Why (Contingentist) Actualists Should Endorse the Barcan Formula

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Abstract: On its usual interpretation, the Barcan Formula – ◊∃x B → ∃x ◊B – tells us that, if it is possible for there to exist something that is such and such a way, then there exists something that possibly is that way. For instance, it says that, if Wittgenstein could have had a son, then there exists something that could have been Wittgenstein’s son. It is traditionally held that the Barcan Formula cannot be accepted by actualists – deniers of mere possibilia – who, in addition, are contingentists, i.e., believers in (the possibility of) contingently existing beings. I shall argue, however, that contingentist actualists should endorse the Barcan Formula. I shall then respond to the concern that, if my argument is successful, it reveals a problem with contingentist actualism. I shall argue that this concern is misplaced.

Key words: The Barcan Formula, quantified modal logic, actualism vs. possibilism, necessitism vs. contingentism

Introduction

On its usual interpretation, the Barcan Formula – ◊∃x B → ∃x ◊B – tells us that, if it is possible for there to exist something that is such and such a way, then there exists something that possibly is that way. For instance, it says that, if Wittgenstein could have had a son, then there exists something that could have been Wittgenstein’s son. The Barcan Formula is easy to accept for possibilists – those who believe that, in addition to what actually exists, there are things that merely possibly exist. It is also easy to accept for necessitists, who believe that (necessarily) whatever exists exists necessarily. But it is traditionally held that the Barcan Formula cannot be accepted by actualists – deniers of mere possibilia – who, in addition, are contingentists, i.e., believers in (the possibility of) contingently existing beings.

I shall argue, however, that even contingentist actualists should endorse the Barcan Formula. I shall then respond to the concern that my argument, if successful, reveals a problem with contingentist actualism. I shall argue that this concern is misplaced.

Before I begin, I want to flag one thing. Above, I write that the Barcan Formula tells us something, given its usual interpretation. But this radically simplifies things. Part of what I’ll do in what follows is flesh out the conditions under which one may reasonably take the Barcan Formula to tell us one thing rather than another. The picture that emerges will have important implications for the question of whether the Barcan Formula should be endorsed or rejected.
1 – The Orthodox Position: Standard Actualists Should Reject the Barcan Formula

Logics can be used for many things. We can use them to construct board games, computer programs, and circuit boards. We can use them to study a number of phenomena – including our thought, our speech, and the norms that govern them. We can even use them to prescribe these norms.

We can also use logics to make discoveries, not just about our thought and speech but about what our thought and speech represent. And there’s a particular way that logics – or certain logics, anyway – allow us to make these discoveries. Certain logics allow us to identify what follows from what. They let us identify which inferences – say, to a proposition Q from a proposition P (or a set S of propositions) are truth preserving, in that the truth of Q is guaranteed by the truth of P (or the collective truths of the members of S). Likewise, by validating certain formulas, these logics let us identify which individual claims are themselves guaranteed to be true. Let us call such logics – and, by extension, the particular formulas such logics validate – “truth-preserving.”

Many philosophers hold that, among the truth-preserving logics, there is at least one that can we can use in reasoning about ontology and (metaphysical) modality, i.e., in reasoning about what must have (or must not have) existed and about what could have (or could not have) existed. A quantified modal logic is a logic whose language includes one or more quantifier and one or more modal operator. On one common way of interpreting these, the quantifiers (typically ∀ and ∃) are treated as ranging unrestrictedly, i.e., as ranging over everything that can be quantified over, and the modal operators (typically □ and ◊) are treated as expressing metaphysical necessity and metaphysical possibility (respectively). Let’s say that to endorse (or to reject) a particular quantified modal logic is to take it to be truth-preserving when the logical operators are interpreted as just specified. And let’s say, by extension, that to endorse (or to reject) a particular formula that a quantified modal logic may validate is to characterize that formula as allowing us, when its logical operators are interpreted as just specified, to identify claims that are guaranteed to be true.

