Using Complex Systems Analysis to Advance Marketing Theory Development: Modeling Heterogeneity Effects on New Product Growth through Stochastic Cellular Automata

Jacob Goldenberg
Hebrew University of Jerusalem

Barak Libai
Israel Institute of Technology

Eitan Muller
Tel Aviv University

This paper is part of a special issue on "Emergent and Co-evolutionary Processes in Marketing" edited by James Wiley. Jacob Goldenberg is Senior Lecturer of Marketing, School of Business Administration, the Hebrew University of Jerusalem, Jerusalem, Israel 91905. Barak Libai is Lecturer of Marketing, Davidson Faculty of Industrial Engineering and Management, Technion - Israel Institute of Technology, Haifa, Israel 32000. Eitan Muller is Professor of Marketing, Leon Recanati Graduate School of Business Administration, Tel Aviv University, Tel Aviv, Israel 69978. The authors would like to thank Moshe Givon, John Hogan, David Mazursky, Jim Wiley and two anonymous reviewers for a number of thoughtful comments and suggestions. This research was supported by the Israeli Science Foundation (ISF) and grants from the K-mart International Center of Marketing and Retailing, The Davidson center at The Hebrew University, The Davidson Center, Technion, and the Israeli Institute for Business Research at Tel Aviv University.

EXECUTIVE SUMMARY

Aggregate level simulation procedures have been used in many areas of marketing. In this paper we show how individual level simulations may be used support marketing theory development. More specifically, we show how a certain type of simulations that is based on *complex systems* studies (in this case Stochastic Cellular Automata) may be used to generalize diffusion theory one of the fundamental theories of new product marketing.

Cellular Automata models are simulations of global consequences, based on local interactions between individual members of a population, that are widely used in complex system analysis across disciplines. In this study we demonstrate how the Cellular Automata approach can help untangle complex marketing research problems. Specifically, we address two major issues facing current theory of innovation diffusion: The first is general lack of data at the individual level, while the second is the resultant inability of marketing researchers to empirically validate the main assumptions used in the aggregate models of innovation diffusion.

Using a computer-based *Cellular Automata Diffusion Simulation*, we demonstrate how such problems can be overcome. More specifically, we show that relaxing the commonly used assumption of homogeneity in the consumers' communication behavior is not a barrier to aggregate modeling. Thus we show that notwithstanding some exceptions, the well-known Bass model performs well on aggregate data when the assumption that that all adopters have a possible equal effect on all other potential adopters is relaxed. Through Cellular Automata we are better able to understand how individual level assumptions influence aggregate level parameter values, and learn the strengths and limitations of the aggregate level analysis. We believe that this study can serve as a demonstration towards a much wider use of *Cellular Automata* models for complex marketing research phenomena.

USING COMPLEX SYSTEMS ANALYSIS TO ADVANCE MARKETING THEORY DEVELOPMENT: MODELING HETEROGENEITY EFFECTS ON NEW PRODUCT GROWTH THROUGH STOCHASTIC CELLULAR AUTOMATA

If the amount of activity in an academic area reflects its importance, then research on the diffusion of innovations is one of the most important areas in the social sciences. With more than 4,000 diffusion publications printed since 1940, it has been declared that "No other field of behavioral science research represents more effort by more scholars in more disciplines in more nations" (Rogers, 1995). The considerable share of the marketing field in the output in this research stream reflects not only the importance of new products, but also the role of diffusion research in helping managers to better plan their entry strategy, target the right consumer, and anticipate demand so as to formulate an efficient and effective promotion, production, and distribution strategy.

Among the challenges that diffusion researchers face, undoubtedly a major one is that of getting the reliable and valid data needed to analyze the diffusion process. The growth of new products is a complex process that typically involves a large body of consumers interacting with one another over a long period of time. Frustratingly, often only aggregate adoption data are available to researchers for analysis, as is generally the case with market-level diffusion models (Sultan, Farley, and Lehmann, 1990; Mahajan, Muller, and Bass, 1990).

Even when collecting individual-level data, diffusion research designs generally consist of correlational analysis data gathered in one-time "snapshot" surveys of consumers, a methodology that amounts to rendering the diffusion process nearly timeless because of its effect of freezing the action of a continuous process (Rogers, 1995).

Consequently, diffusion research often cannot tell us much about the process of diffusion over time, other than what can be reconstituted from respondents' recall data. However, given that recall measures have often been shown not to be that accurate, even for the basic information of time of adoption (Coughenour, 1965), the ability to reliably reconstruct communication and influence patterns over time from such data is very low.

Hence, it is not surprising that much of the theoretical basis for the diffusion of innovations is predicated on a repeatedly analyzed small number of data sets. In these data sets, researchers could actually follow the diffusion process within small social systems, such as in the cases of the diffusion of hybrid corn among farmers in Iowa (Ryan and Gross, 1943), antibiotics among US physicians (Coleman, Katz, and Menzel, 1966) or family planning in Korean villages (Rogers and Kincaid, 1981). While the impressive contribution of these studies is evident, the implementation of new tools should be considered for analyzing the fast-changing and complex environment of new product growth.

The small set of available individual-based data constitutes another research dilemma: The lack of cases prevents our being offered a broad view of how a collective behavior emerges from changes in the individual characteristics. The span of individual-level parameters is too narrow to allow for comprising any explanation of their relationships to the diffusion parameters, or for predicting them.

In this paper, we propose the use of a simulation tool called *Stochastic Cellular Automata*, whose use, we believe, can help us to rigorously examine the process of new product growth, investigating assumptions and conducting studies in a manner not possible otherwise. Basically, Cellular Automata models are simulations of global consequences, based on local interactions between individual members of a population (for a detailed description see Wolfram, 1984).

These individual members may represent plants and animals in ecosystems, vehicles in traffic, people in crowds, or autonomous units in a physical system. The models typically consist of an environment or framework in which the interactions occur between various types of individuals that are defined in terms of their behaviors (procedural rules) and typical parameters. The solution of such models consists of tracking the characteristics of each individual through time.

This stands in contrast to modeling techniques, where the characteristics of the population are averaged together and the model attempts to simulate changes in these averaged characteristics for the entire population being studied.

This view is addressed and implemented in a recent publication by Wilkinson, Wiley and Aizhong (2000) who explore Industrial Market Systems (IMS). In this research they make use of computer models that aim to mimic the evolutionary process of IMS. Their model offer insights and understanding of the processes that govern the evolution of industrial firms along with their networks. The authors adopt the recent developments in the science of complex adaptive systems that show how structures emerge in *bottom-up, self-organizing* ways, rather than top-down fashion. The most adequate way to apply this approach is through computer-based simulation techniques to model the evolution of relationships in IMS. Another related work on relationships in industrial marketing and the formation of network is reported in Easton, Wilkinson, and Georgieva (1997).

Consistent with these two papers, we advocate this kind of approach for two intrinsic reasons: First, the complex interacting processes that take place in such systems is typically beyond the scope of traditional analytical techniques. Second, the data from the real world that are collected for research in marketing, no matter how rich, are only a sample of the phenomena that could arise in practice and in many cases are not diverse enough for scientific methods.

In this paper, we focus on a community of consumers who are potential adopters of an innovation. We use the computer-based *Cellular Automata Diffusion Simulation* (hereafter, CADS), which we developed in order to simulate the diffusion process within a social system. In a sense, CADS creates a "virtual community" in which information flows and a probability exists of potential adopters being affected by it, and in turn adopting, consequently affecting the process in the following period.

