Empirical Evidence of Long-Run Order in Retail Industry Dynamics

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EXECUTIVE SUMMARY

I describe an extension to a recently reported interacting-agent model of retail competition and innovation, which displays a subtle type of order known as self-organized criticality, or SOC. SOC is a stochastic steady state that is "poised" so that a small exogenous perturbation may result in anything from no response, to a system-wide avalanche-like response. Such poised systems have been shown to be superior on some measure (such as "fitness" in ecology) for the system as a whole.

The extension is designed to validate the model by generating falsifiable predictions about retail industry dynamics. Specifically, the extension allows firms to exit, while preserving the self-organized critical state of the entire system. The model predicts that exit data should show the power-law behavior that is the footprint of a critical system. Drawing on work that relates the characteristics of individual events in a SOC system to an aggregate time series, I predict the functional form of the power spectrum of exits for firms in a spatially competing industry. I then show that Canadian retail bankruptcies follow this form, implying that the Canadian retail industry can indeed be described as a critical system—neither stable and in equilibrium, nor completely unpredictable.

One of the requirements to generate SOC behaviour in the model is spatially localized competition, a feature which is much more predominant in retailing than in other industries. An important and predictable result, therefore, is that bankruptcy data for all Canadian industries combined do not show this behaviour. Implications for the state of the retail industry, public policy, and future research are discussed, including the intriguing possibility that a poised state may be the economically healthiest state for an industry.

Keywords: Competition, Market Structure, Complexity, Self-Organization, Retailing, Innovation, Estimation and Other Statistical Techniques, Networks and Marketing

Empirical Evidence of Optimal Long-Run Order in Retail Industry Dynamics

The search for regularities in the complex dynamic evolution of competition in retailing—the focus of this paper—has long attracted the interest of academics. Interacting-agent models and complexity theory seem particularly appropriate research tools for this endeavor. One reason is that the complex dynamics and rapid changes in retailing worldwide make equilibrium models somewhat suspect. Another is that retail competition has a strong spatial component, which translates to *local interactions* among firms (e.g., Rust and Donthu, 1995). Interacting-agent models (a type of cellular automata) are ideally suited to handle both of these characteristics. Furthermore, interacting-agent models can show the connection between simple micro-level behavior of individual firms (or their managers), and emergent, often surprising, macro-level structure at the industry level. They allow the researcher to conduct simulation experiments to rapidly identify factors that determine macro industry outcomes, whether these are in the realm of the purely descriptive (such as “wheels” or “accordions” of retailing), static equilibria, dynamic steady states, or no discernible order whatsoever.

Complexity paradigms have become established in organizational behavior research (e.g. Anderson et al., 1999), and are becoming important in financial economics. Indeed, Nobel Laureate Harry Markowitz stated in a review of a recent complexity book “Microscopic Simulation of Financial Markets” (Levy, Levy, and Solomon, 2000), that it “points us towards the future of financial economics. If we restrict ourselves to models which can be solved analytically, we will be modeling for our mutual entertainment, not to maximize explanatory or predictive power." That statement applies to marketing as well. Early examples include Hibbert and Wilkinson (1994), and the complexity modeling approach is now appearing more frequently in the mainstream marketing literature (e.g., Wilkinson and Young, 2002). In particular, Goldenberg, Libai, and Muller (2001; 2002) have recently demonstrated the usefulness of interacting agent cellular automata models to study diffusion processes, and to explain observed unusual aggregate diffusion patterns.

I study an extension of a recently reported model of dynamic spatial competition (Krider and Weinberg 1997; KW hereafter). The original KW model generates discrete waves of innovation; the extension studied here allows for bankruptcy as an alternative to successful innovation, and generates temporal clusters of bankruptcies. These waves or clusters appear random and unordered (compare, for example, Modis and Debecker, 1992). This matches the observed dynamism in retailing.

The model also reveals underlying structure, in the form of power-law distributions of system properties. These distributions are a falsifiable model prediction, and I confirm their existence in spectral analysis of retail bankruptcy data.

Power Laws

The objective of this research is to establish the possibility of Self-Organized Criticality as a steady state in retail competition. The empirical results hinge on detecting power law distributions. In this section, I will provide a brief overview of power law distributions and why they are interesting in their own right. (Note, however, that explaining power laws per se is not a purpose of this research).

Economic power laws were first observed by Pareto (1897, as cited in Mandelbrot, 1960), who noted that the fraction of the population with wealth w is proportional to a power of w: $P(w) \sim w^{-1-\alpha}$. Over the last 100 years, α has remained consistently in the neighborhood of 3/2 in capitalist economies. The origin of the Pareto law remained a mystery for much of that time. Some early attempts to explain it include Champernowe (1953), who provided an explanation based on a logarithmic random walk, and Mandelbrot (1960), who used the theory of stable Levy distributions. Mandelbrot gave a central-limit argument, showing that the sum of random shocks with infinite variance could converge to a distribution with Pareto-like tails. More recently, Reed (2003) has presented a model which describes both tails of the Pareto income distribution, by incorporating population age distribution dynamics.

While income distribution regularities have received attention in the literature, other economic power laws have only recently been explored. Developments in complexity science, grounded in statistical physics (and the emerging field of econo-physics), have begun to shed some light on this issue, primarily through the use of cellular automata simulations. Very recently, a few analytic models have appeared that make connections between economic micro-dynamics and some financial market power law distributions (e.g., Solomon and Richmond 2001; Louzoun and Solomon 2001; Gabaix et al 2003). The long history of empirical power laws and the beginnings of understanding of their origins are one reason that power law distributions are interesting.

A second reason is that power laws in physics are often associated with a transition region between two stable states—a classic example being the phase transition between solid and liquid. On either side of the transition, some dynamic properties tend to follow exponential distributions, with very rapid declines. As a system goes through the transition, the tails of the distributions become "fat", and events that were rare become less so. Yet, this only occurs in the very special, narrow transition regions. In this context, power laws are interesting because their very occurrence should be unusual. Furthermore, the shift from thin tails to fat tails as the transition region is crossed means that otherwise rare events in the system become commonplace.

The final and most compelling reason that the appearance of power laws is interesting, however, is that when they occur in competitive systems where individual agents (or species in ecology) are attempting to maximize some individual characteristic, the appearance of power laws can be associated with a maximum in system characteristics, such as highest overall fitness in ecological systems. This is discussed in more detail in the final section of the article (also see Kauffman and Johnson, 1992).

In the following sections, I first review a model of dynamic spatial competition among retail outlets (KW, 1997). The discussion emphasizes why the model outcome is surprising. I next describe a variation that allows entry and exit, and an analysis which predicts that the power spectrum of exits for firms in a spatially competing industry will follow a power law with exponent –2. Finally, evidence is presented that shows that this is indeed the case for retail bankruptcies. The article concludes with a discussion of implications and future research directions. The overall approach shows the usefulness of one area of complexity theory in understanding retail competition.

