneous or subsequent purchases, and into variation along different product attributes. These decompositions may provide fundamental insights into the product's competitive market position, and prove particularly useful as a basis for marketing decisions like assortment composition, sales promotion planning and inventory management.

1 Mechanisms and Measures of Purchase Variation

1.1. Mechanisms of Purchase variation

From the marketing and psychological literature, we know that consumers engage in variety seeking behavior for different reasons. A first motive is satiation (Pessemier, 1978, McAlister, 1979, McAlister and Pessemier, 1982): consumption of several units of the same product may create an 'internal attribute stock' for some product attributes, which makes consumers want to switch away to items with different characteristics. Drinking a series of sweet soft drinks can, for example, produce an internal 'sugar inventory', and lead to a desire to switch to products with lower sugar content. Alternatively, variety seeking can result from the consumers' need to avoid boredom and to experience stimulation from the environment (see e.g. Raju, 1980, 1981, and 1984, Steenkamp and Baumgartner, 1992). For instance, watching the same movie repeatedly, can become boring and lead to an increase in appeal of alternative activities with higher stimulation potential. Yet, the loss of attractiveness resulting from boredom or satiation is temporary. This is true because, as a result of physical processes and fading experience, internal attribute inventories are depleted over time and items that were not consumed for some time gradually regain their initial stimulation potential.

Given the mechanisms just described, a proper way to quantify the variation in a purchase history is to confront items adopted at a given point in time with preceding purchases, thus accounting for the *temporal aspect of variation*. Comparisons with recently purchased items should receive a larger weight than comparisons with items further away in the purchase history. This notion of 'discounted comparisons' is based on satiation and stimulation theories and has gained widespread acceptance in the marketing literature (e.g. McAlister and Pessemier, 1982, Raju, 1981). In addition, the comparison should be attribute-based rather than a mere recording of whether the items are identical or not. The effect of internal attribute stocks (satiation) and boredom on the utility of items similar in nature to the one previously consumed implies that variety seeking is attribute driven : consumers' evaluations of an item depend on the features it shares with previously purchased items. The *structural difference* (degree of dissimilarity) between items bought must therefore be included in the measure. Finally, while assessing a household's average tendency to seek variation in a product category is worthwhile (Trivedi et al. 1994), valuable insights are gained from analyzing the *heterogeneity* in purchase variation across purchase occasions and attributes. Consumers with a given variety drive may exhibit divergent purchasing patterns: some will buy different products from the category simultaneously, others primarily switch among different items over time (see, eg, Walsh 1995). Also, various contributions emphasize that consumers' category perceptions are organized along product attributes (see, e.g. Fader and Hardie 1996, Boatwright and Nunes 1999, Hoch et al, 1998). The degree of variety sought by a consumer may vary over these attributes as a result of differences in ideal attribute levels and in the speed with which internal stocks are depleted (Lattin and McAlister 1985, Lattin 1987). Breaking down overall household variation into variation across shopping trips, or variation sought along different attributes, is likely to produce additional -managerially relevant- insights.

1.2. Measures of Purchase Variation

Existing behavioral measures of variation range from simple counts like the number of different brands or items bought by the household, over summary measures like Entropy, the Hirshman-Herfindahl index and the Variance in Quantities Consumed, to more complete indices like Pessemier and Handelsman's 'Index of Temporal Variety' (ITV) or the 'Varied Behavior Measure' (VBM) suggested by Handelsman (Pessemier and Handelsman 1984, Handelsman 1987). An interesting empirical evaluation of these measures can be found in Van Trijp and Steenkamp (1990). Table 1 summarizes the components, expressions, and drawbacks of the behavioral measures. Though useful as first indicators of purchase variation, existing measures suffer from a number of limitations. Disadvantages of the more complete measures are that they combine variation components in an ad hoc fashion (ITV), have low reliability (VBM) and cannot be applied to multiple purchase data (ITV, VBM). Yet, the degree of variety seeking behavior has been found to be positively related to the occurrence of multiple purchases. In addition, the methods cannot be 'decomposed' into more detailed indicators of variation along product attributes or into variation within versus between purchase occasions.

An interesting source of inspiration to assess time-based and attribute-driven variation in purchase behavior, are the 'purchase event feedback' or 'loyalty' variables introduced by Guadagni and Little (1983). These variables quantify a household's loyalty to a given brand or size, as an exponentially smoothed average of the household's past purchases (1) or non-purchases (0) of that brand or size. Even though these measures have a different focus - they concentrate on loyalty to an item, not on variety seeking behavior in a category – and are designed for single purchase sequences, they can be adapted and extended to quantify variation in (multiple) purchase sequences, as indicated below.

2 The MOP Measure of Variation in Purchases

2.1 General Structure

Consider a household's purchases over subsequent shopping trips in a given product category. On each purchase occasion *n*, the household makes a selection from the available *item*s in the product category ('item' refers to a specific brand and variety of the product category). The resulting *purchase set* is characterized by the number of *unit*s (packages) of each item that is included in the set. The *purchase set size* on occasion *n* equals the total number of units from the category purchased on that occasion. A purchase set may include units of only one item, or it may contain a combination of different items in the category. The former is denoted as a 'single purchase', the latter is called 'multiple purchase set', or simply 'multiple purchase'.

Figure 1 clarifies how the MOP measure is constructed. MOP_n represents the degree of variation brought about by purchases on occasion *n*. As indicated in Figure 1, this variation results from two possible sources: variation within the set of products bought on occasion *n* itself, and stimulation caused by differences between currently and previously bought items.