One of the most contested issues in the debate over which quantified modal logic to endorse surrounds the Barcan Formula, i.e.:

\[ \Diamond \exists x \mathcal{B} \rightarrow \exists x \Diamond \mathcal{B} \]

The Barcan Formula (along with its converse) is named after Ruth Barcan Marcus, who first studied the formula explicitly in [10]. What Marcus initially presents is slightly different in that it is a strict rather than material conditional.
Here and throughout, I use ‘→’ as the symbol for material implication, $x$ as a metalanguage variable ranging over individual variables in an object language, and $B$ a metalanguage variable ranging over well-formed formulas in an object language. The Barcan Formula, then, is really a schema. For example, it has, as an instance, the following:

$$\Diamond \exists x \ Fx \rightarrow \exists x \ \Diamond Fx$$

The Barcan Formula is validated (i.e., every instance of the Barcan Formula is validated) by the simplest and most powerful quantified modal logics. But the Barcan Formula – when its modal operator is interpreted as expressing metaphysical possibility – is controversial, and the controversy surrounds instances of the Barcan Formula like the above-presented instance.

In particular, it is traditionally held, standard actualists must deny the Barcan Formula. Actualism is the view that everything that there is is actual, i.e., actually exists. Actualism thus opposes possibilism, the view that there are mere possibilia, entities that do not actually exist but that could have existed. By ‘standard actualism’ I mean the view that combines actualism with contingentism, the claim that (possibly) there are contingent beings, beings that exist but that could have failed to exist. Contingentism opposes necessitism, the claim that (necessarily) everything that exists exists necessarily.

To see why it is traditionally held that standard actualists must deny the Barcan Formula, consider the following prima facie plausible claim:

**Aliens.** Aliens possibly exist.

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2 This includes the aptly-titled “Simplest Quantified Modal Logic” (SQML), which results from combing Classical Quantification Theory with the modal propositional logic $K$. See Linsky and Zalta [7], Menzel [14], and Sider [17] for helpful presentations and discussions of SQML.

3 Possibilists include Fine [4], Linsky and Zalta [8-9], and Salmon [16]. (Lewis [6] also famously champions “possibilism,” but what he calls “possibilism” is different from possibilism as presented here.) Williamson [20-21] endorses necessitism (in part) as a consequence of his commitment to the Barcan Formula. Adams [1], Kripke [5], Menzel [12], Plantinga [14], and Prior [15] are all contingentist actualists who, as such, are led to reject the Barcan Formula. (Marcus [11] is the one contingentist actualist I am aware of who endorses the Barcan Formula.)

4 My presentation of the orthodox position – and of the motivation for this position – roughly follows Menzel [13].
Let ‘the actual existents’ be a rigid designator jointly referring to the things that actually exist. An alien feature is simply a feature that none of the actual existents could have exemplified. For instance, many philosophers hold that – since Wittgenstein actually had no children – no actually existing thing could have been Wittgenstein’s son. If so, then being Wittgenstein’s son is an alien feature. An alien is just something that has such a feature. So, by definition, no alien actually exists. Aliens tells us, simply, that an alien could have existed, i.e., that there could have existed something exemplifying an alien feature – e.g., there could have existed something with the feature of being Wittgenstein’s son even though, plausibly, no actually existing thing could have had this feature.

Philosophers traditionally hold that actualists, if they accept Aliens, must thereby deny (some instances of) the Barcan Formula. Consider, again, the following instance of the Barcan Formula: ◊∃x Fx → ∃x ◊Fx. Here, ‘F’ is an arbitrary predicate constant, so it can be treated as a stand-in for any predicate whatsoever. So, it seems, actualists should interpret ◊∃x Fx → ∃x ◊Fx as telling us that, for any property whatsoever, if it is possible for there to exist something that instantiates that property, then there actually exists something that possibly instantiates that property. But, by definition, no actually existing thing could have instantiated an alien property. So, it is traditionally held, actualists must interpret ◊∃x Fx → ∃x ◊Fx as implying that Aliens is false.