THE KW MODEL

The Retail Context

The KW model builds on the oft-stated intuition that retailing is a highly competitive and rapidly changing industry. Retail managers must continually be on guard to defend their markets—and revenues—from erosion due to both environmental and competitive forces. Successful retailers respond with continual innovation, with the result that we regularly see waves of major change in the industry (Corstjens and Doyle, 1989). New innovative formats sweep retailing, with colorful labels such as category killers, power retailers, and supercenters. On a smaller scale, retailers continually rearrange their assortment and adjust their positioning strategies, in an attempt to draw more customers than their competition. In spite of this continual change, retailing, on some level, has long given researchers the impression of underlying orderliness. Many attempts have been made to define this underlying order, typically in terms of cycles (also with colorfully descriptive names, such as the wheel of retailing and the accordion of retailing). The KW model is much in the spirit of the long history of attempts to find order in the apparent disorder of retailing dynamics.

A general challenge in applying methods from complexity science to competitive dynamics should first be noted. Researchers would like, as much as possible, to derive their models from the substantive field of interest, as opposed to simply re-interpreting models appropriated from other fields. The latter is usually a less desirable, if common, approach. In the early stages of such research, this is a particularly difficult challenge, as it is precisely the limitations of existing models that motivate researchers to turn to new paradigms. The KW model is appealing (and perhaps unusual) in this respect because it is built on established substantive theories and models of bounded rationality, and of spatial competition in retailing. Complexity science methods are used only for analysis of the model.

KW model details are briefly reviewed next. Further details can be found in Appendix A.

The KW Model's Micro-level Behavior

The competition for customers is captured with a Multiplicative Competitive Interaction model (e.g., Cooper and Nakanishi, 1988; Hansen and Weinberg, 1979), with trading areas limited by customers' reservation distance, *R*. (e.g., Carpenter, 1989; Ghosh and Craig, 1991). Managerial decision-making is grounded in Simon's (1955, 1965) theory of bounded rationality. Two features of bounded rationality are particularly relevant: first, that the search for better solutions is undertaken only when it is observed that goals are not being met (reaction), and second, that the decision mode is one of implementing a solution which at least meets the goals, even though it may not be the best possible solution (satisficing). The premise that managers must rapidly deal with large amounts of complex information is also consistent with observational studies that characterize managerial behavior as varied, brief and fragmented (see for example Martinko and Gardner, 1990; Mintzberg, 1971). Finally, the model incorporates environmental adversity in the form of shocks to a store-specific environment term in the share attraction model.[1] A vivid example of such adversity and subsequent reaction was Nordstrom's response to the slowdown in consumer spending in the U.S. after September 11, 2001: "Nordstrom is slashing prices by as much as 60%, in its first early-autumn sale in 40 years" (The Economist, Oct 6, 2001). Other sources of environmental adversity are demographic changes (locally or globally), labour problems, supplier conflicts, closure of nearby complementing stores (as in a bakery complementing a butcher shop), even burglary and fire.

A reduction in firm revenues may or may not (depending on the severity of the reduction) cause the satisficing manager to react with an innovation that increases the firm's competitiveness. If the manager does react, the retail outlet captures market share (and hence revenues) from any competitors that have overlapping trading areas with its own trading area. This causes a reduction in revenues of those stores, some of which may be driven to respond by innovating, and in turn taking market share from their nearby competitors.

The model is stylised in a way that keeps the dynamics simple, while capturing essential elements of spatial interaction and satisficing. It does not specify precisely which marketing mix variables are invoked to improve revenues, only that something can be changed for the better. Similarly, exogenous shocks are always depressing. It is conceivable that an exogenous shock could be positive as well. This possibility could be studied easily enough in the future. However, if the *net* effect of shocks is negative, (an assumption with some face validity) the same results should occur. Net *positive* shocks would imply net growth but the finite market would eventually limit that growth. A growing *market* is another possibility worth investigating. Many of the recent financial models previously discussed study growth and could provide some insights. A final criticism of KW is that it is a simulation. The micro-behaviour is not connected to the macro behaviour analytically. This is normal progress in this type of research. After many researchers have worked with a cellular automaton on a particular problem, enough insight develops that it sometimes becomes possible to consider analytic connections. I offer up this model as a likely prospect, and provide some empirical validation in this article, primarily as an encouragement for others to pursue either different scenarios, which the model can easily incorporate, or perhaps the more difficult analytic task of deriving the macro dynamics from the micro dynamics. As previously noted, this is just now beginning to happen in modelling financial market distributions that display power laws.

1. This term is typically used to introduce elasticity into share attraction models. See, for example, Choi, DeSarbo, and Harker (1990, 1992).

The KW Model's Macro-level Behavior

As shocks are repeatedly delivered and firms respond, the system eventually converges to a steady state, confined to a bounded region in revenue state-space, regardless of initial conditions. To this point, the model owes nothing to complexity theory or any of its precursors, such as chaos theory, fractals, or self-organization. To *analyze* the steady state, however, the first such appeal is made. KW follow other researchers who have studied complex geographic spatial structures (e.g., White and Engelen, 1993; Batty et al., 1989). This work shows how simple models can produce complex spatial "fractals."[2] A simple approach to detecting fractal structure is to plot frequency distributions of characteristics of the system under investigation. KW focus on the distribution of the number of stores that innovate after a shock. A priori, one can identify (at least) four possible steady state response distributions:

1. Each shock produces a ripple that sweeps the entire system. The plot of the distribution will be a spike at the system size (i.e., all stores innovate).
2. Only one store innovates with each shock. The distribution will be a spike at the response size of 1.
3. If a shock has an exponentially declining probability of affecting a store as distance of the store from the shock increases, the distribution will show many small events (a few stores responding), and an exponential decline with increasing response size.
4. The shock produces a size distribution that follows a power law. The probability of affecting a store declines with distance, but much more slowly than exponentially.

The first two possibilities define the extremes. The last two are intermediate cases. As discussed in the following theory section, the last possibility seems to be a rather special outcome.