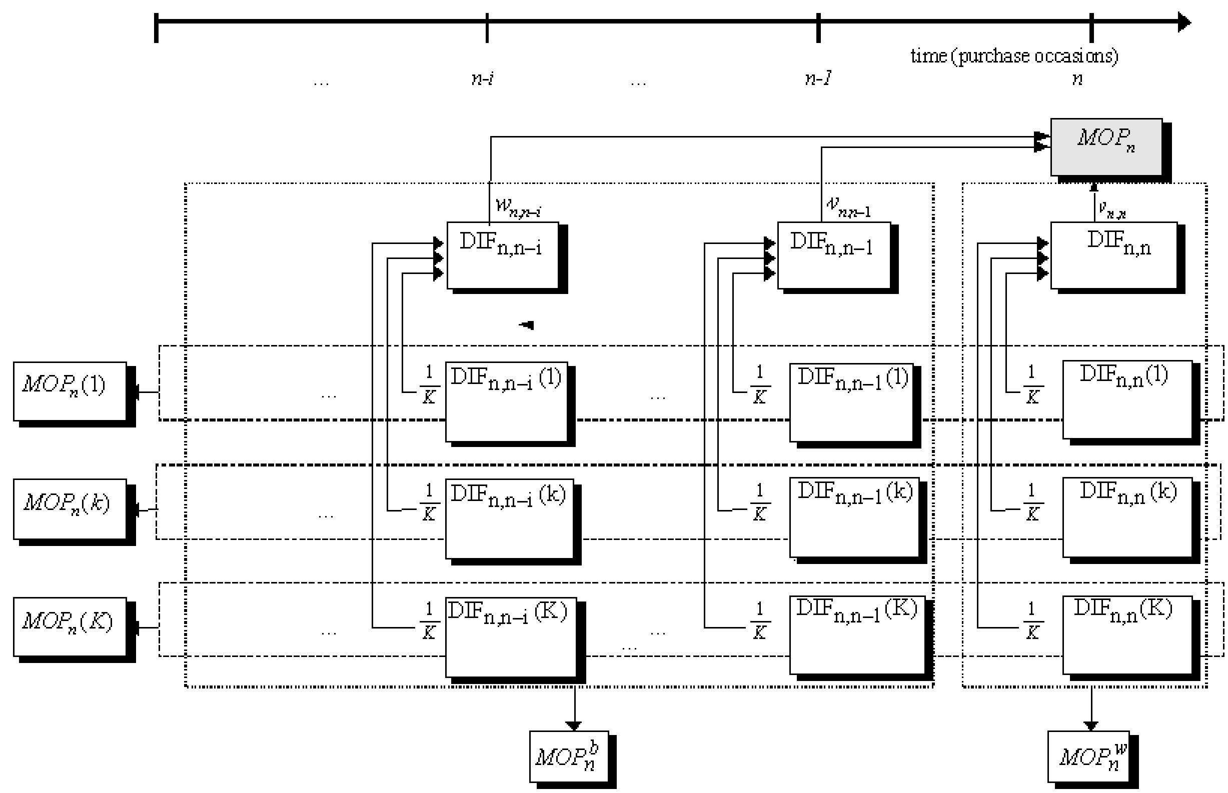
$DIF_{n,n}$ refers to the average difference between products simultaneously bought in *n*. If only one unit is bought on shopping trip *n*, $DIF_{n,n}$ is set to zero. $DIF_{n,n-i}$, in turn, represents the average difference between products bought in *n*, and those purchased *i* shopping trips earlier. These average product differences contribute more or less strongly to the consumers' variation level reached in *n*, to the extent that (i) they cover more consumption units (larger quantities), and (ii) they are 'more closely together' in the purchase sequence. The weights $w_{n,n}$ and $w_{n,n-i}$ in Figure 1 capture the effect of both consumption importance and discounted experience. As shown in Appendix 1, they depend on purchase set size, and decline as more units have been consumed 'in between' through appropriate use of a discount factor α. When consumers buy several items simultaneously, consumption nor 'purchase' order are known, and several consumption sequences may underly the purchase history. The weights $w_{n,n}$ for within set differences are constructed in such a way that the weighted within set variation approximates the average variation of all possible underlying consumption sequences. The interested reader is referred to Appendix 1 for more details. The resulting mathematical expressions are summarized in Table 2.

TABLE 1
Characteristics of Behavioral Purchase Variation Measures

Measure	Components	Mathematical Expression	Evaluation
NUM_i (NUM_b)	Number of different items (brands) bought		Weakly related to overall variation in behavior (Van Trijp and Steenkamp 1990). Does not account for structural variation, temporal variation (number of switches), purchase frequency
Hirshman Herfindahl Index	Distribution of purchases over available items	$HH = -\sum_{j=1}^{J}(p_j)^2$ p_j = share of item j in category purchases	Does not account for structural variation or temporal variation (number of switches)
Entropy	Distribution of purchases over available items.	$ENTR = \frac{ENT}{ENTMAX}$ $ENT = \sum_{j=1}^{J}(-p_j ln(p_j))$ $ENTMAX = \frac{L * ln(N^*)}{N^*}$ L = number of available items $N^* = min(L, N)$ N = overall category purchases	Does not account for structural variation or temporal variation (number of switches)
Varquan	Distribution of purchases over available items.	$VARQUAN = -\sum_{j=1}^{L}\frac{(x_j-\bar{x})^2}{L}$ x_j = quantity bought of item j $\bar{x}$ = average of x_j	Does not account for structural variation or temporal variation (number of switches)
ITV	Distribution of purchases over available items. Dissimilarity of chosen items. Number of switches	$ITV = PRD + PRE + RNB$ PRD = % of realized Dissimilarity PRE = % of realized Entropy RNB = Relative nonbunching	Ad hoc integration of the 3 variation components (structural variation, distribution of purchases and number of switches). Cannot be applied to multiple purchase data
VBM	Dissimilarity of chosen items. Time between successive purchases of the same item	$VBM = \sum_{n=2}^{N}\frac{D_n E_{j,m}}{n-1}$ D_n = relative realized structural variety $E_{j,m}$ = Experience factor	Low reliability (Van Trijp and Steenkamp 1990). Cannot be applied to multiple purchase data
MOP	Dissimilarity of chosen items. Variation within and between purchase sets. Discounted experience	see Table 2	Accounts for all variation components. Components are integrated in a meaningful way. Can be applied to multiple purchase data. Can be decomposed.

FIGURE 1
Structure of *MOP*$_n$

Figure 1: Structure of MOP_n



This results in the following expression for MOP_n, measuring the stimulation brought about by the purchase set bought on occasion n:

$$MOP_n = \left[\sum_{i=1}^{\infty} DIF_{n,n-i} * w_{n,n-i}\right] + DIF_{n,n} * w_{n,n} \qquad (1)$$

In most product categories, products can be described in terms of a small set of discrete attributes or dimensions (Fader and Hardie 1996, Boatwright and Nunes 1998, Broniarcyzk et al 1998). In line with this observation, the values $DIF_{n,n}$ and $DIF_{n,n-i}$ are constructed as summary measures of differences occurring on various product attributes. Similar to the approach taken by Fader and Hardie (1996) in modelling item choice, or by Boatwright and Nunes (1999) in assessing the variety of an assortment, we require these product attributes to be (i) relevant to the consumer ('consumer recognizable'), (ii) collectively exhaustive (applicable to every product in the category), and (iii) separable (representing separate dimensions). Given our interest in building a widely applicable measure of variety seeking, attributes to be included in

$DIF_{n,n}$ and $DIF_{n,n-i}$ should also be (iv) meaningful across a wide range of categories, and (v) represent dimensions along which variation is sought. The 1993 Food Marketing Report on variety and duplication identifies the following key product characteristics answering those requirements: brand, flavor, package form and type[1]. We therefore suggest to consider these attributes for constructing the MOP measure.