Possibilists, meanwhile, needn’t interpret ◊∃x Fx → ∃x ◊Fx as implying that Aliens is false. Possibilists can (and should) interpret it as saying, simply, the following: for any property, if it is possible for something to instantiate that property, then there is something that possibly instantiates that property. Possibilists can endorse this claim while endorsing Aliens, namely by taking it that, for any alien property that could be instantiated, the thing that possibly instantiates that property is a mere possibile. For instance, possibilists can insist that, even though no actually existing thing could have instantiated the property of being Wittgenstein’s son, there is a mere possibile that could have (existed and) instantiated this property.

5 Moreover, there never was such a thing. Here and going forward, for ease of exposition, I shall assume an eternalist ontology, according to which the past, present, and future are all equally real. (So, for instance, given eternalism, Caesar exists but is simply located in the past.)

6 To be clear, then, to say that aliens possibly exist is not to say that, possibly, there is something with features that can’t be exemplified. It is to say that, for some feature Φ such that none of the things that actually exist could have exemplified Φ, there could have existed some Φ-exemplifying thing.
So, while possibilists can endorse the Barcan Formula and accommodate Aliens, actualists cannot. Moreover, it is traditionally held, standard actualists – i.e., contingentist actualists – must accept Aliens. To see why, consider first what necessitist actualists will say of Aliens. Since necessitism implies that nothing could exist beyond what actually exists, necessitism implies that Aliens is false. But necessitists needn’t infer that, e.g., Wittgenstein couldn’t have had a son. For necessitists can (and do) acknowledge that, while the facts about what exists are not contingent, the facts about what is concrete are. Necessitists thus have independent motivation to posit contingently nonconcrete entities – entities that are not actually located in space and time but that could have been. Among these entities, necessitists will say, are things that could have been sons of Wittgenstein. Meanwhile, since contingentists believe in (the possibility of) contingently existing things, it would be at best ad hoc for contingentist actualists to posit a realm of contingently nonconcrete entities. Rather, contingentist actualists should say, nothing actually exists that could have been Wittgenstein’s son, though there could have existed a son of Wittgenstein.

So actualists should accept Aliens, and to do so they must deny the Barcan Formula. Or so it is traditionally held.

2 – Actualism, the Barcan Formula, and Quantification
I have just presented the traditional account of why standard actualists should reject the Barcan Formula. This account relies on the following claim:

**Existentialism.** Since actualists hold that whatever there is actually exists, in deciding whether to endorse or to reject the Barcan Formula (◊∃x B → ∃x ◊B), actualists must interpret the quantifier – if it is outside the scope of a modal operator – as existentially loaded (i.e., they must interpret ‘∃x’, when outside the scope of a modal operator, as there actually exists some x; ‘∃y’, when outside the scope of a modal operator, as there actually exists some y; etc.).

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7 Williamson [20-21] clarifies that, when he says everything that exists exists necessarily, he is using ‘exists’ in the “broadly logical” sense, to mean has being. One might wonder, then, whether there is a non-terminological difference between possibilism and actualist necessitism. (Perhaps the difference has to do with the possibilist’s belief in non-actual things, but one might also worry that the difference here is terminological.) For my project’s sake, however, what matters is that there is a clear difference between, on the one hand, standard (contingentist) actualists and, on the other hand, possibilists/ necessitist actualists. The latter accept – and the former deny (at least setting aside the orthogonal issue of whether there are abstracta) – a realm of being that outstrips the realm of the contingently concrete.
In fact, not only does the argument presented (though not endorsed) in §1 rely on Existentialism, but if Existentialism is false then standard actualists (like necessitist actualists and possibilists) should *endorse* the Barcan Formula. For, if standard actualists *needn’t* interpret the quantifier (when outside the scope of a modal operator) as existentially loaded, then they *shouldn’t*. For, if they *can* endorse the Barcan Formula without being committed to a (supposedly) problematic claim, then they *should*. For, again, the simplest and most powerful quantified modal logics validate the Barcan Formula. And this is a defeasible reason to endorse the Barcan Formula.