Avalanche Size Distribution

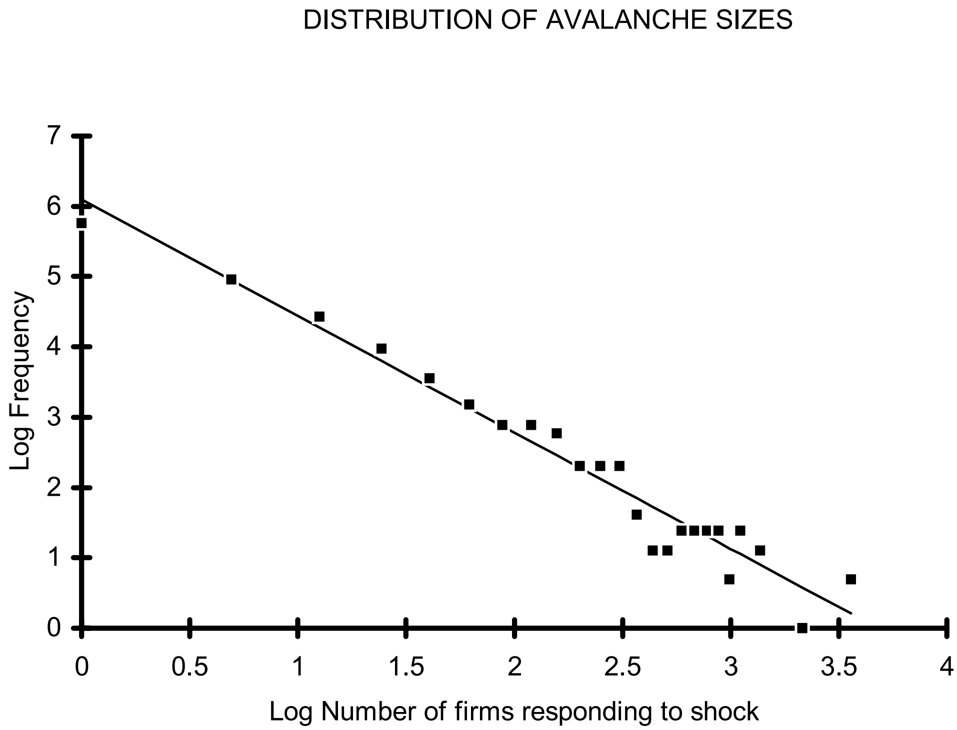
Figure 1 is a log-log plot of the size distribution of the avalanches at steady state. The striking feature is the apparent power law behavior. The probability of large responses does not decline exponentially, as in possibility 3 above. It decreases much more slowly, with a "fat tail" decline, characterized by a power law (possibility 4 above), with an exponent of about -1.6 (the slope of the regression line, superimposed on the plot). That is, a large number of firms innovate simultaneously more frequently than would be expected if the number of innovators were normally distributed.

Summary of Model Behavior

Casual observation of the evolution of the model's steady state suggests a high degree of irregularity. Random shocks occur, with usually very few stores responding; occasionally, large numbers—possibly all stores in the system—respond to a single shock. The number of stores involved in a wave of innovation is highly variable, and resonates with our intuitions of real world retailing. In spite of such irregularity, the model displays order in the form of self-similar distributions of events. KW identify the order principle operating, and study its characteristics.

2. A fractal structure is self-similar at any scale: no matter how much—or little—one magnifies the structure, it looks the same. Self-similarity is indicated quantitatively by power-law distributions of various measures of the structure.

FIGURE 1
Size Distribution of Avalanches in the Steady State and Regression Line



THEORY

Shocks, Fluctuations, and Criticality

To lay the groundwork for identifying an order principle, KW turn to two other fields—economics and statistical physics—to address an important question: should we be surprised by this macro-level result? Much more detail is available in that article, but here I will briefly review the main points. As well, the reader can refer to a recent overview article which admirably summarizes the large field of complexity science and its relevance to marketing in this special issue of AMSR (Holbrook, 2003).

The power law distribution means that the probability of large events is much higher than in a normal distribution (or in other distributions in the exponential family). Bak, et al. (1992) note that obvious micro-dynamics, such as simply adding random shocks, would be subject to the central limit theorem, which in turn would lead to a much more rapid decay in the distribution's tail. To that extent, the power law *is* surprising. One other scientific field where large fluctuations occur with precisely power law distributions is critical phenomena in statistical physics. The observed power laws are "critical" in the sense that they only occur at very special combinations of parameters. And, once again, to observe power laws without *explicitly controlling these parameters* would be very surprising. The question thus arises as to whether the observed power law behavior of the KW avalanche distribution results from a fortuitous combination of parameters, or if it is more robust—and the answer is that it is surprisingly robust, not only to parameter values, but even to some changes in the model structure! Even more interestingly, it appears that a certain minimal level of model complexity is required. The fact that a minimal level of complexity in the model is required, after which the behavior of interest is robust to subsequent model changes, increases the likelihood of observing the behavior in the field[3].

In a seminal article entitled "Self-Organized Criticality" (SOC), Bak, Tang, and Weisenfeld (1988) described a similar system, which has both the unstable characteristics of criticality, and the robust characteristics of a dynamic self-organizing system. I will briefly note some of the characteristics of the self-organized critical state:

- stability: *the steady state is an attractor for the system, in the sense that after any large shock, the system will return to the same (stochastic) steady state.*
- extreme robustness: *the steady state power law behavior is robust to changes in initial conditions, parameter values, and, most strikingly, even to some changes in model structure.*
- power laws: *at steady state, the distributions of system variables follow power laws.*
- poised: *at steady state, a small localized disturbance can (but does not have to) cause an avalanche of response through the entire system.*
- Weakly chaotic: *neither highly predictable nor highly unpredictable.* [4]

The first two points above justify the label "*self-organized*". The last three points are characteristics of *criticality*. I also note that, other than the local interactions and exogenous shocks, the structure of the KW model has *nothing obviously in common with existing models that display SOC*. KW conducted a series of numerical experiments to establish SOC as the relevant "order principle" operating in the model.

While there is no animation available for the KW model, several animations of variations on the classic sandpile models are available on the web. These may provide the reader some sense of the superficially disordered appearance and the underlying order of these models.

1. a one-dimensional model which shows the detail of stylized tumbling sand grains; http://www.maths.nottingham.ac.uk/personal/etzkih/anim_start.html
2. A two-dimensional model, where the grains of sand are added to the centre of a table: http://theorie.physik.uni-wuerzburg.de/~kinzel/compphys/pattern.html
3. A two-dimensional model where grains of sand are added at random locations on a table top. This model is the closest to the KW model in that shocks are administered to random geographic locations. However, the nature of the shocks, the interactions between neighboring locations and the boundary conditions are entirely different http://theorie.physik.uni-wuerzburg.de/~kinzel/compphys/sandpile.html

Innovate or Exit

A major limitation of the KW model is the assumption that whenever a firm starts to get into trouble, it can always get out of trouble—firms never exit. Retailers, however, *do* fail. When revenues drop, some firms will be unable to recover, and will instead go bankrupt. The most parsimonious failure assumption that can be incorporated into the model is that for every avalanche of innovation in the original model, a proportion of marginal firms are actually unable to innovate, and therefore fail. To maintain the system, I also assume that this firm is replaced by another firm, which, at least initially, is competitive enough to generate revenues adequate for survival.

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3. Our hope that the real world is complex enough to be represented by a model is, interestingly, the reverse of the usual modelling exercise, where the modeller hopes the world is *simple* enough to be represented by a model!