As consumers do not necessarily have the same need for variation along all these dimensions, it might be instructive to write $DIF_{n,n}$ and $DIF_{n,n-i}$, as a (weighted) sum of differences along the separate product attributes[2]. This is again illustrated in Figure 1, where $DIF_{n,n}(k)$ and $DIF_{n,n-i}(k)$ represent average product differences within and between purchase sets, resp., *along dimension k* (see Appendix 2 for more details).

Finally, the value of MOP_n assesses variation brought about by a particular purchase occasion *n*. As the consumers' need for variation may change over time (see e.g. Trivedi et al. 1994), averaging MOP_n over a number of subsequent occasions in the history provides a better picture of the household's basic variety drive in the product category:

$$MOP = \sum_n \frac{MOP_n}{N} \quad (2)$$

By construction, the average product differences $DIF_{n,n}$ and $DIF_{n,n-i}$ lie between zero and one, and so do the weights $w_{n,n}$ and $w_{n,n-i}$. Moreover, these weights sum to one over purchase occasions. It follows that the overall measure of variation, *MOP*, also lies between zero and one. MOP equals zero in case of complete 'absence' of variation: the consumer bought only one item from the category throughout his purchase history. Alternatively, it approximates one when the consumer has bought a sequence of maximally different items from a category.

In comparing MOP across categories, researchers may be interested not only in observed levels of purchase variation as such (quantified in MOP), but also in observed variation relative to what is attainable given the category's available assortment. Note that the MOP measure in equation (1) does not yet account for possible restrictions on a consumer's purchase variation imposed by the assortment at hand. When the number of available items in the product class is limited, even extreme variety seeking consumers will end up buying a product they have ever bought before, and MOP will be below one. In such instances, the researcher may be interested in using a 'relative' MOP (MOP^r), obtained by dividing the absolute MOP value by MOP^{max}, the maximally attainable level of purchase variation given assortment characteristics. Appendix 3 provides details on the computation of MOP^{max} . The value of the relative measure MOP^r becomes 1 if the consumer exhibits as much variation in behavior as is possible within the limitations of the assortment (extreme variety seeking behavior), and is equal to 0 when the household always purchases the same item (extreme loyal behavior). This relative measure may be particularly useful for the comparison across categories with very different assortment depths.

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1 Note that the FMI report also articulates a fifth key characteristic, namely size. Given our interest in variety seeking, we decided – inspired by one reviewer's suggestion – not to include size in our measure, as changes in size alone cannot constitute a meaningful source of stimulation or help avoid satiation. Package form and type, on the other hand, can be relevant or not for measurement of variety seeking behavior, depending on whether this characteristic is expressive rather than functional in nature. Whereas expressive item characteristics – such as an attractive package design – may provide some type of stimulation, functional package characteristics are not likely to influence the consumer's boredom or satiation level (see Hirschman and Holbrook 1982).

2 In combining differences along various attributes, two options are available. One is to use equal weights for all attributes. The other is to assign differential weights reflecting the importance of the attributes to the consumer – where attribute saliences could be obtained through primary data collection among consumers, or based on subjective assessment. Both approaches are valid, but serve a somewhat different purpose. In the 'equal weights' option, the MOP measure simply summarizes observed purchase changes in attribute space. It then lets the data 'speak for themselves' in the sense that MOP decomposition along attributes shows how much variety seeking actually occurs along different dimensions. As the same weights are used across consumers and categories, this MOP value allows to compare levels of purchase variation – whatever the underlying reason. In the differential weights option, the summary MOP measure intends to maximally reflect the stimulation experienced by a particular consumer in a particular category, by incorporating tailored a priori info on attribute salience. Note that previous measures including structural variation, like ITV and VBM, also necessitate a choice of attribute weights. To our knowledge, empirical applications of these measures have used the 'equal weights' approach, an option also taken in our empirical section.

2.2 Measure Decomposition

Higher levels of variation in behavior can be the result of consumers buying different items on subsequent occasions, and/or of buying different items simultaneously. The overall MOP can be conveniently decomposed into variation caused by differences within or between purchase sets:

$$MOP = MOP^{w} + MOP^{b} \quad (3)$$

where MOP^{w} groups the components of *MOP* related to within purchase set differences and their associated weights, and MOP^{b} summarizes between purchase set differences and their saliences (see Table 2 for more details).

Alternatively, a distinction can be made between variation components along various product attributes:

$$MOP = \sum_{k} MOP(k) \quad (4)$$

where *MOP(k)* comprises the components of *MOP* linked with attribute *k*.

TABLE 2
Notation and Mathematical Expressions of the MOP Measure and its Components

Notation	**Description &Mathematical Expression**
K	Number of attributes, k:1--> K
N	Number of purchase occasions (purchase sets) in the purchase history, n:1-->N
α	Discount factor (between zero and one)
P_n	Number of units (packages) bought by the household on occasion n (size of the purchase set bought in n)
Ps_n	Index set, comprising all items purchased on occasion n
$\delta_{p,q}(k)$	Dissociation between items p and q on attribute k, equals 1 if items p and q differ on attribute k, and 0 otherwise
cum(n,i)	number of units bought after time n-i (or, going back in time, before reaching occasion n-i) $$cum(n,i) = \sum_{j=0}^{i-1} P_{n-j}$$
$DIF_{n,n-i}(k)$	Difference in attribute k between units bought on occasion n, and units bought on occasion n-i $$DIF_{n,n-i}(k) = \frac{\sum_{p \in PS_n} \sum_{q \in PS_{n-i}} \delta_{p,q}(k)}{P_n * P_{n-i}}$$

TABLE 2 (cont)
Notation and Mathematical Expressions of the MOP Measure and its Components

