Measuring an Unknown, A Laboratory Problem for Basic Electronics

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Abstract

This paper will focus on the problem of identifying an RLC (resistor-inductor-capacitor) combination as a laboratory exercise providing students an opportunity to synthesize basic electronics concepts. Reasons for the exercise and the mathematics behind it will be explored. The problem is suitable for those students that have had at least trigonometry. An understanding of complexed numbers would be helpful. Laboratory equipment should include a multi-meter, dual channel oscilloscope, and an AC signal generator. Calculators such as the HP-48 or the TI-85/86 that can handle complexed number notation as a phasor are helpful when working examples.

Introduction

Mastery of electronic laboratory test equipment would indicate the ability to understand parameters, equipment measurement limits, and relationships between devices. However, most electronics courses teach one concept for each exercise (Boylestad, Tocci, Hope) without opportunities to synthesize those concepts.

Traditional laboratory lessons include:

- Current in a series circuit is the same in all components.
- Voltage in a parallel circuit is the same across all components.
- Superposition as a method to analyze circuits with multiple power supplies.
- Thevinen's equivalent circuit to simplify circuits or identify interaction.
- Changes in frequency changing the impedance values of inductors and capacitors. Increasing frequency causing inductive reactance to increase and capacitive reactance to decrease.

These lessons should ideally be followed by opportunities for students to see the effect of changes in combinations of input variables. However, the process of teaching individual concepts sequentially is constrained by time. Although it has seemed to be a reasonable approach from an instructor's perspective, students often fail to see the relationships between exercises thereby becoming overloaded and lost. This paper suggests a method of combining traditional laboratory activities to reenforce the relationship between the mathematics and those concepts that are expected from the mathematical derivations. The laboratory problem proposed in this paper would follow exercises on simple DC circuits and observations of an AC system after the student has observed that AC systems work like DC systems with a sensitivity to frequency. After students have used a dual channel oscilloscope to measure phase angle, they should be ready to combine the effects of variations in the circuits, elements, and equipment limitations.

After completing these introductory electronics laboratories, students know that:

- The impedance of a capacitor will decrease with increased signal frequency.
- The impedance of an inductor will increase with increased frequency.
- Components act differently in series and parallel configurations.
- Basic algebra or phasor notation will mathematically describe a circuit.

However, synthesis of the material brings up other questions. Do the students understand:

- The limits of what can be measured or how to work the problem backward?
- The significance of the phase shift between the source and the load voltage?
- How to set up an oscilloscope to measure phase shift?
- Where to measure the load current in the circuit?
- When phase shift information is needed?
- The frequency and the data limitations that can be displayed by a multi-meter?
- That a multi-meter cannot see phase angle because it has no reference for phase?
- That a multi-meter has a published high frequency roll off just like the oscilloscope?
- The significance of increasing positive phase shift as frequency is increased?
- What will happen to phase shift with a parallel combination of a resistor and capacitor as frequency is increased?
- Where in the circuit to look for answers?
- Which frequency to use to find answers?

All of these concepts have been discussed in some laboratory within the electronics instructional materials. However, it is still possible that the cause and effect relationship is not understood because of the sequential approach used in teaching concept exercises, probably because the student

has merely followed the directions and has never been prompted to ask why. Studying for a comprehensive exam may prompt students to synthesize course data, that would not have otherwise been reenforced in the laboratory. A better approach would be to design a problem combining concepts of circuits, components, mathematics, and instrumentation that would allow the overall concepts to come together. A problem should be constructed that would include all of the basics of circuit analysis. The problem would demonstrate the difference between what can be calculated versus what can be measured, as well as point out the limitations of the equipment / instrumentation being used.