4. Chaotic systems are strongly sensitive to initial conditions, in that two systems that start close together in state space will diverge at an exponential rate. In contrast, two stable systems will converge. SOC models typically diverge with power law rates, that is, more slowly than exponentially. The KW model diverges at a linear rate, much more slowly than common chaotic systems. Its evolution is neither highly predictable, nor highly unpredictable.

Let $F = kn^{-a}$ be the power law distribution of the number of firms n whose revenues drop low enough after a shock that they must either innovate, or exit, with parameters k and a, given that at least the shocked firm drops below its threshold. This corresponds to the distribution of firms in the original model that are involved in an avalanche of innovation. Let the number of firms b which exit be a fixed proportion p of the total number of firms n whose revenues drop below the threshold after a shock, so that $b = pn$. The distribution of the number of firms exiting is thus $F = kp^a b^{-a}$. Under this parsimonious structure, bankruptcies also have power law distributions, with the same exponent as the original model's innovation distributions, shifted down by a constant proportion p^a relative to the innovation distribution.

As KW, and others (e.g., Bak et al. 1992), have argued, the appearance of power laws in the distribution of macroeconomic events—bankruptcies in this case—does not, on the face of it, appear to be a likely outcome. Exponentially declining distributions, such as the normal distribution, would seem more likely. Power-law structures are thus a falsifiable prediction of the model.

Hypothesis 1: The distribution of clusters of retail bankruptcies is power-law.

Bankruptcy data is in the form of number of bankruptcies per time period. A crucial problem, therefore, is that in such data there is no obvious way to determine what constitutes a single wave-of-bankruptcy event, and hence measure its size and then determine the distribution of sizes of responses. An approach to handling the problem has been developed by Christensen et al. (1991). When a long time series is produced by an overlapping superposition of short time series, or pulses, with specific characteristics, the resulting long time series will have features that depend on the characteristics of the short pulses. Even though we can no longer identify an individual pulse within the series, we can look for their footprints in the full series. In our case, we can model the bankruptcy time series as the summation of overlapping waves of bankruptcies, each with power law size distribution. In the single full series, we cannot examine the pulse size distribution, but we can use spectral analysis to examine how the energy in the series varies with frequency ω. Christensen et al. (1991) showed that the power spectrum of the full series carries information on the size distribution of the individual pulses. The objective here, then, is to determine if the power spectrum of the bankruptcy data shows the predicted footprint of SOC. The logic of Christensen et al. (1991) is as follows.

Let $\delta(\boldsymbol{x},t)$ be an indicator function, which takes the value 1 if a bankruptcy occurs at location $\boldsymbol{x}$ at time t. The bankruptcy rate $f(t)$ associated with a particular response avalanche, which is triggered by a single shock, is given by integrating over the locations in the system,

$$f(t) = \int \delta(\mathbf{x},t)\, d\mathbf{x} \qquad \textbf{(1)}$$

where $f(t)$ is zero before the triggering shock occurs at time t_0, and zero again after the system stabilizes, at time $t_0 + T$. (Note that each location will respond at most once to a single shock). Consider two avalanches to be of the same type if and only if they produce the same trace as defined above, except for a shift in start time t_0. Indicate the type of avalanche by A, and the rate trace associated with this type as $f_A(t)$. The power spectrum of this type of rate trace is calculated by decomposing the time series into constituent additive traces, each consisting of a single wavelength, or frequency ω, using the Fourier transform. The square of the Fourier transform results in a spectrum—a function showing the power in the series at each frequency—much as the power in an audio signal can be broken into the power at low, midrange, and high frequencies (see, for example, Brockwell and Davis, 1991). The power spectrum of the type A trace, $S_A(\omega)$, is defined as

$$S_A(\omega) = \left\| \int f_A(t) \exp(i\omega t) dt \right\|^2 \quad \textbf{(2)}$$

Most statistical software packages and all time series software will perform this calculation quickly. Define *P(A)* to be the probability of an event of type *A*.

If a time series is constructed by superimposing a large number of individual events $f_A(t)$, with random start times, *Christensen* et al. (1991) show that the power spectrum of the time series, $S(\omega)$, can be expressed as a weighted average of the spectra of the individual types of avalanches, weighted by the probability of occurrence:

$$S(\omega) = v \sum_A P(A) \cdot S_A(\omega) \quad \textbf{(3)}$$

where *v* is the rate at which events are triggered, and *P(A)* is the probability of an event of type *A*. Assume that signals $f_A(t)$ associated with each avalanche event can be characterized by the individual lifetime *T* and size n; that is, $A = (n,T)$. This allows the weighting probability *P(A)* to be expressed in terms of the joint distribution of n and T, and the single avalanche spectrum $S_A(\omega)$ to be expressed in terms of the spectrum of an elementary event, a single bankruptcy. Christensen et al. (1991) then go on to show that for a wide variety of models that display SOC, as determined by power law behavior in the distribution of event sizes and durations, the power spectrum is *also* a power law function of the *frequency*, and that the exponent is -2. Chau and Cheng (1992) confirm this result for a large class of discrete models. Thus, we can test Hypothesis 1 by calculating the power spectra of the bankruptcies. But with the above results, we can go further, and specify what the exponent of the distribution should be if the system is self-organized critical.

Hypothesis 2: The power spectrum of bankruptcies follows a power law, with exponent -2.

DATA AND METHOD

Twenty-one years of data, starting in 1980, of the number of Canadian trade bankruptcies each month was analyzed[5]. Figure 2 shows the bankruptcy time series (Statistics Canada, CANSIM database). The power spectrum is the square of the Fourier Transform of this series. There are two technical problems, however, that first must be dealt with.

The first is that this calculation is inherently very noisy. The variances of such estimates do not decrease with the length of the series. There are a number of methods of dealing with this, such as pre-smoothing the original time series and sacrificing resolution. In certain physical applications, it is also possible to take repeated measurements of a series and average the results to reduce the variance. This method is not normally possible with econometric data; however, in this case, the series for the individual provinces, which sum to the Canadian series, are also available. It is therefore possible to reduce the variance by calculating the Fourier Transform for individual provinces and summing them. The second problem is that a linear trend in the data will contribute power to the single-cycle component (frequency = 1). The data is therefore detrended before transforming.

5. The "trade" category consists mainly of retailers. From 1990 on, data was available on wholesale and retail bankruptcies separately. For these 11 years, retail bankruptcies averaged 78% (s.e. 3%) of the total trade figures.