The manner in which the information is transferred from one individual to another and the criteria that affect adoption are determined by predefined *transition rules*. This approach enables us to perform studies on the diffusion process and to examine possible outcomes of a change in the assumptions made regarding the diffusion process and its aggregate results. As we demonstrate, the transition rules give us the ability to explore assumptions not examined in previous research due to the complexity of gathering the data (see an illustration of a complexity approach to marketing in Goldenberg, Libai and Muller, 2001).

This study demonstrates how CADS can allow us to examine the effect of heterogeneity in the communication behavior of adopters on the aggregate adoption level, effects which are typically analyzed in aggregate diffusion models such as the Bass model and its extensions (Mahajan, Muller, and Bass, 1990). More specifically, aggregate diffusion models make simplifying assumptions that presuppose homogeneity in the communication behavior of adopters. However, while concern regarding this issue has been expressed throughout the diffusion literature (e.g., Mahajan and Peterson, 1985), because of the nature of the aggregate data available to researchers, limited options were available to those who wanted to examine these assumptions and their implications.

The rest of the paper continues as follows: First we present the Cellular Automata framework, and its adaptation to the diffusion phenomena. We then present a Cellular Automata model for a homogeneous market that corresponds to simple diffusion model assumptions. Study 1 examines how robust the aggregate diffusion modeling approach is to these conditions.

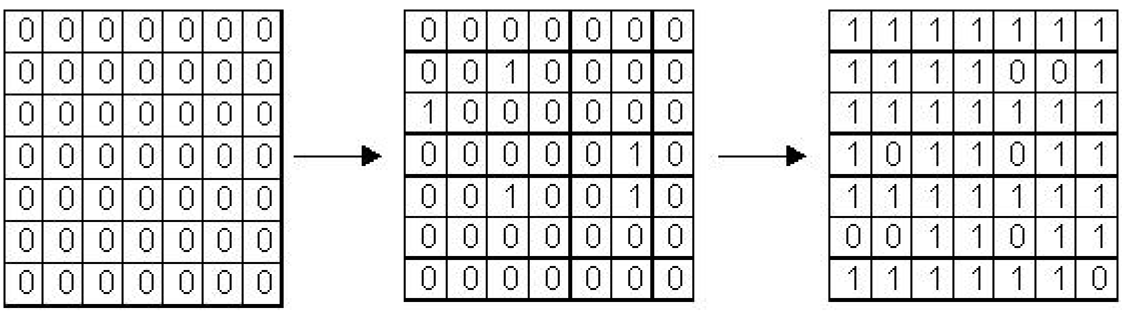
Next, we relax the homogeneity assumption and examine two cases of heterogeneous communications processes. Study 2 examines the situation wherein adopters differ in their propensity to be affected by internal and external information, but all adopters can communicate with all other adopters. The aim is to see how robust the aggregate-level model is to this heterogeneity assumption. Before we conclude, a further analysis of significant effects related to the two studies is presented. Finally, we offer our conclusions.

Background

The Cellular Automata model type was originally conceived by Ulam and Von Neumann in the 1940s to provide a formal framework for investigating the behavior of complex, extended systems (Von Neumann, 1966). A cellular automaton consists of a grid of cells, each of which can be in one of a finite number of k possible states, updated in discrete time steps according to a local interaction rule. This rule (also known as the *transition function* or *interaction rule)* normally operates on each of the cells. Hence, the state of any single cell is determined by the previous states of the cells in its neighborhood (Wolfram, 1984; Toffoli and Margolus, 1987).

In order to illustrate the principles of the Cellular Automata approach, consider the classic research on the propagation of forest fires. In these similar fundamental models, individuals (trees in forest fires) can spontaneously change their condition (start burning) due to external conditions (heat). The probability for this spontaneous change differs from tree to tree. We can denote this transition by a "0" state altered to a "1" state. According to these models, individuals in "1" state interact with those individuals in "0" state: They cause at some level of probability a fire in trees. We can view the propagation of the fire- spreading "1" states in Figure 1. A thorough description and illustration can be found in http://www.mirwoj.opus.chelm.pl/ as well as the cellular automata simulators that can be downloaded for personal exploration. Note what Figure 1 illustrates is not a regular forest fire – the fire here is not leaping from tree to tree (as it might first appear), but rather represents a case with few sources and many spontaneous tree fires. We have selected this case because it s similarity to marketing context as will be demonstrated shortly.

FIGURE 1
An Illustration of Simulated Fire



The emergence of recognizable collective phenomena emerging from the interactions of individuals is a recurring theme in *complex systems studies*. During the past three decades, the use of Cellular Automata models and their computational solutions became part of the rigorous theory development practice. In particular in *Complex System Study* (the analysis and study of systems which include a large number of interacting individuals), the computational solution became the dominant tool. Within the basis of that approach is the realization that the numerical solution of models is powerful, and most of the time is even more accurate than the traditional equation solving (Casti, 1989). Consequently, in most of the exact sciences, Cellular Automata became a recognized proof and theory development tool.

Because in this type of modeling the visualization and the dynamical nature of the solution are observable, there exist quite a few home-pages and websites that contain theory and online simulations. In Appendix A we offer a few interesting URLs in which the reader may find complete presentations, details and even online simulators to "play with" that can expose the interesting and rich dynamics allowed by these models.

Computer simulation models such as Cellular Automata are also increasingly being used in social science fields such as sociology (Hummon, 1990), economics (Wirl, 1998), geography (Batty, 1998), psychology (Nowak and Vallacher, 1998) and technological forecasting (Bhargava, Kumar, and Mukherjee, 1993; Goldenberg, Libai, Sorin, Neam and Stauffer, 2000; Goldenberg, Libai and Muller, 2001).

Cellular Automata can help us to examine fundamental questions such as: How do the interactions of individuals produce structures (e.g., markets, organizations, cultures, and languages), which in turn shape those individuals into action patterns that recreate the structure? Hanneman, Collins, and Mordt (1995) argue effectively for the use of such computer-based simulations for formulating and examining social science theories, often as a relatively simple alternative to explicit mathematical modeling.

They point to the ability to dynamically examine theories, taking time into account. Cellular Automata tools can also reveal how important brute quantitative conditions are to shaping the overall patterns of social actions.

A long tradition exists of using simulations to explore marketing phenomena, with recent examples including examining the robustness and validity of multi-dimensional scaling (Bijmolt and Wedel, 1999), logistic choice analysis models (Andrews and Manrai, 1998), or an analysis of the robustness of nonlinear promotion models (Christen et al, 1997).

In most marketing research simulations, however, the simulation is carried out on some averaged characteristics of the population (i.e., parameters) and not at the individual consumer level. In a notable exception, Frenzen and Nakamoto (1993) provide a fine example of how individual-level simulation modeling can lead to a better understanding of aggregate-level phenomena. Granovetter and Song (1986) have demonstrated the adoption and switching (un-adoption) among different products which shows complex dynamics (especially when interactions with suppliers is included).

Generally, there is a sparse attention to Cellular Automata applications in the classical marketing literature. One related application to innovation diffusion comes from the technological forecasting field, where a Cellular Automata system involving one hundred individuals demonstrated a possible use for forecasting purposes (Bhargava, Kumar, and Mukherjee, 1993). For more recent applications of complex systems models to marketing problems see Krider and Weinberg (1997) and Goldenberg, Libai and Muller (2001).