Moreover, Existentialism isn’t *obviously* true. Actualism is a claim about what there is. It is not, itself, a claim about how to interpret quantifiers. And a quantifier is just that – a quantifier. It does not *automatically* have to do with existence. So it isn’t obvious that actualists are *forced* to interpret ∃ (or ∀) as existentially loaded.

I’ll now argue that the truth of Existentialism is conditional on actualists’ facing certain limitations on how they can coherently interpret the quantifier. First, Existentialism is true only if:

**Maximum.** We can quantify *at most* over what there is.

While Maximum is plausible, it isn’t obviously true. Suppose that I say, “Among the characters in Victor Hugo’s *Les Misérables*, some are real and some are not.” One might plausibly say that, in this case, I am quantifying both over entities and over non-entities (e.g., I am quantifying over Napoleon Bonaparte and over Éponine Thénardier). If so, then Maximum is false and, consequently, even actualists are free to treat their quantifiers as ranging both over what actually exists and over what merely could exist. And, if so, then actualists are free to interpret, e.g., ◊∃x Fx → ∃x ◊Fx as telling us, simply, the following: if it is possible for something to be an F, then something possibly is an F (where ‘something’ does not *mean* some entity, i.e., some part of reality).

Second, Existentialism is true only if

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8 One may interpret van Inwagen [18, pgs. 492 – 499] as arguing that it does since, according to van Inwagen, the formal quantifiers of first-order logic are meant to do what the quantifiers of natural languages do. But, first, it seems to me that van Inwagen’s argument is based on his conflating the process of formalizing something with the process of translating something. And, second, not even natural language quantifiers (at least in English) automatically have to do with existence. (See the example below regarding Hugo’s *Les Misérables*.)

9 Crane [3, pgs. 28-51] provides a similar example.
Minimum. In order to quantify unrestrictedly, the quantifier – when it falls outside of the scope of a modal operator – must have within its domain at least everything that actually exists.

Let me begin by clarifying what Minimum is saying. On a standard Tarski semantics, formulas of a formal language are evaluated as true or false only with respect to a model of a particular logic. Such models include the domains over which the quantifiers of the language in question range. The idea behind Minimum is that, if we are to interpret a quantifier as genuinely quantifying unrestrictedly, then the domain over which the quantifier ranges must include (at least) everything that actually exists. The alternative to Minimum says that, in order to interpret a quantifier as quantifying unrestrictedly, it is enough that we populate the domain with enough things to serve as proxies for everything that actually exists.10 So, for instance, we might populate the domain with some subset of the natural numbers and then just stipulate that, nevertheless, the quantifier should be interpreted as quantifying unrestrictedly.

To see that Existentialism is true only if Minimum is true, assume that Minimum is false. Again, if Minimum is false, it’s because one can coherently interpret one’s quantifiers as quantifying unrestrictedly regardless of which entities literally populate the domains of one’s quantifiers. But, if so, then we needn’t interpret expressions of the form $\exists x \mathcal{B}$ as existentially committal in the sense that they express claims only about what actually exists. But, then, Minimum is false.