FIGURE 2
Monthly Canadian Trade Bankruptcies, January 1980 to March 2002

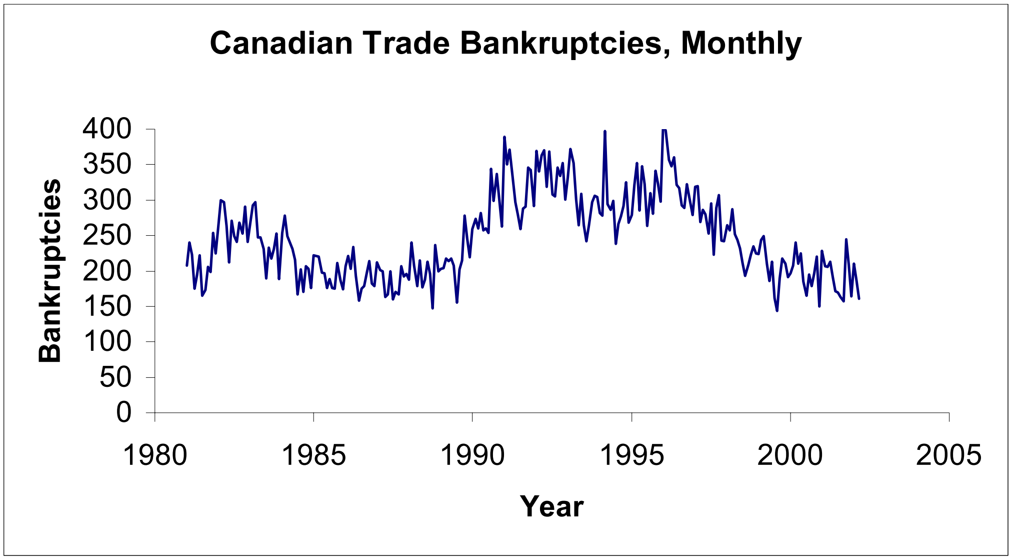
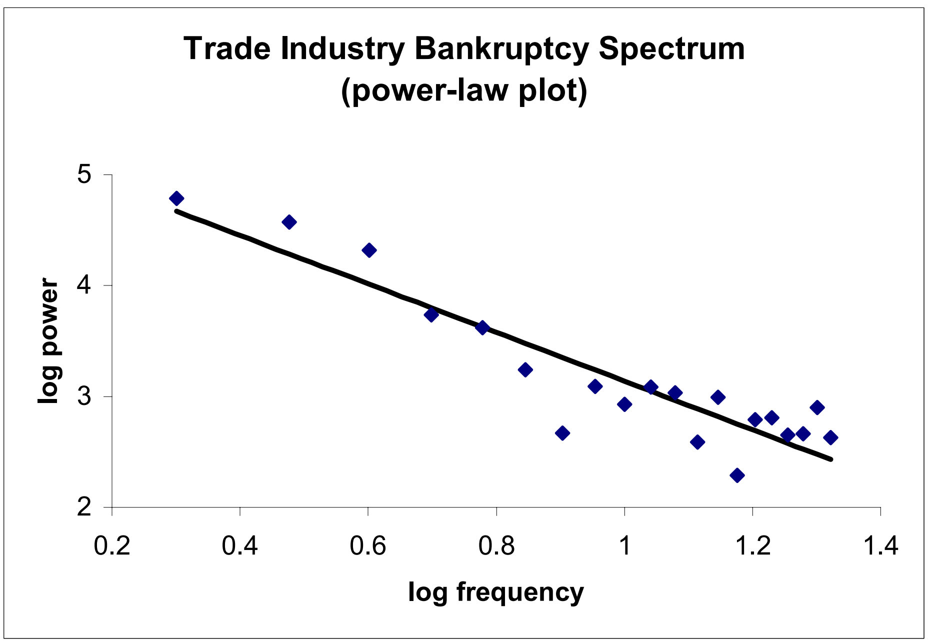


FIGURE 3
Log-log plot of the power spectrum of the bankruptcies in Figure 2 and superimposed regression line. The linear decline in log power corresponds to a power law spectrum, with exponent -2.2 (s.e 0.22; t-stat = -9.9; adjusted R^2 = 0.84)



Results

Figure 3 shows the power spectrum of the detrended series, on a log-log scale, with a superimposed regression line. The plot shows the power in the series at frequencies ranging from 1 to 20 cycles over the 22-year period[6]. The approximate power law behavior is apparent. Hypothesis 1 is supported.

6. Here, we measure frequency in cycles per 22 years. Audio frequencies, for comparison, are measured in units of Hertz, or cycles per second. (Note: the period is the time between successive peaks, and the frequency is the inverse of the period).

Furthermore, the slope of the regression line is -2.2, with a 95% confidence interval of ±0.43. This is consistent with Hypothesis 2, which predicted a value of two for the exponent.

Table 1 provides benchmarks for this result in three ways. First, regressing the frequency against the log power tests an exponential law, a common structural form and plausible alternate hypothesis. The result indicates that this is an inferior model. A second plausible hypothesis is that the decline may simply be the tail of the ubiquitous normal distribution. By regressing the frequency against the square root of the log power spectrum, we see that this is also inferior. In short, the tail of this distribution is "fat" compared to exponential and normal distributions. Third, the total Canadian bankruptcies for *all* industries (not just trade) are analysed by the same methods. Recall that the hypotheses are derived from a cellular automata model whose interactions are described by long-standing and empirically validated models of retail competition. There is no compelling a priori reason why we would see similar results in non-retailing industries (except the possibility that non-retailing industries may have competitive interactions similar to retailing, except in product space). Hence we would expect a poorer fit of a power law to the data (compare the left and right sides of the top row of the table), which is also the case.

TABLE 1
Benchmarking the Power-Law Behavior of Retail Dynamics Against an Exponential-Law Model, a Normal-Law Model, and Against All Other Industries

	Retail Bankruptcy Power Spectrum			All Industry Bankruptcy Power Spectrum		
	Slope	t-stat	Adj. R^2	Slope	t-stat	Adj. R^2
Power Law regression	**-2.19**	**-9.9**	**0.84**	-3.16	-5.0	0.56
Exponential Law Regression	-0.09	-5.8	0.63			
Normal Tail Regression	-0.02	-5.9	0.64			

CONCLUDING REMARKS

Corstjens and Doyle (1989) describe the central dynamic of retailing as the continual erosion "by maturing markets and aggressive competitors," with the response being either minor repositioning, or, occasionally, a major change. The Economist (1995), in a special report on retailing stated, "In the past 15 years, retailing has undergone a many-sided revolution from which it has emerged as a leader in business *innovation* and the management of *complexity* [emphasis added]." Technology evolution, most notably the Internet, and political shocks such as 9/11, affect retailers strongly and quickly.