Instrumentation is used in the real world that doesn't display what the student has seen in calculations. The student knows that it is important to look at the phase shift of the load current and that current cannot be measured without disturbing the circuit. Current is typically measured as a voltage drop across a sensing element (R_s in Figure 1) placed in series with the component to be tested. He knows that the sensing resistor must be lower in value by a factor of 100 or its presence will measurably disturb the circuit. The student can see phase shift by using a dual channel oscilloscope across both the source and the sensing resistor. The student may not realize why he is using a sensing resistor. Students start the course using the multi-meter and often fall back to old trusted equipment when lost. The multi-meter will measure volts RMS and cannot measure phase shift since it only has one input and phase shift is a relationship between two values. The multi-meter is also limited in frequency response. Oscilloscopes are labeled with their frequency limitations on the front panel. Multi-meters have limitations published in their specification data sheets. Response time would indicate how long the meter needs to see a signal before it will be displayed. Transients will not show on most multi-meters. The upper roll-off point will indicate where a constant frequency will be attenuated similarly to the limiting frequency on a oscilloscope.

The oscilloscope must be used when working with frequency variable components so that frequency can be calculated from the measurements. The student knows that a capacitive circuit will have a negative impedance angle. The student may not realize that the measurement of current phase angle is a measurement of the impedance angle with the opposite sign. Capacitive circuits cause a positive current angle just like the old saying; "ELI the ICE man" indicates. The student may not remember that it is important to include a sensing resistor so that current can be measured using the voltage drop across a known resistor. Measuring the current via a voltage drop will allow the user to limit the change in the circuit being measured by using high impedance voltmeters and low resistance sensing resistors. Without the sensing resistor, there are too many unknowns to make calculations.

The frequency of measurement must be found by experimentation. The realization that a 45 degree angle phase shift must be found if the resistive and reactive components are to be separated must be discovered during calculations since most students will not discover this during directed experiments. To find the value of an unknown impedance, the student must connect the oscilloscope in its dual channel mode and change the frequency while he observes whether the phase shift is increasing or decreasing. This is done on a point by point basis because changing the frequency requires that the oscilloscope time base be constantly adjusted and the angle between source and sensing voltage be recalculated.

The student must then remember the expected impedance changes caused by capacitors and inductors. For series connections where the resistive zero angle and the reactive ± angle are added (equation 1), the student must remember that increasing frequency will increase the angle of an inductive load and decrease the angle with a capacitive load. That means that the current angle will go toward a negative 90 degrees with a series inductive load as the frequency in-

creases and the current angle will go toward a positive 90 degrees as the frequency decreases with a series capacitive load.

The parallel connection is more difficult to picture. Most exercises do not look at multiple frequency phase angles in parallel because of the difficulty. In the series situation, we were looking for changes from zero to ±90 degrees. Now the student needs to think through the parallel effect. Components in parallel are dominated by the smallest value. Parallel impedance is calculated (equation 5) by adding the inverse of the component impedance values and then inverting. Observation of a parallel combination may require data at different frequencies as we observe impedance and phase angle changes.

If we were to take the resistor - inductor case in parallel, we would expect an increase in impedance and a phase angle going from +90 to zero as frequency is increased. When the resistor and the inductor are the same magnitude, we would see a phase angle of 45 degrees. As the inductor increases in magnitude, the resistor takes over control of the total impedance value and the phase shift will go toward zero while the total impedance magnitude rises and approaches the value of the resistor.

The resistor - capacitor case will be similar. At low frequency, the reactance of the capacitor is large and doesn't show in a parallel circuit so the resistor will set the total impedance and phase angle. As the frequency increases, the phase angle will move from zero toward -45 degrees as the capacitor approaches the same magnitude as the resistor. Remember this will show on the oscilloscope as a positive 45 degrees when measuring sensing resistor voltage to get current. As the frequency increases, the total impedance will be controlled by the capacitor which is getting smaller in magnitude and approaching -90 degrees as its limit.

The Problem
The unknown component is a problem that will require a syntheses of past material. It will show limitations of

equipment, and will show the limitations of measurement versus design calculations. The student is asked to measure and describe as component values some unknown combination of RLC components. Unknowns are configured, soldered, and wrapped in tape so that the student sees only two leads coming out of the tape. Possible configurations are; RL (resistor - inductor) in series or parallel and RC (resistor - capacitor) in series or parallel.