Our thrust is to utilize Cellular Automata mainly as a research tool to more deeply understand and examine theoretical and practical marketing phenomena. Naturally, when there are more parameters or more individuals involved, the number may increase considerably. Cellular Automata models have the potential to become an important market analysis tool in many areas. In that sense, we believe that the application of Cellular Automata in this paper can be viewed as a demonstration of future wider applications in many marketing research areas.

The CADS system used here is composed of two parts.

1. The first part is a data-generating system composed of computer programs written in C language. The user indicates to the program the level of the initial parameters and how s/he wants them changed if a study is carried out. The data-generating system enables the tracking of individual-level adoption while producing aggregate-level adoption curves.
2. The second part is data analysis, in which the individual-level adoption and the aggregate-levels are read by an SAS program that performs the analysis needed (e.g., NLS).

Throughout this paper, we use a social system numbering 1,000 adopters in order to understand the adoption phenomena. In choosing the system size, we aimed to ensure a system large enough to enable complex pattern analysis, yet timely enough to enable analysis of many cases (we created and analyzed close to 5,000 diffusion processes in this study). We examined a few larger social systems as well, in order to compare system size results, and the similarity of the results convinced us that a 1,000-member system is adequate.

Diffusion Theory and Modeling Through Cellular Automata

The modeling of the diffusion of new products lies between two extremes, namely aggregate level and individual level adoption. Aggregate-, or market-level, diffusion models, such as the Bass (1969) model, are based on market-level data, and generally assume a large degree of homogeneity in the population of adopters. Basically, they are rooted in the assumption that communication behavior is central to a new product's growth, following the theory of the diffusion of innovations.

One of the advantages of aggregate models is that they provide a relatively easy and efficient way to look at the entire market and interpret its behavior. Further, aggregate models are parsimonious yet based on a rich and empirically grounded theory. Another advantage is that quite often, the market level is also the level in which managers will be mostly interested. Finally, aggregate models can be estimated with market-level data, such as number of adoptions in a given year or average price, which are relatively easy to obtain.

This simplicity is also associated with some critique of the aggregate approach to diffusion. One of the shortcomings of the approach is that the models make strong and simplifying assumptions regarding the behavior of individuals. For example, the lack of heterogeneity among adopters is often cited as a major shortcoming of the aggregate level analysis. Also, the ability to test the assumptions that these models make with very limited data at the aggregate level can be questioned (Parker, 1994).

Individual-level models, on the other hand, acknowledge differences between consumers (e.g., differences in utility among potential adopters and their affect on adoption). Generally they follow economic theories (e.g., Lancaster, 1966), assume that individuals maximize some personal objective function such as utility of the product, and may update their beliefs as more information arrives at the market. Thus, individual-level models can be viewed as more behavior-based than aggregate models.

Aiming at explaining aggregate adoptions at the market level, restrictions on the heterogeneity in behavior among individuals are sometimes introduced, and individual-level models are aggregated to provide an explicit diffusion function at the market level (e.g., Chatterjee and Eliashberg, 1990). However, the use of market-level data to calibrate individual-level models is still not common, partly because the very limited aggregate-level data do not really allow for individual-level testing, as opposed to the case in the traditional diffusion models.

Our study synthesizes individual- and aggregate-level modeling in a way which may help to overcome some of the above-described barriers. We follow diffusion theory and its emphasis on communication behavior as a driver of new product growth, and generate a variety of possible dynamics to explore their influence at the aggregate level. CADS enables us to perform sensitivity analyses and examine the effect that changes in the parameters on the individual level have on the aggregate level, thus overcoming some of the limitations that follow the use of few data points at the aggregate level.

As a case study of this concept, we concentrate on the Bass (1969) diffusion model, the most popular product growth model in marketing, and the infrastructure to a large number of extensions. This modeling research school includes many extensions incorporating assumptions regarding issues such as the effect of marketing mix, competition, repeat purchase, and technological substitution (see, for example, reviews in Mahajan, Muller, and Bass, 1990; Parker, 1994; Mahajan, Muller and Wind 2000).

Here we explore the fundamental assumption in most of these models: Homogeneity at the communication level among adopters. We note that the Cellular Automata method presented here is probably also very relevant for testing and examining various extensions suggested for the basic Bass model. However, we leave that as an option for future research.

The modeling of the aggregate penetration of new products in the marketing literature generally follows the Bass (1969) model. The Bass model in turn follows Rogers' diffusion of innovations theory, which emphasizes the role of communication methods—external influence (e.g., advertising, mass media) and internal influence (e.g., WOM)—as driving the product adoption pattern. Thus, an individual's probability of new product adoption at time t (given that s/he has not yet adopted) depends in the Bass model linearly on two forces: a force which is not related to previous adopters, represented by the parameter of external influence p, and a force that is related to the number of previous adopters, or the parameter of internal influence q. The hazard model that describes the conditional probability of adoption at time t is:

$$(1) \qquad \frac{f(t)}{[1-F(t)]} = [P + Q \cdot F(t)]$$

Regarding the interpretation of the communication parameters, P and Q (hereafter, we use upper-case letters to denote the Bass model parameters and to distinguish them from individual-level parameters in lower-case letters that will be presented shortly), generally P represents the effect of external influences, i.e., factors not related to the number of previous adopters, such as advertising; Q represents the effect of internal influence coming from previous adopters.

How can these parameters be interpreted at the individual level? The answer does not come directly from the Bass model equation, where P and Q are the product of the aggregate level analysis. These parameters have been generally labeled as "force" (Bass, Krishnan, and Jain, 1994) or "influence" (Mahajan, Muller, and Bass, 1990) and not as probabilities. This study will aid in the understanding of the relationship between the macro-level parameters (P and Q) and the micro-level probabilities (*p* and *q*).

The Simulation Model

We begin by creating a Cellular Automata model that constitutes a realization of the assumptions in the basic Bass model. Consistent with Cellular Automata paradigm, our models consist of cells, each one representing a potential adopter, which can accept discrete values representing various states. Following the Bass model, our Cellular Automata model cells are binary, meaning that each potential customer can exist in two phases representing two situations: a) state "0" – the potential customer that did not adopt the innovation, and b) state "1" – the potential customer that has already adopted the new product. Also, irreversibility of transition is assumed, so that customers cannot "un-adopt" after adoption. An underlying assumption is that all potential adopters are capable of interaction with each other.

The mechanisms that govern the transitions of potential adopters can be classified into two types: 1) <u>External influence</u>: There is some probability *p* that in a certain time period, an individual will be influenced by external influence mechanisms such as advertising or mass media to adopt the innovation. We begin by setting this probability to be constant across potential adopters and time, and 2) <u>Internal influences</u>: There is some probability that during a single time period, a person will be affected by interactions with others who have already adopted the product. We represent the probability that a person will be affected by an interaction with <u>one</u> other person as *q*. In the case of the homogeneous market, *q* is constant for all potential adopters.

Thus, a time-dependent individual probability of adoption, PA(t), given that s/he has not yet adopted, is based on the binomial formula

$$(2) \quad PA(t) = [1 - (1-p)\,(1-q)^{k(t)}]$$

where $k(t)$ is the number of previous adopters during time period t.