The KW model extension captures the notion that, in response to continual erosion by both competitive and non-competitive forces, retail managers "scramble astutely," or fail. The model evolves to a poised state, where the probability of a large response to a shock declines—surprisingly—according to a power law. The likelihood of a system-wide wave of response, in the form of either innovation or exit, in response to a small shock is much higher than the exponential decline one would expect from simple smoothing mechanisms. The result is not a static industry, but one that continually undergoes a "many sided-revolution" while remaining innovative and strong. In fact, in other contexts where individual entities interact and attempt to improve their survival chances, or fitness, the critical state has the highest *system* fitness.

On the basis of the model, I predict power law distribution of bankruptcy power spectra in Canadian trade data, and find support for the prediction. Furthermore, the magnitude of the exponent is consistent with other work showing that discrete SOC systems should have $1/f^2$ power spectra (Christensen et al. ,1991; Chau and Cheng, 1992). The implication is that the Canadian retailing industry is "poised at the edge of chaos." Any disturbance may cause nothing to happen, or sometimes cause a few retailers to respond or, occasionally, may cause a large wave of change to sweep across it. Marginal retailers, susceptible to these changes, show up in bankruptcy statistics. The observed power spectrum of the bankruptcy time series may be the footprint of a system self-organized to a weakly chaotic critical state.

More generally, I have taken an intuitively appealing model that combined established models of spatial competition and of managerial behavior, and analysis techniques from complexity science, and extended it so that it could be used to generate falsifiable predictions. This allows testing the model against data, an important step in establishing the boundaries of applications of new paradigms. The positive results suggest that self-organized criticality is a useful paradigm to understand dynamic spatial competition, and that more work on the implications is well-justified.

Power law distributions and correlations are currently the subject of intense study in a number of fields. A stream of research related to the current work has focused on explaining the power laws in financial economics, such as the Pareto law of individual wealth distribution, and the Levy-stable distribution of volatility in stock markets. These phenomena have been investigated primarily using simulations of many agents interacting according to micro-level rules to produce macro-level outcomes with intermittent fluctuations and stable distributions. Some success has recently been achieved in proving analytically that power law behaviour arises from stochastic generalizations of the Lotka-Voltera equations (e.g., Solomon and Levy 1996; Louzoun and Solomon 2001). These formalisms have some similar characteristics to the KW model, in that many agents are driven by multiplicative interactions with each other and by random shocks to individual agents. It appears that stochastic multiplicative dynamics may be the source of many naturally occurring power laws. On the other hand, these models differ in that an individual agent—nominally an investor in this stream—interacts with the system as a whole (a "mean field" effect), rather than with a spatially close neighbour. See Levy, Levy, and Solomon (2000) for detailed review of the simulation approach to financial markets.

Optimality of the SOC State

For retail managers, the terms, "critical" and "edge of chaos" may have a negative connotation. Retailing as an industry, however, is consistently healthy. The Economist's (1995) report on retailing states that retailers "exert enormous power and influence over manufacturers and consumers—and over urban, suburban and rural environments the world over…Many of the star stock market performers of the past two decades have been retailers." The past five-year stock returns of the retail sector (as reported by Morningstar.com) far exceed those of any other sector listed. Is this consistent with a poised system? The likely answer is 'yes'. The reason is simply that neither a static, unchanging system such as a centrally planned economy, nor a completely unregulated, anarchic system is likely to be optimal. The best place, from a system wide perspective, should be somewhere between the two. This still leaves a wide range of possibilities. For insight on precisely *where* between these two extremes is best, we can look to research in theoretical biology, such as Kauffman and Johnson (1992), who study co-evolution in a model that incorporates short-term localized optimising behavior. They describe a range of systems that evolve to a continuum of states, from static Nash equilibria, through to completely chaotic states, as system parameters are changed. Systems (nominally ecological) with the *highest fitness levels* occur at the critical region, poised on the edge of chaos. Routledge (1994) developed an interesting example of an economic system involving many agents playing a repeated prisoner's dilemma game on a torus, with evolving strategies. He also found that maximal system wide fitness occurred in an intermediate zone between frozen Nash equilibria and wildly fluctuating states—that is, at the edge of chaos. Gabaix et al. (2003) published a model in Nature that shows "that the power laws observed in financial data arise when the trading behaviour is performed in an optimal way." Is it also true that, in economic competition, the fittest industries, with, for example, highest industry-wide profits or greatest return on equity, also occur at the edge of chaos? This is certainly consistent with our observation that retailing is a relatively strong industry by these measures, and that it appears to be self-organized critical. Furthermore, if it is true, there are important policy implications: how much and what kind of control does a system need to achieve the critical state? Or, if left alone, would it self-organize to that state, simply by virtue of system profits being highest? These and other questions now appear to be tractable, and remain for future research.

REFERENCES

Anderson, P., A. Meyer, K. Eisenhardt, K. Carley, A. Pettigrew 1999. "Introduction to the Special Issue: Applications of Complexity Theory to Organization Science." *Organization Science*, 10 (May-June): 233-6.

Bak, P., K. Chen, J. Scheinkman, and M. Woodford 1992. "Aggregate Fluctuations From Independent Sectoral Shocks: Self-Organized Criticality in a Model of Production and Inventory Dynamics." Working Paper No. 4241, National Bureau of Economic Research, Inc.

_________, C. Tang, and K. Weisenfeld 1988. "Self-organized Criticality." *Physical Review A*, 384: 364-374.

Batty, M., P. Longley, and S. Fotheringham 1989. "Urban Growth and Form: Scaling, Fractal Geometry, and Diffusion-Limited Aggregation." *Environment and Planning A*, 21: 1447-1472

Brockwell, P.J., and Richard A. Davis. 1991. *Time Series: Theory and Methods*. New York: Springer Verlag

Carpenter, G.S. 1989. "Perceptual Position and Competitive Brand Strategy in a Two-Dimensional, Two-Brand Market." *Management Science*, 359: 1029-1044.

Champernowne, D.G. 1953. "The Dynamics of Income." *Economic Journal,* 63, 250: 318-351.

Chau, H.F. and K.S. Cheng 1992. "Relations of 1/f and 1/f^2 Power Spectra to Self-Organized Criticality." *Physical Review A*, 466: R2981-R2983.

Cheng, Y.T. and A.H. Van de Ven 1996. "Learning the Innovation Journey: Order out of Chaos?" *Organization Science*, 7, 6: 593-614.

Choi, S.C., W.S. DeSarbo, and P.T. Harker 1990. "Product Positioning Under Price Competition." *Management Science* 362: 175-199.

________, _______, _______ 1992. "A Numerical Approach to Deriving Long-Run Equilibrium Solutions in Spatial Positioning Models". *Management Science*, 381: 75-86.

Christensen, K., H.C. Fogedby, and H.J. Jensen 1991.Dynamical and Spatial Aspects of Sandpile Cellular Automata. *Journal of Statistical Physics*, 63, (3 / 4): 653-683.

Cooper, L.G. and M. Nakanishi 1988. *Market-Share Analysis*. Boston: Kluwer Academic Publishers.