Configuration of Unknowns

It is important that the instructor configure the unknown combinations so they can be diagnosed. If the frequency required is too high or too low for the equipment available, the student will be frustrated and give up. If the components used contain values to close to the sensing elements required, the results will be affected and the student's solution will indicate his failure to properly diagnose. If the non-ideal values, such as the resistance of an inductor, are large, some configurations will be impossible to diagnose. Select components with matching impedances at frequencies in the 300 hz to 300K hz range. Low frequencies flash because of the oscilloscope recycle rates and are hard to read without a digital or persistence type oscilloscope. The high end frequency needs to be low enough that the student can test to 100 times that value with the equipment in the laboratory. Select component values that will cause impedances in the 1K to 10K ohm range. This will allow the student to select a sensing resistor that is lower than any tested value by a factor of 100. That will allow the student to ignore the sensing resistor in the calculations.

Non-ideal properties of inductors can be a problem. The ideal inductor would have no resistance. All inductors have resistance because they need a length of wire wrapped around a core to create the inductance phenomenon. If an inductor is chosen that has a high relative value of resistance and that component is put in parallel with a resistor of similar value, there is no way for the student to separate the two components. If the inductor is used in series, just add the inductor's resistance to the resistor value and that will be what the student can find.

Instructor's Setup

L and C components should be chosen so that X_C or X_L (equation 10 & 19) will be the same magnitude as the resistance portion of the unknown when the frequency is between 300 hz and 300K hz. The technique used requires that the student find a frequency (f_{45}) that has a phase shift of 45 degrees. 45 degrees allows the magnitude of the RLC components to be the same relative size on the complexed number triangle and allows the student to separate the components through trigonometry or phasors. Since the real resistance value will be on the real (horizontal) axis and the reactive component value will be on the vertical axis, the complexed number will show both impedance values. It is not reasonable to put both inductors and capacitors in the same unknown. The student would see a combination of their effects and would be unable to separate the components.

Example unknowns

If the real resistance value (R_U) is chosen to be above 1K ohm, choose the inductors internal resistance (R_l) to be less than 100 ohms to limit its error in the final solution to less than 10%. For example a RC pair that would use f_{45}=2K hz could be R=8k ohms and C=.01µF. A RL pair at 2K hz needs a large inductor. A RL at 20K hz could be R=1200 ohms and L=10mH. A RC at 10K hz could be R=1600 ohms and C=.01µF. These would be good for either series or parallel. Students worked well at middle frequencies in my classes. Large and small values were difficult for the students to breakout. Stay away from significant values of R_l (the resistance component of inductance caused by the wire used).

Student Setup

Analysis of the unknown will require the use of a signal generator, a multi-meter to measure resistance, a dual channel oscilloscope, and a known 10 ohm resistor to use as a sensing resistor (R_S). Construct the test circuit (figure 1). Connect channel 1 of the oscilloscope across V_{ba} (the source) and channel 2 across V_{ca} (the sensing resistor). This will allow both the

source and the sensing resistor voltage to be seen and will allow a single-ended oscilloscope to be used without a ground-loop problem. Notice that the channel grounds are both on point "a" between the source and the sensing resistor. If the AC supply source is grounded, be sure it is connected so that its ground is at "a" also.

Set the signal generator to 2V or a magnitude high enough to show on the oscilloscope. Set the frequency (f_{45}) to a value that will show a phase shift as close to 45 degrees as can be measured. It is more important that the signal period and thus frequency is accurately identified than it is to get exactly 45 degrees phase angle. Record the source (V_{ba}) and the sensing resistor (V_{ca}) as phasors remembering to call the angle of V_{ca} positive if it starts up before V_{ba} and negative if the reverse is true. V_{ba} is the source voltage and should be considered to be a zero angle phasor. Waveforms that start up before the reference are positive and those that start after the reference are negative. Trigger control on the oscilloscope should be on internal, AC, channel 1.