DIF $_{n,n}$(k)	Difference in attribute k between units simultaneously bought on occasion n $$DIF_{n,n}(k) = \frac{\sum_{p \in PS_n} \sum_{q \in PS_n, q>p} d^2_{p,q}(k)}{P_n * (P_n - 1)/2}$$
$DIF_{n,n-i}$	Average difference between units bought on occasion n, and units bought on occasion n-i, over all attributes $$DIF_{n\ n-i} = \sum_{k=1}^{K} \frac{DIF_{n\ n-i}(k)}{K}$$
DIF $_{n,n}$	Average difference within purchase set n, over all attributes $$DIF_{n\ n} = \sum_{k=1}^{K} \frac{DIF_{n\ n}(k)}{K}$$
MOP_n	Variation brought about by purchase occasion n $$MOP_n = \left[\sum_{i=1}^{\infty} DIF_{n\ n-i} * w_{n\ n-i}\right] + DIF_{n\ n} * w_{n\ n}$$
$w_{n,n}$	Weights for units bought simultaneously on occasion n $$w_{n\ n} = \frac{(P_n - 1 - \frac{\alpha}{(1-\alpha)} . (1 - \alpha^{P_n - 1}))}{P_n}$$
$w_{n,n-i}$	Weights for comparison of units bought on occasion n, to units bought previously (i purchase occasions ago) $$w_{n\ n-i} = \frac{\alpha^{cum(n,i) - P_n} . (1 - \alpha^{P_n}) . (1 - \alpha^{P_{n-i}})}{(1 - \alpha) P_n}$$
MOP^b	Temporal variation in the purchase history (brought about by differences between purchase sets) $$MOP^b = \frac{\sum_{n=1}^{N} \sum_{i=1}^{\infty} DIF_{n\ n-i} * w_{n\ n-i}}{N}$$
MOP^w	Simultaneous variation (caused by differences within purchase sets) $$MOP^w = \frac{\sum_{n=1}^{N} DIF_{n\ n} * w_{n\ n}}{N}$$

TABLE 2 (cont)
Notation and Mathematical Expressions of the MOP Measure and its Components

MOP(k)	Variation observed in the purchase history along attribute k $$MOP(k)=\sum_{n=1}^{N}\frac{MOP_n(k)}{N}=\sum_{n=1}^{N}\left(\frac{1}{N*K}\right)\cdot\left[w_{n,n}(DIF_{n,n}(k))+\sum_{i}w_{n,n-i}(DIF_{n,n-i}(k))\right]$$
MOP	Overall variation in the purchase history $$MOP=\frac{1}{N}*\left[\sum_{n=1}^{N}\left(\sum_{i=1}^{\infty}DIF_{n,n-i}*w_{n,n-i}+DIF_{n,n}*w_{n,n}\right)\right]$$ $$=MOP^{b}+MOP^{w}=\sum_{k}MOP(k)=\sum_{n}\frac{MOP_n}{N}$$

Turning back to the criteria used for evaluating the existing measures in Table 1, we conclude that MOP is more complete than the ‘simple’ variation measures. It accounts for both *temporal* variation (the distribution of purchases over different items as well as their sequence) and *structural* variation (the degree of dissimilarity between products bought). Moreover, it integrates these variation components in a more meaningful way than the existing ‘complete’ measures like ITV and VBM, and can be readily applied to multiple purchase data. Also, being confined to the (0,1) range makes the measure easily interpretable. The last row of Table 1 summarizes the properties of the MOP measure.

3 Empirical Validation

3.1 Data

Scanner panel receipt data are available for a sample of customers of a large supermarket chain, for 29 consecutive weeks and 4 product categories with a quite deep assortment (from 62 to 231 items). Of the potentially relevant attributes (brand, package type, flavor and form, see section 2.1), only those are retained along which items actually differ in the category’s assortment (see Boatwright and Nunes 1999 for a similar approach). For margarine and milk, available products differ in brand, and form (light/not light, and low/medium/high fat respectively)[3]. For crackers and jam, items belong to different brands, flavors and forms (light/not light; and jam/jelly and light/not light respectively). Only households with a minimum number of purchases (10 units) are retained for analysis in a product category, leading to 149, 88, 38 and 41 usable households for margarine, milk, crackers and jam respectively.

3.2 Choice of α.

The discount factor α used to calculate the weights in MOP is conceptually 'identical' to the loyalty constants in classical brand and size loyalty measures (Guadagni and Little 1983)[4]. A wide range of earlier studies lead to surprisingly similar ‘optimal’ values for the smoothing constant (see, e.g., Guadagni and Little 1983, and Lattin 1987)[5]. These studies point to α levels close to .7, and show that model outcomes are relatively insensitive to changes in the loyalty constants. Capitalizing upon these findings and a sensitivity analysis of our measure and dataset, we set α equal to .7. Comparison of results obtained with α equal to .7 with those obtained with α equal to .5 demonstrates that MOP values are not overly

3 Differences in package type (aluminum foil or not) were not taken into account, because they are functional rather than expressive in nature. Changes from one package type to the other are therefore likely to have been incited by another motivation than boredom or satiation (cf. note 1).

4 In fact, in the simple case with only two items in the category, one attribute, and a single purchase sequence, the MOP measure reduces exactly to these loyalty variables.

5 In these papers, the loyalty variables are introduced in choice models, and α is chosen so as to maximize model fit.

sensitive to changes in the discount factor: the 'average' level of MOP in each category remains largely unaffected, and correlations between household MOP values for α =.5 and α .=.7 range from .976 (jam) to .990 (margarine).

3.3 Multiple Purchases and Overall Purchase Variation.

In the data set, multiple purchase sets account for up to 24% of all purchase occasions (crackers). Moreover, between 29% (milk) and 70% (margarine) of consumers engage in at least one multiple purchase. Disregarding sequences containing a multiple purchase thus leads to a considerable loss of observations. More importantly, though, omitting such observations produces a biased picture of purchase variation in a product category.

TABLE 3
Average Level of Purchase Variation for Respondents Without (A) and With (B) Multiple Purchases

	margarine		**milk**		**crackers**		**jam**	
	(a)	**(b)**	**(a)**	**(b)**	**(a)**	**(b)**	**(a)**	**(b)**
ENT	.67	1.20	.36	.89	.86	1.53	1.01	1.82
HH	-.63	-.39	-.80	-.51	-.56	-.30	-.49	-.21
NUMb	2.02	3.32	1.72	3.02	1.74	2.57	1.62	2.40
NUMi	2.70	5.15	1.90	3.68	3.21	6.76	3.75	7.40
ITVRK	0.96	1.46	0.73	1.29	1.21	1.59	1.38	1.69
MOP	.16	.36	.09	.31	.24	.29	.09	0.22

This is clear from Table 3, which compares average levels of purchase variation for sequences with and without multiple purchase occasions, and for different measures. The ITVRK measure in Table 3 is a generalized version of Pessemier and Handelsman's ITV index adapted to accommodate for multiple purchase sequences[6]. The table reveals that, depending on the product category and variation measure, purchase variation in multiple purchase sequences is found to be between 1.5 and 2 times as large as in single purchase sequences. We therefore conclude that MOP's ability to treat multiple purchases offers a true advantage over measures that do not possess this property.