An example may help to demonstrate this. Alvin Plantinga [14] is a contingentist actualist who interprets the quantifier, when it is inside of the scope of a modal operator, as having as its domain the haecceities – individual essences – of whatever actually or merely possibly exists. Accordingly, he interprets the quantifier, whenever it is outside of the scope of a modal operator, as having as its domain the haecceities simply of whatever actually exists. He then interprets wffs of the form $\exists x \Diamond \Phi x$ as saying that there is exemplified some individual haecceity that, possibly, is coexemplified with $\Phi$, and in light of this he rejects the Barcan Formula. But Plantinga, instead, could have interpreted wffs of the form $\exists x \Diamond \Phi x$ as saying that there exists some individual haecceity (which may or may not be exemplified) that is possibly coexemplified with $\Phi$. This

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10 These proxies may be actual existents, but the domain of proxies needn’t be populated by everything that actually exists.
would have allowed him to adopt a logic that validates the Barcan Formula but that is friendly both to (standard) actualism and to Aliens.\textsuperscript{11}

So Existentialism is true only if Minimum and Maximum are both true. And, again, if Existentialism is false then actualists (contingentist and necessitist) \emph{should} endorse the Barcan Formula. So, if either Minimum or Maximum is false, then actualists should endorse the Barcan Formula.

Actualists should also endorse the Barcan Formula provided that Minimum and Maximum are both true.\textsuperscript{12} Consider, again, the Barcan Formula: $\square \exists x \, B \rightarrow \exists x \, \diamond B$. Any sensible quantified modal logic that invalidates (at least one instance of) this schema will be one on which the domain of the quantifier, when the quantifier is inside the scope of a modal operator, has as a member something that is not a member of the domain of the quantifier when the quantifier is outside the scope of a modal operator.\textsuperscript{13} But, if Maximum is true and actualism is true, then – regardless of whether the quantifier is outside the scope of a modal operator – nothing can appear in the domain

\textsuperscript{11} Linksy and Zalta [8, §2] make a similar observation.

\textsuperscript{12} The argument to follow is inspired by the well-acknowledged difficulty that actualists have faced, in the wake of Kripke [5], of constructing models that falsify the Barcan Formula without appeal to mere possibilia. Linsky and Zalta [8] and Menzel [13] provide helpful overviews of the discussion surrounding this difficulty.

\textsuperscript{13} For the sake of space, I limit the formal explanation of this to a footnote. Let $L_{QML}$ be a standard language of quantified modal logic, and let $L_\alpha$ be some quantified modal logic whose semantics consists of a model theory and a definition of truth in a model. A model $M_{L_\alpha}$ of $L_\alpha$ is an ordered set $\{W, w_0, D, Q, R, V\}$ such that $W$ is a nonempty set (intuitively, the set of all possible worlds), $w_0$ is a designated member of $W$ (intuitively, the actual world), $D$ is a nonempty set (intuitively, the maximal domain of quantification), $Q$ is a function from members of $W$ to non-empty subsets of $D$, $R$ is a function characterizing accessibility relations between worlds, and $V$ is $M_{L_\alpha}$'s valuation function. \textit{There is one important restriction on $Q$: for any model $M_{L_\alpha}$ of $L_\alpha$, every member $w$ of $W$ is such that $Q(w) \subseteq Q(w_0)$}. Truth conditions of simple wffs are determined, in the usual way, with respect to a model $M_{L_\alpha}$, world $w$, and assignment $f$ on $M_{L_\alpha}$. Truth conditions of wffs with respect to a model $M_{L_\alpha}$ are determined recursively, in the usual way. Now assume, for reductio, that $L_\alpha$ fails to validate some instance of the Barcan Formula. Then, for some wff $B$ and variable $x$, there’s a model $M_{L_\alpha}$ of $L_\alpha$ such that – for some assignment $f$ on that model $M_{L_\alpha}$ – $w_0, f \not\models \square \exists x \, B$ even though $M_{L_\alpha}, w_0, f \models \exists x \, \diamond B$. In this case, there’s a world $w$ accessible to $w_0$ such that, for some member $q$ of $Q(w)$, $M_{L_\alpha}, w, f[x/q] \not\models B$, but there’s no member $q^*$ of $Q(w_0)$ such that, for some world $w^*$ accessible to $w_0$, $M_{L_\alpha}, w^*, f[x/q^*] \not\models B$. But then the member $q$ of $Q(w)$ just mentioned is \textit{not} a member of $Q(w_0)$. So $Q(w)$ is not a subset of $Q(w_0)$. But, since $M_{L_\alpha}$ is a model of $L_\alpha$, $Q(w)$ is by stipulation a subset of $Q(w_0)$. So we have a contradiction. So our assumption for \textit{reductio} is false. Moreover, no sensible quantified modal logic will differ from $L_\alpha$ in a way that will lead to its invalidating the Barcan Formula unless the modification has to do with the restriction that, for any model $M_{L_\alpha}$ of $L_\alpha$, every member $w$ of $W$ is such that $Q(w) \subseteq Q(w_0)$. But this restriction is what ensures that there is nothing in the domain of the quantifier, when the quantifier is inside the scope of a modal operator, that is not in the domain of the quantifier when the quantifier is outside the scope of a modal operator.
of that quantifier other than what actually exists. Moreover, if Minimum is true, then actualists are committed to endorsing a logic on which the domain of the quantifier, at least when outside the scope of the modal operator, contains everything that actually exists. So, if Minimum and Maximum are true, actualists must endorse a logic on which everything included in the domain of the quantifier when it is \textit{inside} the scope of a modal operator is also in the domain of the quantifier when it is \textit{outside} the scope of a modal operator. So, if Minimum and Maximum are true, actualists must endorse a logic that validates the Barcan Formula.