In the following sections, we examine how the individual-level model can help us to understand aggregate-level diffusion results. We start by analyzing the simple homogeneous model described above, labeling this case a *homogeneous market*. Then, we relax the communications homogeneity assumption, and consider the case of a *heterogeneous market*.

Before we continue this exploration let us add a parenthetical remark. In this type of modeling, there are different ways to get closer to reality. One possible way calls for a matrix (of *n* rows and *n* columns) of probabilities of communications, where *n* is the number of individuals in the population. Then, the $PA(t)$ value could be calculated by brute force, with the $(1-q)^{k(t)}$ term being replaced by the product of $n-1$ terms for each individual. This alternative is less parsimonious then the one used here and in addition requires more parameters. We leave this option for a future research. Another alternative is to have a few segments in the model with two values of *q* for individuals in each segment, one (higher) value if communication is coming from within the segment, and a lower one if it is not (this network effects is discussed in Goldenberg, Libai and Muller, 2001).

OVERVIEW OF THE STUDIES CONDUCTED

We start our exploration in a homogeneous market. We will generate various diffusion processes using various *p* and *q* parameters (with identical value for all consumers). Using that database, we want to explore how individual-level behavior corresponds to aggregate-level parameters in the simple, homogeneous case. Specifically, our research questions are:

A.1: *To what extent would the aggregate-level model (Bass) capture an individual-level generated process in a homogeneous market?*

A.2: *How do the levels of individual parameters p and q affect the homogeneous market diffusion process, specifically the aggregate-level parameters P and Q?*

After examining the case of homogeneous markets, we will address the changes in a heterogeneous market, one in which individuals differ in their propensities to be affected by external and internal influences. Our interest is in seeing to what extent the aggregate-level results are robust to the introduction of variance at the individual level.

B1: *To* what *extent would the variance in individual parameters p and q affect the aggregate-level explanatory power of the diffusion process?*

B2: *What is the effect of the heterogeneity in the individual level on the aggregate-level parameters P and Q?*

We will conclude this exploration by presenting a further analysis, shedding light on some complex processes that are part of the diffusion processes.

STUDY I: THE CASE OF HOMOGENEOUS MARKETS

We begin with the homogeneous case, where our aim will be to understand how *p* and *q* at the individual level govern the values of the Bass model parameters P and Q.

Study 1: Method

CADS was run using a social system of 1,000 potential adopters, varying p and q. We want to generally examine values of p and q which are consistent with the already known values for communication effects in the diffusion process. Following previous research on the values of the Bass model parameters (Sultan, Farley, and Lehman, 1990; Parker, 1994), and assuming that the individual-level parameter will be related to the aggregate-level one, we manipulate p between 0.004 and 0.04.

The case of *q* is more complicated; Q, the aggregate-level internal effect parameter, relates to the overall internal effect on a given potential adopter. However, *q* at the individual level is the probability that *one* previous adopter will affect the potential adopter. When there are many potential adopters, *q* should be much lower than Q in order to achieve a similar effect. It is expected that the relationship between the two will be proportional to m, (m = 1,000 in our case). Thus, we set the boundaries of *q* to range from 0.00001 up to 0.001. For example in a case in which q = 0.0005, the probability that a person will be affected by internal influence when there are 500 previous adopters (half of the population) will be:

(3) $[1-(1-0.0005)^{500}]=0.22$

This parameter is consistent with typical values of Q for the Bass model (Sultan, Farley, and Lehman, 1990).

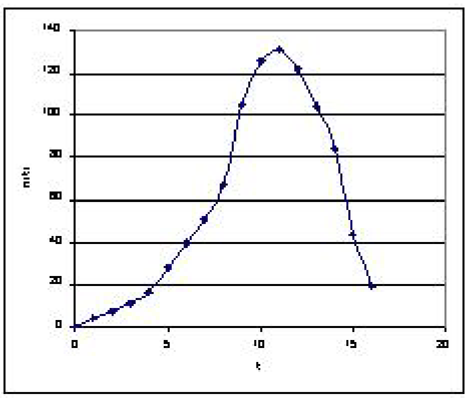
Study 1: Results

Result A1. *The Bass model fits the individual-level generated process well, with the exception of the case of individual-level internal influence q being under a very low critical value.*

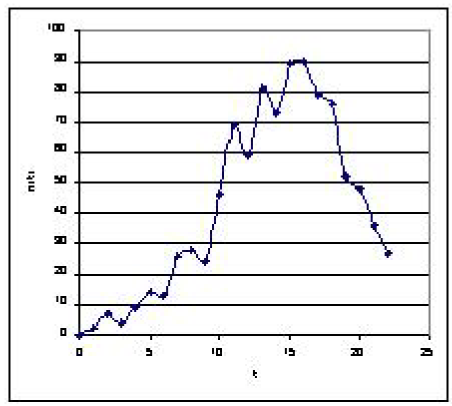
As a first step, we checked for the existence of parameter values on the individual level for which we do not get the classic bell-shaped curve expected for a diffusion process. Our results indicate that for all values of p in the range, the diffusion curve generally has a bell shape.

In the case of q, however, the effect on the shape of the diffusion curve is more apparent. Figure 2 a-c illustrates the influence of q on the diffusion process for three representing values of q. When q is large enough (e.g., q = 0.00061, Figure 2a) the adoption curve is smooth, similar to what we might expect from a Bass-type process. When q decreases, the process changes from smooth to a more fluctuating one, as shown in Figure 2b (q = 0.00037). Note that at this stage (which is a representative case of a large range of examined values of q), the adoption curve still maintains its general bell shape. However, further decrease in q (e.g., q = 0.00001) is followed by a collapse from the familiar bell shape to a different, high volatility-based regime (see Figure 2c).

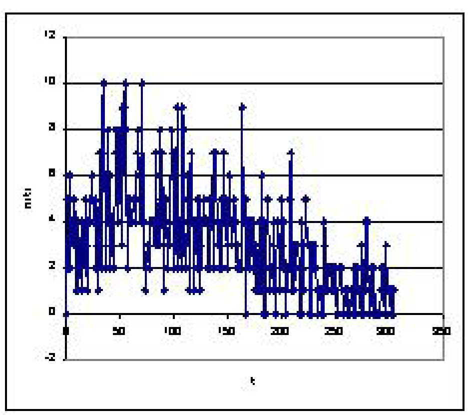
FIGURE 2
The Influence of *q* on the Diffusion Process
(p = 0.004 in al three cases)



Plot for q=0.00061



Plot for q=0.00037



Plot for q= 0.00001

The rationale behind this result may be related to the role of internal influence in the diffusion process. Generally, the main force of the diffusion process has to do with the effect of previous adopters on potential ones, where this effect has a crucial role in "driving" the bell-shaped curve (Rogers, 1995). We find that beyond a critical point, because the driver for the diffusion process as we know it does not exist, the expected result—the bell-shaped curve—does not exist either.

Interestingly, the dynamics of this change in the diffusion process has a critical nature: A "phase transition" was observed in the change of the diffusion curve. While up to about q = 0.0001, the diffusion curve maintained its bell-shaped structure, below this critical value, the diffusion process collapses, as shown in Figure 2c.