Corstjens, M. and P. Doyle 1989. "Evaluating Alternative Retail Repositioning Strategies.' *Marketing Science*, **8**2: 170-180.

The Economist 2001, "Nightmare on Fifth Avenue." October 6, 2001, 361 8242: 58-61.

Frank, M.Z. and T. Stengos 1988. "Some Evidence Concerning Macroeconomic Chaos." *Journal of Monetary Economics*, 22: 422-438.

Gabaix X, Gopikrishnan P, Plerou V, Stanley HE 2003. “A theory of power-law distributions in financial market fluctuations.” *Nature,* 423, May 15 6937: 267-270

Ghosh, A. and C.S. Craig 1991. “FRANSYS: A Franchise Location Model.” *Journal of Retailing*, 67: 212-34.

Goldenberg J., B. Libai, and E. Muller Eitan, “2001.Talk of the Network: A Complex Systems Look at the Underlying Process of Word-of-Mouth.” *Marketing Letters* 12(3): 209-21.

_____________________ 2002. “Riding the Saddle, How cross-Market Communications Creates a Major Slump in Sales.” *Journal of Marketing*, Vol. 66 April: 1-16

Hansen, M.H. and C.B. Weinberg 1979. ‘Retail Market Share in a Competitive Market.” *Journal of Retailing*, 551: 37-46.

Hibbert, B. and I. F. Wilkinson, Ian F 1994. **“**Chaos Theory and the Dynamics of Marketing Systems.” *Journal of the Academy of Marketing Science*. Greenvale: Summer, Vol. 22, 3: 218-234.

Holbrook, Morris B. 2003. “Adventures in Complexity: An Essay on Dynamic Open Complex Adaptive Systems, Butterfly Effects, Self-Organizing Order, Coevolution, the Ecological Perspective, Fitness Landscapes, Market Spaces, Emergent Beauty at the Edge of Chaos, and All That Jazz." *Academy of Marketing Science Review* [Online] 2003 6 Available: http://www.amsreview.org/articles/holbrook06-2003.pdf

Kauffman, S. A. and S. Johnson 1992. “Coevolution to the Edge of Chaos: Coupled Fitness Landscapes, Poised States, and Coevolutionary Avalanches.” *Journal of Theoretical Biology*, 149: 467-505.

Krider, R.E., and C.B. Weinberg 1997. “Spatial Competition and Bounded Rationality: Retailing at the Edge of Chaos.” *Geographical Analysis*, 29, 1: 16-35.

Levy, M., H. Levy, and S. Solomon. 2000. *Microscopic Simulation of Financial Markets*. Academic Press, New York. 2000.

Louzon, Y. and S. Solomon 2001. ”Volatility Driven Market in a Generalized Lotka-Voltera Formalism.” *Physica A* 302: 220-223.

Mandelbrot, B. 1960. “The Pareto_Levy Law and the Distribution of Income.” *International Economic Review* 1,2: 79-106

Martinko, M.J. and W.L. Gardner 1990. “Structured Observation of Managerial Work: A Replication and Synthesis.” *Journal of Management Studies*, 273: 329-357.

Mintzberg, H. 1971. “Managerial Work: An Analysis From Observation.” *Management Science B*, 182: B97-B111.

Modis, T. and A Debecker 1992. “Chaos-like States Can be Expected Before and After Logistic Growth.” *Technological Forecasting and Social Change*, 412: 111-120.

Phillips, F. and N. Kim 1996. “Implications of Chaos Research for New Product Forecasting.” *Technological Forecasting and Social Change*, 53, 3: 239-261.

Reed, William J. 2003. “The Pareto law of incomes—an explanation and an extension.” *Physica A*, 319: 469-486.

Routledge, Bryan R. 1994. "Co-Evolution and Spatial Interaction" - Working Paper. Carnegie Melon University

Rust, Roland T. and Naveen Donthu 1995. "Capturing Geographically Localized Misspecification Error in Retail Store Choice Models." *Journal of Marketing Research*, February: 103-110.

Simon, H. 1955. "A Behavioral Model of Rational Choice." *Quarterly Journal of Economics*, 691: 99-118.

_______ 1965. *Administrative Behavior*, 2nd edition. New York: MacMillan.

Solomon, S. and M. Levy 1996. "Power laws are logarithmic Boltzmann laws." *International Journal of Modern Physics C* **7**4: 595-601

__________ and P. Richmond 2001. "Power Laws of Wealth, Market order Volumes and Market Returns." *Physica A* 299: 188-197.

Statistics Canada. Matrix 0136, CANSIM database. Canadian socio-economic information management system. Ottawa Ont: Statistics Canada.

White, R. and G. Engelen 1993. "Cellular Automata and Fractal Urban Form: A Cellular Modelling Approach to the Evolution of Urban Land-Use Patterns." *Environment and Planning A*, 25: 1175-1199.

Wilkinson, I. and L. Young 2002. "On cooperating: Firms, relations and networks." *Journal of Business Research*, 55, 123-132.

APPENDIX

KW Model Details

Spatial Competition

Competition is modelled by a Multiplicative Competitive Interaction model (e.g., Cooper and Nakanishi, 1988; Hansen and Weinberg, 1979). The j^{th} store's share of the i^{th} customer's purchases is given by

$$M_{ij} = \frac{S_j^{-\alpha} D_{ij}^{-\beta}}{\sum_k S_k^{-\alpha} D_{ik}^{-\beta} + K_j} \quad , \quad D_{ij} \leq R$$

$$= \quad 0 \quad , \quad D_{ij} > R \tag{A1}$$

where $0 \leq \alpha$, ß and the summation is over all stores within customer *i*'s reservation distance, *R*. Analogous to reservation price, this is a distance beyond which customers will not travel to patronize the firm.
The term K_j represents a no-purchase option, typically used to ensure elastic total demand in equilibrium share modelling. In the context of this research, its role is to capture store-specific environmental effects that impact share without any change in competitors' behavior. The store-specific attractiveness parameter S_j may be store size or any number of other quality measures (such as product valuation minus price). Distance D_{ij} is Euclidean distance.