So, to conclude: if either Minimum or Maximum is false, actualists should endorse the Barcan Formula; if Minimum and Maximum are both true, actualists should (indeed, must) endorse the Barcan Formula. So actualists (both contingentist and necessitist) should endorse the Barcan Formula.

\textbf{3 – Why My Argument Should Not Worry Standard Actualists}

So much the worse for standard actualism, one might conclude. For, one might insist, Minimum and Maximum are true. If so, then – one might argue – actualists really must deny the Barcan Formula in order to accommodate Aliens, and furthermore standard actualists must endorse Aliens. It follows, given my argument in §2, that standard actualists are committed both to accepting \textit{and} to denying Aliens.

Or so one might argue. I reject this argument. For, setting the plausibility of Minimum and Maximum aside, it seems to me that – pace the traditional argument presented in §1 – it is perfectly plausible for standard actualists to deny Aliens. To see why I say this, consider the following two claims:

\textbf{Serious Actualism.} Necessarily, whatever exemplifies a property exists.

\textbf{Possibility.} The facts about what is possible are fully grounded in the facts about what properties are actually instantiated.

Unless one is a possibilist, Serious Actualism is especially hard to deny. Moreover, Possibility is independently plausible, and Possibility is easy for contingentist actualists (as opposed to necessitist actualists or possibilists) to accommodate. Together, Serious Actualism and Possibility imply that the facts about what properties actual existents actually instantiate fully ground the facts about what is possible. Now, it does not strictly speaking \textit{follow} from this that alien properties could not be instantiated. But, if the possibility of some property F’s being instantiated is grounded
in the *actual* property instantiations of *actual* existents, then it is hard to see how that property could be so different from the properties that are actually instantiated so as to be not even *possibly* instantiated by an actual existent. So there is a plausible picture of what grounds possibility (i) that actualists should find attractive, even setting aside the above-discussed considerations regarding the Barcan Formula, and (ii) that rules out Aliens.\textsuperscript{14}

**Conclusion**

It is traditionally held that standard (i.e., contingentist) actualists should deny the Barcan Formula. I have argued, however, that actualists, standard or otherwise, should endorse the Barcan Formula. I have then responded to the concern that my argument, if successful, reveals a problem with contingentist actualism. I have explained why this concern is misplaced.

**References**


\textsuperscript{14} But must actualists who endorse this picture say that Wittgenstein couldn’t have had a son? Not necessarily. Instead, they might argue that, while nothing could have exemplified the property of *being Wittgenstein’s son*, Wittgenstein could have exemplified the property of *having a son*. Making sense of this difference is part of a larger project.