In the following analyses, we restricted ourselves to values of *q* that correspond to a diffusion process in which internal influence is a major driver of adoptions, or one that generally produces bell-shaped curves (though clearly not perfect ones, as can be seen from Figure 2b). Consequently, *q* values above 0.0001 were manipulated.

Study 1: The Relationship Between Individual and Bass Model Parameters

Next, the relationship between the aggregate-level Bass model and the homogeneous individual-model parameters were examined. We started by examining the effects of *p* and *q* on the fit of the Bass model to aggregate diffusion data. We again varied *p* and *q* to produce a total of 100 combinations. For each simulated diffusion process, we performed Non-Linear Least Squares (Srinivasan and Mason, 1986) in order to estimate the Bass models P and Q. We found that overall, the Bass model fits well with an average R^2 of 0.95.

At a later stage, we performed OLS regression to determine the relationship between *p* and *q* to the Bass model parameters P and Q. The results are presented in Table 1, where parameter results are standardized.

TABLE 1
Standardized Effect of Individual-Level Parameters on Aggregate Ones

	Individual-level independent variables		**R^2**
Dependent variable	***p***	***q***	
P (Bass)	0.90 *	-0.29 *	0.91
Q (Bass)	0.015	0.99 *	0.99

* significant at the 0.05 level

As seen in Table 1, Q (Bass) is mainly generated by *q* (the probability of being affected by another person), while *p* does not affect Q at all. As for aggregate external effect: *p* has a strong significant effect on P, however *q* also has a significant negative effect on P, though it is considerably lower than that of *p*.

The fact that the NLS procedure correlates the aggregate level P also to individual level *q* may be attributed to the nature of the estimation procedure and the diffusion process. As we later discuss, P is mostly responsible to the shape of the diffusion curve in the early parts of the diffusion while Q strongly determines the shape after the takeoff. However, since the magnitude of observations after the takeoff and especially near the peak are much larger than that in the beginning, the estimation procedure will focus on these observations when minimizing least squares. Thus, the estimation of P may be not as "clean" as that of Q. Furthermore, when *q* is higher, the difference between the early and later parts is larger which may cause an under estimation of P, as can be seen in the negative sign of the correlation of q and P.

These above are summarized in the following two results:

Result A2: *Aggregate-level internal effect parameter Q is strongly generated by individual-level internal effect.*

Result A3: *While aggregate-level external effect parameter P is mostly generated by individual-level external effect, its value may also be affected by individual-level internal effect.*

The Case of Heterogeneous Markets

Because of the centrality of the communication between individuals, homogeneity assumption of the ability of market-level models to present an efficient, yet faithful representation of the market, is generally underlying most of the extensions to the Bass model. In some cases, heterogeneity in the communication behavior was introduced by dividing the market into a few large segments, where each segment has different communication behavior, but may interact with members of other segments (e.g., Gore and Lavaraj, 1987; Tanny and Derzko, 1988; Kalish, Mahajan, and Muller, 1995). Homogeneity was assumed within these segments.

We begin by introducing individual-level heterogeneity, where various members of social systems will be affected in various ways both by external and internal communications sources. Our aim is to understand how such heterogeneity in the communication pattern affects aggregate-level results, and how robust the process is to the homogeneity assumption. Accordingly, we allow variety in the way people are affected by external and internal influences.

STUDY II: MARKET HETEROGENEITY

Here we assume that *q* and *p* derive from a normal distribution, each with its own mean and variance. Hereafter, we term the standard deviation of *q* by *sigma q*, and that of *p, sigma p*.

In order to analyze the diversity effects on the diffusion process, individual-level parameters (*p, q), sigma p*, and *sigma q* were manipulated, each on seven levels to produce $7^4 = 2{,}401$ data sets of simulated diffusion processes. We varied *sigma p* and *sigma q* to include cases of both low and high variance relative to the mean. *p* ranged from 0.004 to 0.4, and *q* from 0.0001 to 0.0007. *Sigma p* ranged from zero to 0.004, and *sigma q* from zero to 0.00014. For each diffusion process, NLS was conducted to fit the Bass model. Then OLS regression was performed to determine the effect of all parameters on the Bass model results. The results of this regression are presented in Table 2.

TABLE 2
The Effect of Heterogeneity on Bass Model Results
(standardized parameters)

	P (Bass)	**Q (Bass)**
Q	-0.32 *	0.98 *
Sigma q	0.07 *	-0.03 *
P	0.89 *	0.02 *
Sigma p	0.006	-0.004
$\mathbf{R^2}$	0.90	0.98

* significant at the 0.05 level

Note that while the effect of *sigma q* on Q is significant, the effect size indicates that this effect is negligible compared to the effect of *q* itself. Similarly, here *p* has a very small but significant effect on Q, which may be attributed to the large number of observations as compared to study 1. Interestingly, *sigma q* has a stronger (yet small) effect on P as compared with Q. As Table 2 clearly indicates *sigma p* had no significant effect on aggregate-level results, including the values of P.

These results lead to the following conclusions:

Result B1. *Heterogeneity in individual-level* internal *influence has a small effect on the aggregate-level results.*

Result B2. *Heterogeneity in individual-level* external *effect has no effect on aggregate-level results.*

MORE COMPLEX EFFECTS AND FURTHER RESEARCH

In addition to the heterogeneity issue, CADS enabled us to relate to a number of issues that we believe are of great interest to diffusion researchers. In the following section, we present two issues that demonstrate the power of CADS and cellular automata modeling approach.

Left and Right Tails to the Diffusion Curve

One result noted when varying parameter values in the homogeneous case is that low values of p create a "left tail" in the diffusion curve. While not examined at the individual level, a possible association between low values of P, the aggregate-level external influence parameter, and a left tail on the diffusion curve, had been pointed out in the past (Kohli, Lehman, and Pae, 1999; Parker, 1994).

A left tail can be created by very small values of either external or internal influence. However, very low internal influence also indicates a very slow diffusion process at a later stage, on occasion even leading to a breakdown of the diffusion process as we know it, as has been shown previously herein. On the other hand, low values of external influence allow for a long left tail followed by a strong "takeoff", as is often witnessed (Golder and Tellis, 1997). This phenomenon is also witnessed in the individual-level analysis.

Less anticipated was the phenomenon of a right tail to the diffusion curve that we found for high values of *sigma q*. This is clearly seen in Figure 3, where a diffusion process with *sigma q* in the size of *q* (both are 0.0004) is presented. After about 30 time periods (where 82% of the population has adopted), we notice an adoption process that produces a very long *right* tail, in more than 200 time periods in this example.

In order to understand the dynamics, Figure 3a is divided into two time periods: before and after Time Period 30. Here we see a bell-shaped diffusion process up to Time Period 30 (Figure 3b), followed by a process that after some time does not resemble a diffusion process (Figure 3c). Note however, that even with a right tail, the Bass model fit to the data may be reasonable, because the first part represents the vast majority of the adopting population.