3.4 Comparison of MOP with other Overall Measures of Purchase Variation

Table 4 reports on correlations between MOP and other measures of purchase variation[7]. All correlations in the table are positive and significantly different from zero, supporting the convergent validity of MOP. At the same time, the correlation coefficients between MOP and ENT, and between MOP and ITVRK suggest that MOP does not perfectly coincide with other measures.

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[6] ITVRK is defined as the sum of ENTR (the relative entropy measure), AD (the average difference between all items in a purchase history; this measure is proportional to Pessemier and Handelsman's structural variation component) and SWITCH (which is an extension of Pessemier and Handelsman's indicator of nonbunching, adapted to purchase sets involving more than one item). To compute SWITCH, we determine, for each purchase set, the number of items it has in common with the immediately preceding set, and divide this number by the minimum of both set sizes. The value thus obtained is averaged over all sets in the purchase history, and subtracted from 1. If all sets are of size 1, SWITCH equals Pessemiers and Handelsman's degree of nonbunching. If some sets include more than 1 unit, SWITCH accounts for switches between subsequent purchase occasions, but still ignores within set switching.

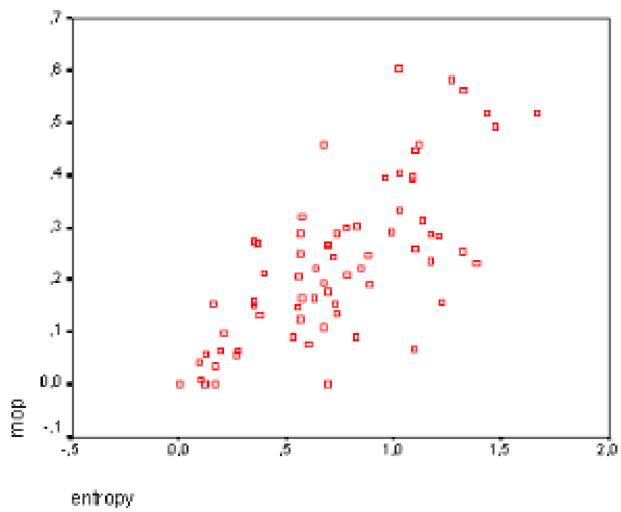
[7] In our data set, MOP^{max} approximates 1 in all categories, such that absolute (MOP) and relative (MOP^{r}) measures are almost equal. We therefore concentrate on the (absolute) MOP measure in the remainder of the discussion.

Table 4: Correlation Between MOP and Alternative Measures of Purchase Variation

	Margarine	Milk	Crackers	Jam
ENT	0.547	0.823	0.804	0.887
HH	0.587	0.807	0.776	0.831
NUMb	0.563	0.737	0.588	0.736
NUMi	0.422	0.730	0.682	0.817
ITVRK	0.737	0.773	0.834	0.786

Scatterplots of MOP and these other measures confirm this[8]. For illustrative purposes, figures 2 to 5 picture the link between MOP and entropy, each observation in the plots corresponding to a particular household (purchase history) in the product category considered.

FIGURE 2
Scatterplot of MOP and Entropy for Milk



8 The other measures - NUM_b, NUM_i and Hirshman-Herfindahl - are not analyzed here. Count measures like NUM_b, NUM_i are overly simple (see, e.g. Van Trijp and Steenkamp, 1990), and would not constitute a 'fair' comparison. Entropy and Hirschman-Herfindahl turn out to be very strongly correlated, such that only one of them is further considered: we choose entropy because it is more established, and offers a better comparison to *ITVRK*.

FIGURE 3
Scatterplot of MOP and Entropy for Crackers

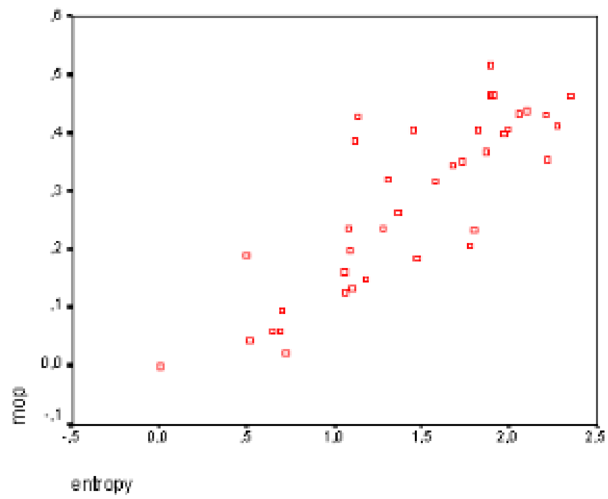


FIGURE 4
Scatterplot of MOP and Entropy for Margarine

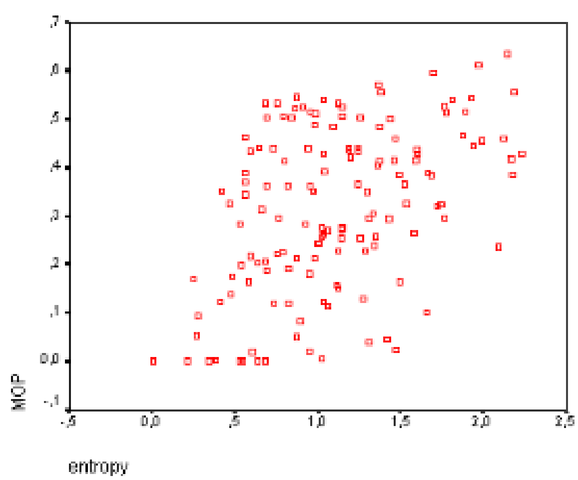
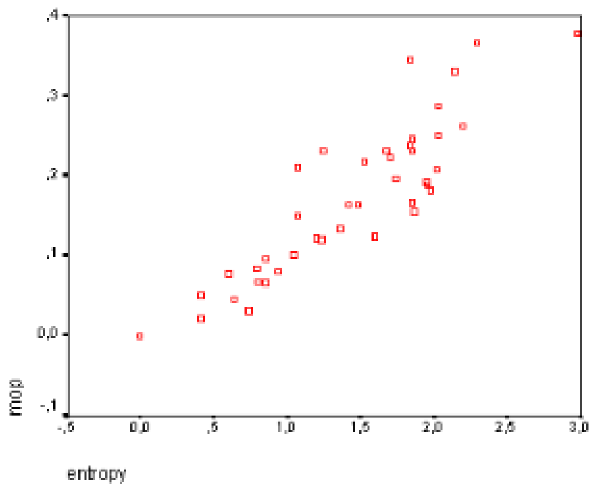


FIGURE 5
Scatterplot of MOP and Entropy for Jam



The plots confirm a strong association between *ENT and MOP* for crackers and jam. However, substantial deviations do occur for margarine and milk. Closer analysis of 'deviant' households points to two major reasons behind these differences. First, the entropy measure simply records whether items are the same or different, without accounting for the *degree* of dissimilarity. The failure of entropy to account for structural differences may distort the link between MOP and entropy, especially in product categories like margarine and milk where some households switch between similar, and others between very different items. This is illustrated in Table 5, which provides values of *ENTR*, *ITVRK* and *MOP* for fictitious purchase histories. *ENTR* fails to pick up the fact that items bought in sequence 2 are more diverse than those in sequence 1.