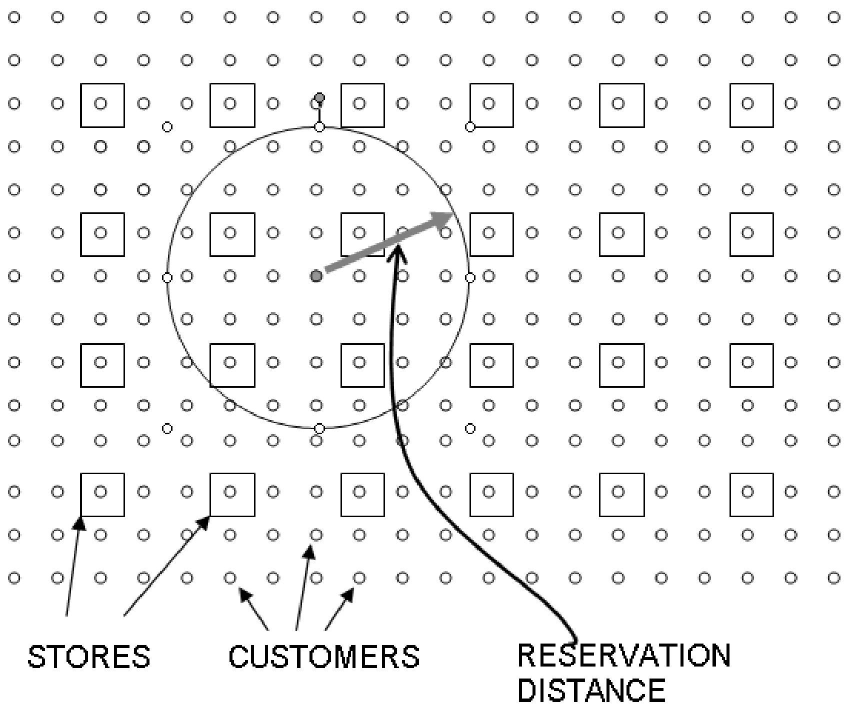
In each period, each firm's revenues are the sum of attracted customer's shares of expenditures:

$$\rho_j = \sum_j M_{ij} \tag{A2}$$

Market Configuration

Customers are uniformly distributed on a rectangular bounded plane, with customer-origin points in a regular 26 x 26 grid. Sixty-four stores are located in a coarser 8 x 8 grid. In each period, each customer has one unit (e.g., dollar) to spend of which a portion, depending on K_j, is allocated to all the stores within the customer's reservation distance according to share of attraction.

Figure A1



Dynamics

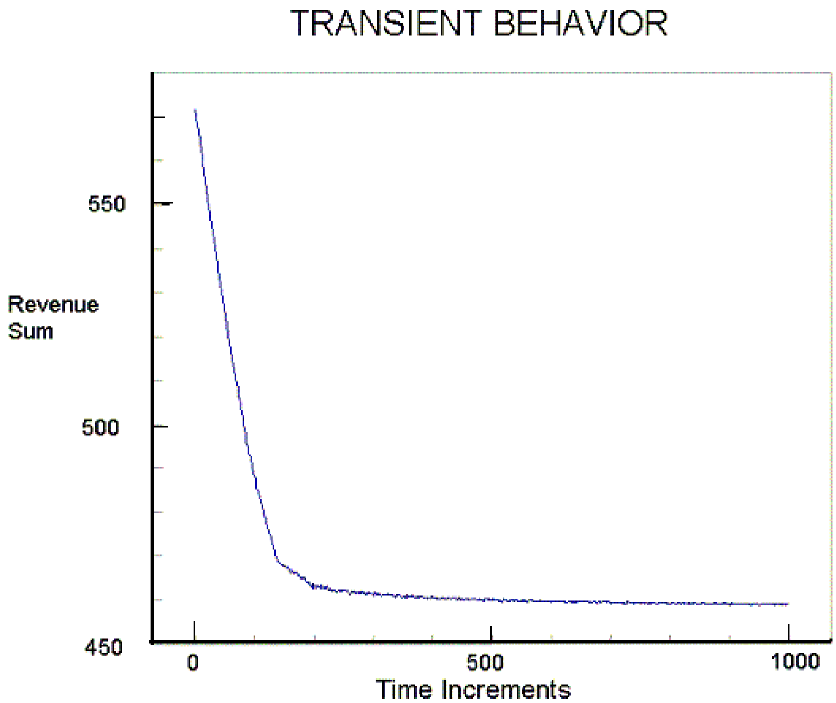
At time t = 0, store sizes and revenue thresholds (the point at which managers take action) are initialized. Store attractivenesses S_j are randomly assigned to all the stores in the plane. Initial revenue is then calculated for each store, and thresholds initialized at some fraction of initial revenue. Stores are shocked by the addition of an increment δk to K_j. Stores either innovate or exit when their revenues drop below their individual revenue thresholds. Innovation is modelled by the addition of an increment δS, to S_j. The algorithm is:

1. Shock a store chosen at random.
2. Calculate revenues of all stores.
3. If all stores have revenues above their threshold, increment time, record the total number of stores that have reacted to the shock, and return to 1; Otherwise
4. All stores whose revenues have dropped below their threshold innovate or exit in fixed proportions.
5. Return to 2.

Transient Behaviour: Convergence to the Attractor

The example below shows all 64 stores starting above their satisficing revenue thresholds and converging to a region near their threshold. In system terms this attractor is a 64-dimensional revenue hypercube.

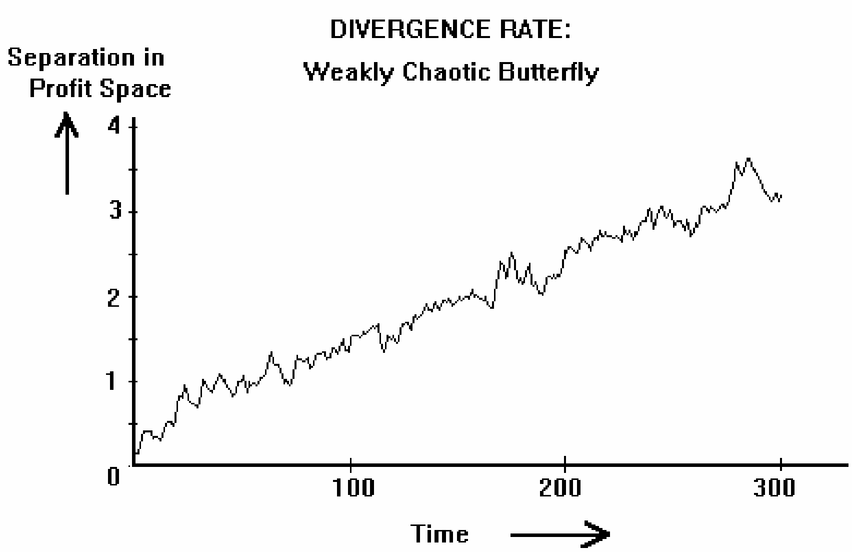
Figure A2



Steady State Behaviour: Divergence Within the Attractor

Once at steady state the system stability within the attractor is explored. Two systems were started with very small state differences and allowed to evolve. Their Euclidean separation in 64 dimensional profit space was measured and plotted below:

Figure A3



While the two systems stay within the attractor (as indicated above) they do not converge to a point, but rather diverge. The divergence rate is, perhaps surprisingly, linear. This is described as "weak chaos" because it is not an exponential divergence. This is further evidence that the system is poised on the edge of chaos.