FIGURE 3
The Effect of High Heterogeneity in *q*
(q = 0.0004, *sigma* q = 0.0004)

Figure 3a

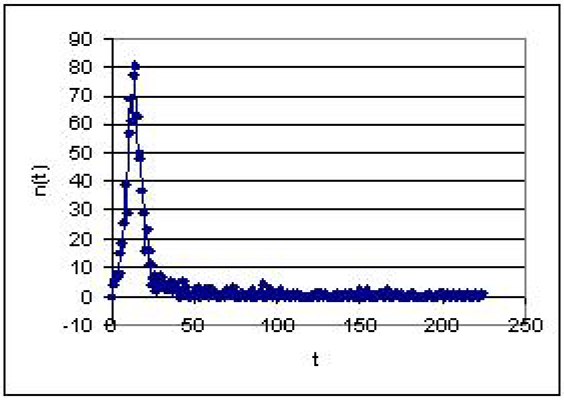


Figure 3b

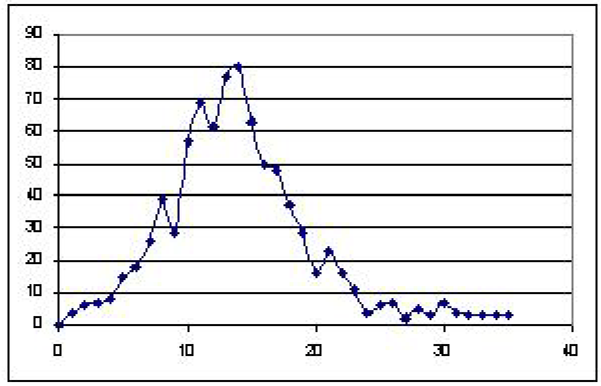
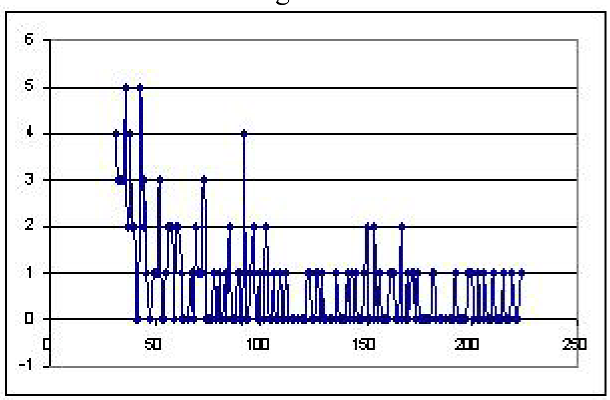
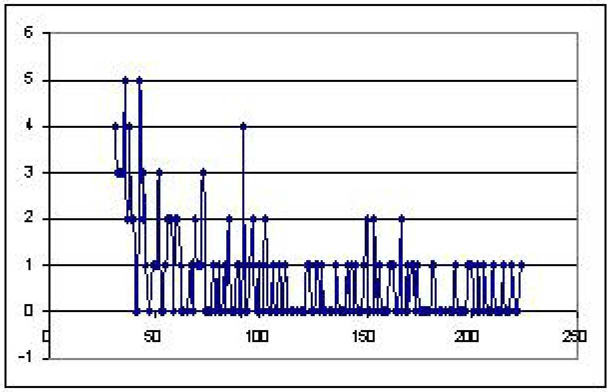


Figure 3c



When can we expect this right tail to appear? A way to quantify this effect is to examine the time needed to reach 95% adoption (denoted hereafter as $t_{95\%}$) under various values of *sigma q*. This result is presented in Figure 4. As can be seen, the $t_{95\%}$ is not a monotonic process: At critical values of diversity (*sigma q*) there is a step function in the levels of the values of $t_{95\%}$. The critical value of *sigma q* is about 60% of the mean. Manipulation of q values revealed that this critical value of the standard deviation of about 60% is stable across q values, implying for criticality nature of the diversity effect.

FIGURE 4
The Phase Transition Type of the Diffusion Structure due to the Heterogeneity in the Parameter *q*



The phenomenon of a long *right* tail has not been a focus of attention, possibly due to measurement problems. For many product categories, adoptions are analyzed up to a point in time that is not much later than the peak, due to the difficulty later in distinguishing between new adoptions, repeat purchases, and multiple purchases. Consequently, there is a relative lack of knowledge on the adoption pattern of later adopters.

Our results shed light on the way the later adoption occurs: A consequence of a high variation in individual internal influence is that while for many adopters, the internal effect is the major driver of diffusion, there is a group of adopters for whom internal effect has little impact. Hence, their adoption is characterized by a long period in which very few will adopt during each time period. Because these adopters will be affected mainly externally, the length of the period depends upon the intensity of external effect. In addition, the long right tail may be very noisy at its end, not resembling "classic" diffusion patterns. Interestingly, the size of the group that produced the right tail in the CADS model (18% in the case we analyzed) is close to Rogers' estimation of the size of the Laggards group (16%).

Does Advertising Work After Takeoff?

An interesting question relates to the possible changing effect of internal influence (e.g., WOM) vs. external ones (e.g., advertising) during the diffusion process. The conventional wisdom is that earlier adopters rely more on external influence in their adoption decision as compared with later adopters (Rogers, 1995; Kotler, 1996).

Rogers (1995) makes the even stronger claim that there is hardly a need for advertising after the product begins to "take off", since at that point, internal influence governs the diffusion process almost by itself (Rogers, 1995). Mahajan, Muller, and Srivastava (1990) show that indeed according to the Bass model, the effect of internal influence increases over time, while external influence effect falls. However, this effect was not examined using individual-level data.

Our *heterogeneous market* analysis (see study 2) allowed us to examine how various diffusion parameters can affect the speed of the diffusion process. We studied the effects on two time points: Peak and Takeoff. Time to peak is a common measure of diffusion speed (e.g., Jain, Mahajan, and Muller, 1995). Identifying the takeoff is more complicated because there is no agreed-upon definition that enables defining it exactly. There have been various attempts to quantify takeoff (Golder and Tellis, 1997), here we follow Rogers' suggestion that takeoff typically occurs when about 16% of the population adopt (Williams, Rice, and Rogers, 1988).

Table 3 presents the effect of various diffusion variables *p, q, sigma p*, and *sigma q* on three time periods: *zero to takeoff*, *takeoff to peak*, and the sum of both, or *time to peak*. We see that as we found before, heterogeneity in external influence has no significant effect on the process. Heterogeneity in internal effect, or *sigma q*, has no effect on time to takeoff, and a relatively small effect on takeoff to peak and time to peak. External effect parameter *p* and internal effect parameter *q* both affect the process, and had a similar effect on time to peak.

Yet, when breaking it down into *time to takeoff* and *takeoff to peak,* we could see the dynamics of effect over time: external influence was more dominant in the period before takeoff, and internal effect dominated in the period from takeoff to peak. However, unlike Rogers' suggestion, we cannot say that following takeoff, external effect nearly disappears, and there is still the considerable effect of external influence. Thus, CADS results suggest that halting advertising, as Rogers implies, may not be justified.

TABLE 3
The Standardized Effect of Heterogeneous Market Parameters on Takeoff and Peak
(n = 2401)

	Time to takeoff (16%)	**Time: Takeoff to peak**	**Time to peak**
Independent variable			
q	-0.45 *	-0.51 *	-0.53 *
Sigma q	-0.003	-0.08 *	-0.04 *
P	- 0.71 *	-0.32 *	-0.59 *
Sigma p	-0.004	0.01	0.004
R^2	0.71	0.37	0.63

* significant at the 0.05 level

DISCUSSION

A major aim of this study was to demonstrate the capabilities inherent in simulation models such as Cellular Automata. Often, because of measurement problems, models that are based on individual-level theory are examined using aggregate-level data that can in fact tell us little about individual-level phenomena. Where obtaining full data at the individual level or performing explicit analytical analysis is difficult, Cellular Automata creates the simulated environment that helps us to perform individual-level "studies" and examine the effect of the assumptions on the aggregate level.