A second reason why household MOP's differ from entropy relates to the temporal aspect of the measures. While entropy computes one level of variation over the whole purchase sequence, MOP attaches more weight to item differences occurring 'close together' in the purchase history. Going back to the illustrative purchase histories in Table 5, we observe that while *MOP* accounts for the increased switching activity in sequence 3 compared to 2, *ENTR* erroneously assigns the same variation to these sequences, ignoring the temporal difference. In our empirical application, the temporal difference becomes more outspoken for margarine and milk where the purchase histories are typically longer.

ITVRK does involve some indication of structural differences (through the average difference component *AD*) and of time sequence (through the non-bunching component *SWITCH*). Even so, scatterplots also reveal substantial differences between *MOP and ITVRK*. This can be attributed to the ad hoc construction of *ITVRK* - and, for that matter, of the ITV measure that inspired it. In *ITVRK*, the temporal and structural variation components (*SWITCH, ENTR* and *AD*) are separately computed for the entire purchase history, and simply added up afterwards. Consequently, while *ITVRK* does account for degrees of item dissimilarity and for switching patterns in an aggregate fashion, it does not capture their interactions in causing consumption stimulation. The stimulation is higher if consumers frequently alternate between highly dissimilar items (in which case *ITVRK* underestimates the variation level), and lower if they purchase in series or blocks of relatively similar items (in which case *ITVRK* overestimates variation, this is a typical source of outliers for milk). This is again illustrated by the values in Table 5. Thanks to the average difference component *AD*, *ITVRK* picks up the difference between sequences 1 and 2, while the non-bunching component *SWITCH* allows it to distinguish 2 from 3. Yet, sequence 4 and 5 are assigned the same variation level, even though it is clear that 5 implies immediate switches between highly contrasting items, and hence causes more consumption stimulation. Another reason behind the lack of association between *MOP* and *ITVRK*, is that the switching measure in *ITVRK* involves a comparison with the immediately preceding purchase set only, and does not account for within set switching. The importance of within set switching and multiple purchases was already dwelt upon in the previous section. Not accounting for within set switching will be more problematic in categories with a large proportion of purchase sets involving several units (crackers), and/or where substantial differences occur between households in the homogeneity of these sets (milk).

TABLE 5
Variation Associated with Fictitious Purchase Sequences for Different Measures

	Sequence[9]					
Occasion	1	2	3	4	5	6
1	Aa1	Aa1	Aa1	Aa1	Aa1	Aa1
2	Ba1	Bb2	Aa1	Aa2	Bb3	Ab1
3	Ca1	Cc3	Aa1	Aa1	Aa2	Ac1
4	Aa1	Aa1	Aa1	Aa2	Bb4	Aa1
5	Ba1	Bb2	Bb2	Aa1	Aa1	Ab1
6	Ca1	Cc3	Bb2	Aa2	Bb3	Ac1
7	Aa1	Aa1	Bb2	Bb3	Aa2	Aa1
8	Ba1	Bb2	Bb2	Bb4	Bb4	Ab1
9	Ca1	Cc3	Cc3	Bb3	Aa1	Ac1
10	Aa1	Aa1	Cc3	Bb4	Bb3	Aa1
11	Ba1	Bb2	Cc3	Bb3	Aa2	Ab1
12	Ca1	Cc3	Cc3	Bb4	Bb4	Ac1
MOP	.260	.781	.609	.567	.685	.260
ENTr	.442	.442	.442	.558	.558	.442
ITVRK	1.685	2.170	1.351	2.376	2.376	1.685
% MOP due to						
attribute 1:	100%	33.3%	33.3%	26.66%	30.57%	0%
attribute 2:	0%	33.3%	33.3%	26.66%	30.57%	100%
attribute 3:	0%	33.3%	33.3%	46.77%	38.85%	0%

[9] Each combination Xxi represents one unit of an item with value X on the first, x on the second, and i on the third attribute

To summarize, compared to alternative variation measures such as entropy and *ITVRK* the *MOP* measure may yield different results and provide additional insight into purchase variation characteristics. This is especially true in markets where consumers tend to purchase multiple items simultaneously, and where consumers differ in the temporal pattern of switching behavior and in degree of structural variation.

In addition to providing an accurate overall assessment of purchase variation, the MOP measure may further our understanding by indicating along which attributes variation is primarily sought. This is, again, illustrated in Table 5, where the bottom row indicates the percentage contribution of each of the three attributes to overall MOP. Comparing sequences 1 and 2, for instance, we find sequence 2 to be substantially more varied (MOP of .780 compared to .260). The decomposition of MOP in the bottom row reveals the underlying reason: while variation in sequence 1 is entirely due to attribute 1, sequence 2 exhibits (comparable) changes along all three product dimensions. This is also true for sequence 3 where overall stimulation is lower than for 2 (MOP of .609 instead of .780), but, like in sequence 2, realized along all three attributes in the same proportion. Other purchase histories have comparable levels of overall variation, but do not derive it from the same attributes. An extreme example is the comparison of sequences 1 and 6, which have exactly the same MOP value, but show variation along entirely different attributes(attribute 1 for sequence 1, and attribute 2 for sequence 6).