Switch leads so that Channel 1shows V_{ab} (source) and Channel 2 shows V_{cb} (the unknown). Record V_{cb} with its angle and sign. The angle will be very small across the unknown.

Positive values of phase angle on V_{ca} (the sensing resistor) indicate a capacitive load. This can be seen by symbolically solving for the current of figure 1 as seen by equations 1, 2, 3 in the appendix.

Measure the unknown with an ohm meter. Resistance measured or an open circuit indicated will be clues to the configuration of the unknowns. An open circuit would indicate a RC series configuration. If RC is in parallel, the resistance measured is the value of R_U. If RL is in series, the resistance reading will be the combination of R_U and R_l. The resistance of the inductor (R_l.) cannot be separated from the unknown resistance (R_U) with this procedure. If RL is in parallel, the measured resistance is the parallel sum of R_U and R_l. When the instructor chooses components, magnitudes of R_l should be small so that the student can find a value for R_U.

Measurements and Calculations for the four listed configurations.

Parallel C

If the phase angle of V_{ca} was positive, it is a capacitive circuit. If the value of resistance measured with an ohm-meter does not show an open circuit, the capacitive circuit is connected in parallel and the measured value is R_U. By working backwards from the parallel impedance formula, X_c can be found and the value of the capacitor can be found. Because R_S is very small and has a zero angle, vector magnitudes and Pythagorean's theorem can be used to calculate component values as shown by equations 4, 5, 6, 7, 8.

R_U was a known value from the ohm-meter test, X_C is now known for the parallel capacitive configuration. Calculating at f_{45}, we can find C from equation 9.

R_U and C are known and the complete solution for the parallel RC configuration.

If the phase angle of V_{ca} was negative indicating inductance or if the ohm-meter shows an open circuit indicating capacitance in series, another data point is needed to identify components. Changing the test frequency to 100 times greater (f_{100}) than that at f_{45} would allow us to isolate the inductor or capacitor from the unknown resistance because the reactance would then be 100 times greater or smaller than the resistance. In the series RC case, the increase in frequency will cause X_C to be small (see equation 10) compared to R_U so that R_U would be the result of the high frequency or f_{100} test.

Series C

Redo the previous test sequence at f_{100} recording V_{ca}' and V_{cb}' as before. Notice the "prime" values for measurements that are done at the f_{100} frequency. The f_{45} frequency was chosen so that the magnitudes of R and C would be similar. The f_{100} frequency should cause the reactance of the capacitor to be 100 times less than previous and therefore total impedance (Z_U') approximately equal to R_U.

Taking the calculated value of R_U (from equation 11 & 12) back to the f_{45} measurements, the series combination can be solved with equations 13, 14, 15, 16, 17.

R_U is known from the f_{100} test, X_c can be calculated using f_{45} measurements. C can be found from X_C at f_{45} from equation 18.

R_U and C are known and the complete solution for the series RC configuration.

As opposed to capacitive reactance, inductive reactance increases with increased frequency as shown by equation 19.

In the case of series RL components the impedance of the unknown would be expected to increase continuously with frequency. Therefore, if frequency is increased by 100 times, the impedance of Z_U would increase by nearly 100 times and its phase angle would approach 90 degrees. In the case of parallel RL components, the increased frequency will cause the X_L portion of the impedance to be so large as to have little effect on the total impedance. Total impedance of parallel components is controlled by the smallest impedance values. The small magnitude of R_U would dominate total impedance and the phase angle would approach zero degrees. F_{100} measurements must be taken. In the series RL configuration, the measurements will be used to calculate X_L by making R_U insignificant and in the parallel RL configuration, the measurements will be used to calculate R_U by making X_L insignificant.