As computer systems become more powerful and easier to use, the ability to simulate large and complex systems enables analyses unheard of only a few years ago. The complexity of the marketing environment, in which so many consumers and merchants interact, points to Cellular Automata as an essential tool for marketing analysis in the coming decade.

The matter of heterogeneity in the communication pattern of a new product's adopters, which we analyzed here, is a good example. The notion that individuals vary in their communication behavior is intuitive and not new. However, the ability to follow a large number of consumers with the goal of understanding the effect of such heterogeneity on aggregate-level results has been limited. In the case of aggregate diffusion models such as the Bass model, this heterogeneity has not been fully examined in spite of the consequent disputable validity of the homogeneous model. Consequently, efforts to explain the ability of the simple Bass model to fit complex market data have focused on the marketing mix issue (Bass, Krishnan, and Jain, 1994), and not on the communication question.

Our results point to another explanation of the ability of the Bass model to describe complex data: heterogeneity robustness. We examined the case of *market heterogeneity*, in which the assumption of full interaction between social system members is preserved, but members differ in their propensity to be affected by external influence and by others. Not only was it found that the Bass model still has good fit to the data, but the estimation results did not change much due to the heterogeneity.

We found, in fact, that the heterogeneity in individual external influence does not affect aggregate results at all. Furthermore, we found that heterogeneity in individual internal influence has a low effect (compared to the mean value) on the aggregate level. It may be that individual-level heterogeneity cancels out the variance between individuals in a way that leaves the mean individual affect as the major source of influence on the aggregate level. The bottom line is that *market heterogeneity* has little effect on the aggregate-level results.

It is important to note that these conclusions are relevant for the cases in which the individual parameters are in a range of values that constitutes a normal diffusion process. We have shown that the diffusion process is subject to collapses, where some of these collapses appear when the individual parameters are below certain critical values (e.g., *q* or *sigma q*). Under such critical conditions, we posit that the diffusion process loses its bell shape, causing known aggregate models to lose their fit and predictive power. Clearly, future research can shed more light on this aspect.

The analysis of the diffusion case demonstrated that Cellular Automata models are capable of shedding light on complex processes. It is our contention that many issues that are of great interest in marketing can be explored efficiently through this approach. Naturally the power of Cellular Automata should be stronger in the analysis of complex systems where different consumers will have some mutual dependence on each other. Such is the case of diffusion of innovations where WOM or imitation plays a central role in the penetration process. Issues such as the analysis of the effects of network externalities on product growth, the dynamics of first and repeat purchase buying on the shape of the diffusion curve, the effect of opinion leaders and negative WOM on a new product’s acceptance can all be analyzed via this approach without altering the basic concept.

Cellular Automata can be useful on a much broader framework than the diffusion process. For example, models in which consumers can be affected by advertising of various brands as well as the positive and negative reactions of other consumers can help to complement the game theory literature on competition. The reactions of consumers and channel members to new kinds of distribution methods can help to better analyze emerging channels such as electronic commerce. "Relationship marketing" strategies in which the marketing mix is tailored to an individual’s taste and profits are measured in the long run, can be optimized.

REFERENCES

Andrews, Rick L. and Manrai, Ajay K. (1998). "Simulation Experiments in Choice Simplification: The Effects of Task and Context on Forecasting Performance," *Journal of Marketing Research*, 35(2), 198-209.

Bass, Frank M., (1969). "A New Product Growth Model for Consumer Durables," *Management Science*, vol.15 (January), 215-27.

_______, Trichy, Krishnan, V., and Jain, Dipak C. (1994). "Why the Bass Model Fits Without Decision Variables," *Marketing Science*, Vol. 13 (3), 203-223.

Batty, M. (1998). "Urban Evolution on the Desktop: Simulation with the Use of Extended Cellular Automata," *Environment and Planning*, 30(11), 1943-67.

Bhargava, S. C., Kumar, Arun, and Mukherjee, A. (1993). "A Stochastic Cellular Automata Model of Innovation Diffusion," *Technological Forecasting and Social Change*, 44, 87-97.

Bijmolt, Tammo H. A. and Wedel, Michel (1999). "A Comparison of Multidimensional Scaling Methods for Perceptual Mapping," *Journal of Marketing Research*, 36(2), 277-285.

Brown, Jacqueline and Reingen, Peter H. (1987). " Social Ties and Word-of-Mouth Referral Behavior," *Journal of Consumer Research*, 14, 350-362.

Bryce, Ryan and Gross, Neal C. (1943). "The Diffusion of Hybrid Seed Corn in Two Iowa Communities," *Rural Sociology*, 8:15-24.

Casti, J. L (1989). *Alternate Realities: Mathematical Models of Nature and Man*. John Wiley & Sons, New York.

Chatterjee, Rabikar and Eliashberg, Jehoshua (1990). " The Innovation Diffusion Process in a Heterogeneous Population: A Micro Modeling Approach," *Management Science*, 36 (9) 1057-74.

Coleman, J.S., Katz, E., and Menzel, H. (1966). *Medical Innovation: A Diffusion Study*. Bobbs-Merrill, New York.

Coughenour, Milton C. (1965). "The Problem of Reliability of Adoption Data in Survey Research," *Rural Sociology*, 30, 184-203.

Easton G., Wilkinson I. and C. Georgieva (1997) "On the Edge of Chaos: Towards Evolutionary Models of Industrial Networks" in Hans Georg Gemunden and Thomas Ritter eds. *Relationships and Networks in International Markets,* Elsevier pp273-294

Frenzen, Jonathan and Nakamoto, Kent (1993). "Structure, Cooperation, and the Flow of Market Information," *Journal of Consumer Research*, 20, 360-375.

Goldenberg Jacob, Libai Barak, Solomon Sorin, Jan, Neam and Dietrich Stauffer (2000(, "Marketing Percolation". *Physica A*, 284, 335-47

Goldenberg Jacob, Libai Barak and Muller Eitan, (2001), "Talk of the Network: A Complex Systems Look at the Underlying Process of Word-of-Mouth", *Marketing Letters,* 12, pp. 209-221.

Golder, Peter N. and Tellis, Gerard T. (1997). "Will it Ever Fly? Modeling the Takeoff of Really New Consumer Durables," *Marketing Science*, vol.16 (3), 256-70.

Gore, A.P and Lavaraj, U.A. (1987). "Innovation Diffusion in a Heterogeneous Population," *Technological Forecasting and Social Change*, 32, 163-168.

Granovetter, Mark (1973). "The Strength of Weak Ties", *American Journal of Sociology*, 78, 1360-1380.

Granovetter, Mark and Roland Song (1986), "Threshold Models of Interpersonal Effects in Consumer Demand " *Journal of Economic Behavior and Organization,* 7, (March) pp. 83-99.

Hanneman, Robert A., Collins, Randall and Mordt, Gabriele (1995). "Discovering Theory Dynamics by Computer Simulation: Experiments on State Legitimacy and Imperialist Capitalism," *Sociological Methodology*, 25, 1-46.

Hummon, Norman P. (1990). "Computer Simulation in Sociology," *Journal of Mathematical Sociology*, 15 (2), 65-66.