This information may provide important cues on how *category changes* are likely to affect consumers. If, for instance, all consumers exhibit a type 6 sequence (that is, do not seek variation along the first attribute), pruning the product line along attribute 1 (say, deleting level A) will very strongly affect some consumers (those always buying A), while being irrelevant for others (those always buying either B or C). If all consumers have, say, a type 2 sequence, eliminating level A will have an impact for each customer, but this impact will be more modest. Decomposition of MOP along attributes may also shed light on the likely impact of *promotional actions*. Cross-promoting products that differ on an attribute is likely to lead to more ‘subsidizing’ when consumers seek variation along this attribute anyway. For instance, adding a coupon for item Ba1 to a pack of item Aa1 will have stronger appeal for consumers with type 1 purchase sequences, but these consumers buy the combination anyway and can profit from the promotion without changing their purchasing pattern. Less subsidizing occurs among consumers with type 6 sequences, who need to alter their behavior to benefit from the coupon. These simple examples illustrate how the decomposition of MOP along attributes may provide additional insights relevant to managers. For sequences involving the purchase of more than one unit, distinguishing within purchase set (simultaneous) from between purchase set (subsequent) variation may provide additional insights into the impact of promotional baskets and timing.

5 Conclusion and Directions for Future Research

An extensive body of research on the mechanisms of variety seeking indicates how households' variation in purchasing behavior could be properly quantified. Capitalizing on this research, the present paper suggests a 'new' measure of purchase variation that is inspired by traditional measures of brand loyalty. This new measure offers an improvement over existing variation measures. A*s an indicator of the overall variation* in a purchase history, it is more logical and complete than existing measures, and applicable to multiple purchase sequences. A particularly interesting feature of MOP is that it does not just offer summary insights per household and category, but automatically produces more detailed information on the sources of variation. *Temporal decomposition* of the MOP measure sheds light on the relative importance of within-purchase set and between-purchase set variation - information that is potentially useful for package size and product bundling decisions, and sales promotion planning. A*ttribute based decomposition* allows to determine the product dimensions for which consumers seek variation, and could be useful for assortment or sales promotion decisions.

Obviously, our approach exhibits a number of limitations. For one, the MOP measure is more complex and tedious to calculate than some traditional measures like Counts, Entropy or the Hirshman-Herfindahl index. Second, like other measures of purchase variation and loyalty, MOP only provides a *description* of actual purchase behavior of the individual or – in case of scanner data- the household. While such a description might be an important source of inspiration for hypotheses about what causes this behavior, the true 'reasons why' can only be uncovered through a more refined causal analysis. Another caveat is the fact that observations based on the MOP measure are conditional upon the characteristics of the offer in the past. Great care should therefore be taken in assessing how a change in the distributor's assortment or promotion strategy will alter the patterns of purchase variation in the future. *Future research* could evolve in two major directions. First, more extensive empirical validation of MOP as an overall measure of variation might be called for. Second, future research could concentrate on the decompositions of MOP to deepen our understanding of dynamic purchasing behavior, and derive managerial implications. Unravelling the attributes along which variety seeking does or does not occur, as well as the distinction between temporal and contemporaneous purchase variation, may help us to detect ‘patterns’ of dynamic purchasing for different product types or household segments, in a variety of circumstances. It goes without saying that manufacturers and retailers in charge of assortment and promotion decisions may profit from such insights.

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APPENDIX
Calculation of Weights

In quantifying the stimulation brought about by shopping trip n, differential weights $w_{n,n}$ and $w_{n,n-i}$ are adopted. These weights reflect the 'salience' of the comparison within purchase set n, and the comparison between purchase occasion n and n-i respectively. Based on the premise that experiences fade out as the number of intermediate consumptions increases, and inspired by classical loyalty measures (Guadagni and Little, 1983, Fader et al, 1995) we use geometrically decaying weights to capture the discounting of past experience.

If the consumer purchases **only one unit at a time**, the purchase sequence can be used as an indicator of the consumption sequence. In assessing the stimulation caused by shopping trip n, the comparison with trip n-1 receives a weight proportional to 1, that with n-2 will get a weight proportional to a smoothing constant or discount factor α (between zero and one), the comparison with n-3 will contribute in proportion to α^2, etc. Higher levels of α imply that purchases have a longer lasting impact on subsequent decisions, extreme cases being those where α =0 (zero order, previous consumption is completely irrelevant) and α approaching 1 (no decay over time, every past consumption experience is equally salient now).

In the more general case where **purchase set sizes larger than one** occur, computing the variation inherent in a purchase history is much more complex. The reason is that the *consumption* sequence of items bought on one shopping trip is unknown. Yet, from the psychological theories underlying variety seeking, we know that the dynamics (consumption sequences) are important. One way to overcome this problem of incomplete information would be to consider all the possibilities of 'spreading out' sets bought at one point in time over a number of subsequent consumptions of individual items, and then use classical measures of variation to compute the variation associated with each of these sequences. The overall variation in consumption for a multiple purchase history could then be the average over all possible associated consumption sequences. However, such a procedure would be extremely tedious. In the recent literature, attempts have been made to circumvent this problem by either ignoring within purchase set variation (Papatla and Krishnamurthi, 1992), or by imposing a consumption order on the items bought in the purchase set (Harlam and Lodish, 1995). Neither approach is very appealing. The measure suggested here approximates the results of a complete enumeration procedure by means of some simple 'average' calculations.

To illustrate this process, consider a situation where purchase set n comprises 5 units, purchase set n-1 covers 3 units, and set n-2 covers 2 units (to simplify the illustration, we limit the comparisons to two past shopping trips). These units will be consumed in a certain (unknown) order. The rows in table A1 correspond with subsequent consumption occasions, a total of 10 occasions (5 + 3 + 2) occurs for the three purchase sets. Assigning the units to rows of A1 is similar to spreading out the observed (multiple) purchases into a 'single purchase' sequence. Each unit in this 'single purchase sequence', or each consumption occasion, may lead to consumption stimulation. Following the procedure for single purchase sequences outlined above, the stimulation caused by unit U1 (of purchase occasion n) is measured through comparison with the unit consumed previously, U2 (weight proportional to 1), as well as with the unit before that, U3 (weight proportional to α), with U4 (weight proportional to α^2), with U5 (weight proportional to α^3), with U6 (weight proportional to α^4), etc. These weights are indicated in the second column of table A1. In a similar fashion, the stimulation brought about by consumption of unit U2 involves a comparison with its preceding unit U3 (with weight proportional to 1), the unit before that U4 (weight proportional to α), etc. These weights are shown in the third column of table A1. The stimulation caused by consumption of U3, U4 and U5 is developed in a similar fashion, and the corresponding weights are indicated in the fourth, fifth and sixth column of table A1, respectively. The total stimulation caused by purchase set n consists of the stimulations over the individual units U1 to U5. As can be derived from table A1, this total stimulation involves 10 comparisons within the purchase set n itself, with a total within purchase set weight *proportional to* $4 + 3\alpha + 2\alpha^2 + \alpha^3$. In MOP, this weight will be applied to the average difference between units in purchase set n, irrespective of their (unknown) consumption order. Confrontation of units bought in n with those bought on occasion n-1 involves 15 comparisons, with a total weight *proportional to* $1+2\alpha +3\alpha^2+ 3\alpha^3+3\alpha^4+ 2\alpha^5 + \alpha^6$. This weight will be applied to the average difference between items bought in n versus n-1. Derivations for the comparison with units of occasion n-2 or earlier occasions proceed in a similar fashion.