Jain, Dipak, Mahajan, Vijay, and Muller, Eitan (1995). "An Approach for Determining Optimal Product Sampling for the Diffusion of a New Product," *Journal of Product Innovation Management*, 12, 124-135.

Kalish, Shlomo Mahajan, Vijay, and Muller, Eitan (1995). "Waterfall and Sprinkler New Product Strategies in Competitive Global Markets, *International Journal of Research in Marketing*, 12 (2) 105-119.

Kohli Rajeev, Lehmann, Donald R., and Pae, Jae (1999). "Extent and Impact Time in New Product Diffusion," *Journal of Product Innovation Management*, 134-44.

Kotler, Philip (1996). *Marketing Management*: *Analysis, Planning, Implementation, and Control*, Prentice-Hall.

Krider, R. and Weinberg, C. (1997), "Spatial Competition and Bounded Rationality: Retailing at the Edge of Chaos," *Geographical Analysis*, 29 (1), 16-34.

Lancaster, Kelvin (1966). "A New Approach to Consumer Theory," *Journal of Political Economy*, 132-57.

Mahajan, Vijay, Muller, Eitan, and Bass, Frank M. (1990). "New Product Diffusion Models in Marketing: A Review and Directions for Research," *Journal of Marketing*, vol. 54 (January), pp. 1-26.

________, _______, and _____ (1995). "Diffusion of New Products: Empirical Generalizations and Managerial Uses," *Marketing Science*, vols. 14, 3, pp. 79-88.

____, _____, and Srivastava, Rajendra K. (1990). "Determination of Adopter Categories by Using Innovation Diffusion Models," *Journal of Marketing Research*, vol. 27 (February), pp. 37-50.

____ and_______ (1998). "When is it Worthwhile Targeting the Majority Instead of the Innovators in a New Product's Launch?," *Journal of Marketing Research,* 35, 488-495..

____ and_______and Yoram Wind (2000). *New Product Diffusion Models*, Kluwer Academic Publishers, Boston MA.

Mahajan, Vijay and Peterson, Robert A. (1985). *Models for Innovation Diffusion*, Sage, Newbury Park, CA.

Morrison, Pamela and Roberts, John H. (1992). "The Effect of Network Structure in Industrial Diffusion Processes," *Research Policy* 21 (6). 533-552.

Nowak, Andrzej, and Vallacher, Robin. R. (1998). "Toward Computational Social Psychology: Cellular Automata and Neural Network Models of Interpersonal Dynamics," in Read, Stephen et al (eds). *Connectionist Models of Social Reasoning and Social Behavior*, Lawrence Erlbaum Associates, Mahwah, NJ, 277-311.

Parker, P. M. (1994)."Aggregate Diffusion Models in Marketing: A Critical Review," *International Journal of Forecasting*, 10, 353-380.

Reingen, Peter H. and Kernan, Jerome B. (1986). "Analysis of Referral Networks in Marketing: Methods and Illustration," *Journal of Marketing Research*, 23 (4) 370-378.

Roberts, John and Lattin, James M. (1999). "Restricted Parameter Individual-Level Diffusion Models", in *New Product Diffusion Models,* Vijay Mahajan, Eitan Muller and Jerry Wind (eds.), Kluwer Academic Publication, 2000.

Rogers, Everette M. (1986), *Communication Technology: The New Media in Society,* New York: Free Press.

______ (1995). *The Diffusion of Innovations*, 4th, New York: Free Press.

_______ and Kincaid, D.L. (1981). *Communication Networks: A New Paradigm for Research*. New York: Free Press.

Srinivasan, V. and Mason, C.H. (1986). "Nonlinear Least Square Estimation of New Product Diffusion Models," *Marketing Science*, vol. 5, 169-78.

Sultan, Fareena, Farley, John U., and Lehmann, Donald R. (1990). "A Meta Analysis of Applications of Diffusion Models," *Journal of Marketing Research*, Vol. 27 (February) 70-77.

Tanny, S. M and Derzko, N. A. (1988). "Innovators and Imitators in Innovation Diffusion Modeling," *Journal of Forecasting*, vol. 7, 225-34.

Toffoli, T. and Margolus, N. (1987) *Cellular Automata Machines*. MIT Press, Cambridge, Massachusetts.

Valente, Thomas W. (1995). *Network Models of the Diffusion of Innovations*, Cresskill, NJ: Hampton Press.

Von Neumann, J. (1966). *Theory of Self-Reproducing Automata*. University of Illinois Press, Champaign-Urbana, Illinois. Edited and completed by A.W. Burks.

Wilkinson I., James B. Wiley and Aizhong Lin 2000)) "Modeling the Structural Dynamics of Industrial Networks" International Conference on Complex Systems, New England Complex Systems Institute, Nashua, NH, USA. *Interjournal* of Complex Systems Brief Article #409.

Williams, Frederick R., Rice, Ronald E., and Rogers, Everett M. (1988). *Research Methods and the New Media*. New York: Free Press.

Wirl, Franz (1998). "Socio-Economic Typologies of Bureaucratic Corruption and Implications," *Journal of Evolutionary Economics*, 8(2), 199-220.

Wolfram, S. (1984), "Cellular Automata as Models of Complexity," *Nature*, 311 October P. 419-24.

APPENDIX

List of URLs with More Details and Personal Experiences with Complexity Models

In this appendix we offer a list of homepages that contain details, theories and simulators that the reader can experience and "play with". Despite the rich dynamics the rules of transition are simple and can be changed by the reader. The interface is friendly and no special programming skills are needed. We recommend visiting these cites in order to gain further insight to complexity.

A general homepage that presents and revisits Artificial Life: http://alife.org/

Exploring the Space of Cellular Automata is possible at: http://alife.santafe.edu:80/alife/topics/ca/caweb/

Discrete Dynamics Lab: Tools for researching discrete dynamical networks - from Cellular Automata to Random Boolean Networks and beyond: http://www.ddlab.com/

A rich homepage on Complex Adaptive Systems and Artificial Life with links to journals, conference simulations and homepages: http://lslwww.epfl.ch/%7Emoshes/caslinks.html

InterJournal is a refereed World Wide Web-based Internet journal that has been developed as part of the activities of the New England Complex Systems Institute. The central journal database consists of abstracts, comments and relevant manuscript information including pointers to the Internet address of the original articles. The papers explore wide range of fields (Physics, biological social systems etc.). Its editorial board includes leading scholars from various fields of science. www.interjournal.org

A compilation of Web links towards system and simulation links: http://www.isima.fr/ecosim/simul/simul.html

An entire universe can be explored by playing around with the simulation on the board below at:
http://www.lycos.com/wguide/tools/pgview.html?wwbestof=Y&wwtitle=Artificial%20Life&wwdoc=http%3a%2f%2fwww.student.nada.kth.se%2f%7ed95-aeh%2flifeeng.html&wwmid=50853&wwdocid=90614&wwprate=0.86&wwdoctype=2

Complete descriptions and illustrations of cellular automata as well as simulators that can be downloaded for personal exploration can be found at: http://www.mirwoj.opus.chelm.pl/
A homepage that focuses on the Ising Model which is the foundation of the complexity science:
http://www.treasure-troves.com/physics/IsingModel.html

Leigh Tesfatsion's personal homepage is a good source of related to complexity references.
http://www.econ.iastate.edu/tesfatsi/