TABLE A1
Development of Weights for Multiple Purchase Sets

	Weight to compute stimulation caused by unit:				
Purchase set n (set size=5):	U1	U2	U3	U4	U5
U1					
U2	1				
U3	α	1			
U4	α^2	α	1		
U5	α^3	α^2	α	1	
Purchase set n-1(set size=3):					
U6	α^4	α^3	α^2	α	1
U7	α^5	α^4	α^3	α^2	α
U8	α^6	α^5	α^4	α^3	α^2
Purchase set n-2(set size=2):					
U9	α^7	α^6	α^5	α^4	α^3
U10	α^8	α^7	α^6	α^5	α^4

(*) U1 to U5 refer to the units in purchase set n, U6 to U8 refer to the units inset n-1, U9 and U10 to those in set n-2.

Generalizing these developments for a purchase set with P^n units, and rescaling to obtain a sum of weigths equal to 1 ($w_{n,n}+\alpha_{\ i}\ \alpha_i\ w_{n,n-i} = 1$), leads to a weight $w_{n,n}$ equal to:

$$w_{n,n} = \frac{\left(P_n - 1 - \frac{\alpha}{(1-\alpha)}.(1-\alpha^{P_n-1})\right)}{P_n}$$

or :

$$(1-\alpha).\frac{1+(1+\alpha)+(1+\alpha+\alpha^2)+\ldots+(1+\alpha+\alpha^2+\ldots+\alpha^{P_n-2})}{P_n}$$

which is the expression given in Table 2.

In a similar fashion, the general weights for comparison between purchase sets can be shown to equal:

$$w_{n,n-i} = \frac{\alpha^{cum(ni)-P_n}.(1-\alpha^{P_n}).(1-\alpha^{P_{n-i}})}{(1-\alpha).P_n}$$

The suggested procedure approaches results obtained when all possible underlying consumption sequences are simulated and the average of the corresponding variation values is computed.

APPENDIX 2
Computation of Average Product Differences

Similar to Lattin and McAlister (1985), we quantify the difference between items in the category on the basis of a number of discrete 'features' or attributes. Let K be the number of attributes, and k an attribute index (k:1→ K). For any product attribute k, we can calculate the average within purchase set difference $DIF_{n,n}(k)$ using Kronecker ’s. All items included in the purchase set are subject to pair-wise comparisons on that product attribute (e.g. brand). Comparisons give rise to a zero if both products are identical on the considered attribute (e.g. they share the same brand name), and to a one otherwise. The average within set difference $DIF_{n,n}(k)$ then simply consists of the ratio of (i) the number of paired comparisons leading to a one, divided by (ii) the total number of paired comparisons in the purchase set. Using the notation clarified in table 2 this leads to:

$$DIF_{nn}(k) = \frac{\sum_{p \in PS_n} \sum_{q \in PS_n, q>p} \delta_{p,q}(k)}{P_n * (P_n - 1) / 2}$$

The between purchase set difference on attribute k is calculated in a similar fashion, the only difference being that items are now compared across purchase sets n and n-i:

$$DIF_{nn\text{-}i}(k) = \frac{\sum_{p \in PS_n} \sum_{q \in PS_{n-i}} \delta_{p,q}(k)}{P_n * P_{n-i}}$$

Overall average difference within the purchase set ($DIF_{n,n}$) is obtained as a weighted sum of attribute-specific differences, with weights equal to the inverse of the number of attributes. Overall average difference between purchase set n and n-i ($DIF_{n,n-i}$) is obtained in a similar fashion. Resulting formulas are given in Table 2.

TABLE A2
Maximum Variation Sequence with three attributes, and 3, 2 and 4 attribute levels, resp.

Period	Attribute level 1	Attribute level 2	Attribute level 3
1	1	1	1
2	2	2	2
3	3	1	3
4	1	2	4
5	2	1	1
6	3	2	2
7	1	1	3
8	2	2	4
9	3	1	1
10	1	2	2
11	2	1	3
12	3	2	4

APPENDIX 3
Computation of MOPmax

For ease of exposition, and without loss of generality, we illustrate the computation of MOP^{max} assuming that the consumer buys one unit at a time (no multiple purchases).

The developments start from a situation where the assortment is 'full profile', that is: where every combination of attribute levels is available to the consumer. Let K be the number of attributes, and C_k the number of levels relevant for attribute k. Consider, for instance, a situation with three relevant product dimensions, where attribute 1 can take on 3 possible levels ($C_1=3$), attribute 2 has 2 levels ($C_2=2$), and attribute 3 has 4 distinct categories($C_3=4$). In that case, maximal variation is reached if, for instance, the consumer buys units in the order illustrated in Table A2.

In the case of such a sequence, attribute k contributes to the calculation of MOP as follows:

$$\frac{1}{K}.(\frac{1}{1-\alpha}-\alpha^{C_k-1}.(1+\alpha^{C_k}+\alpha^{2C_k}+\alpha^{3C_k}+...)).(1-\alpha)=\frac{1}{K}.((\frac{1}{1-\alpha}-\frac{\alpha^{C_k-1}}{1-\alpha^{C_k}}).(1-\alpha)$$

Summing this value over attributes, one obtains the following value for MOP under maximal variation:

$$\sum_k \frac{1}{K}.(1-\frac{\alpha^{C_k-1}}{1-\alpha^{C_k}}.(1-\alpha))$$

The developments above show that for full profile assortments, the maximum attainable MOP is easy to compute, even if the assortment is large. In view of the difficulties encountered by other variation measures – such as VBM and ITV - in the computation of maximal structural variation (see Handelsman 1987, and Pessemier and Handelsman 1984), this is an appealing finding. If the assortment is less than full profile, the value of MOP^{max} developed above still provides a good approximation as long as the number of unavailable combinations remains reasonable, and/or the assortment is sufficiently large. For limited assortments where many attribute combinations are not available, the approximation may become unacceptable. Yet, in those cases, the maximum variation sequence is easy to detect among (the limited number of) all possible sequences, and its variation can be calculated using the MOP formula in equation (1).