Series L

The series configuration must first be confirmed with the f_{100} measurement values. Total impedance must increase (decreased current) with increased frequency and phase angle must proceed toward 90 degrees to indicate a series configuration. Phase angle will be most easily seen by monitoring the sensor resistor (V_{ca}) with respect to the source voltage (V_{ba}). If V_{ca} approaches -90 degrees, then impedance is as follows (see equations 20 & 21) and X_L' is much larger than R_U' so the calculation of Z_U' is approximately equal to X_L' and L is found from equation 22.

Solving for the inductor value at f_{100}. X_L at f_{45} can be calculated and used in equation 23 so that R_U can be calculated using equations 24, 25, 26, 27.

R_U and L are known and the complete solution for the series RL configuration.

Parallel L

The parallel configuration must be confirmed with the f_{100} frequency measurements. Total impedance is expected to be limited by the value of R_U and the phase angle is expected to approach zero instead of 90 degrees. Phase angle will be most easily seen by monitoring the sensing resistor at V_{ca} with respect to the reference voltage V_{ba}. If V_{ca} approaches zero degrees at f_{100}, impedance is found from equations 28 & 29.

Taking R_U back to the f_{45} measurements, the parallel configuration gives us L from equations 30, 31, 32, 33, 34.

R_U and L are known and the complete solution for the parallel RL configuration.

Conclusions

The ability to calculate the components present in an unknown load shows a knowledge of the relationships between impedance and frequency and a synthesis of exercises presented in basic electronics courses. Measurements made using a multi-meter and oscilloscope require an understanding of equipment limitations and data collection techniques. Mathematically presenting measured values of magnitude and angle representing current in an AC system shows a high level of concept integration. The goal of the this laboratory is to assist in the integration of theory and component configuration concepts. The presented problem includes all of the basic circuit configurations and basic passive components.

References

Boylestad, *"Experiments in Circuit Analysis"*, Prentice Hall, 2000

Hope, *"Experiments in Circuit Analysis"*, West Publishing Co., 1996

Tocci, *"Digital Systems Principles & Applications"*, Prentice Hall, 1995

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Figure 1

FIGURE 1
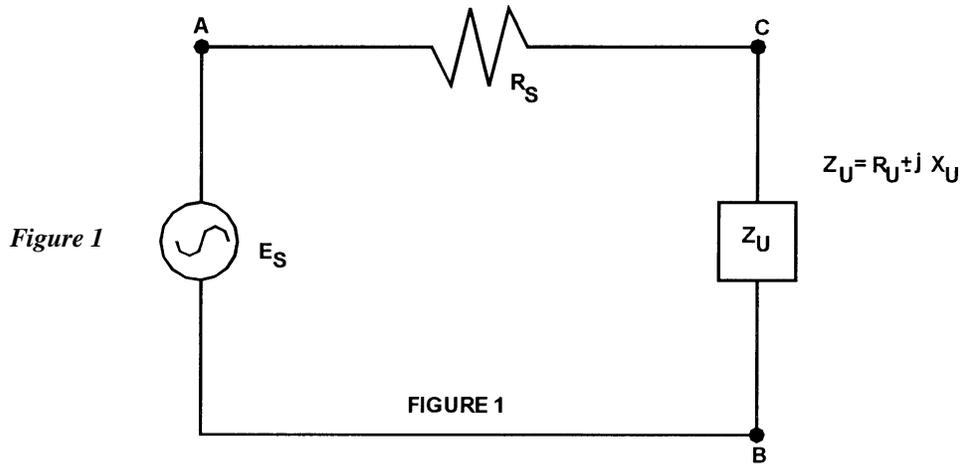
Equation 1

$$\mathbf{Z}_T = \mathbf{R}\angle 0 + \mathbf{X}_C\angle\text{-}90$$

Equation 2

$$\mathbf{I} = \frac{|\mathbf{Es}|\angle 0}{|\mathbf{Z_T}|\angle\textbf{ negative}} = \frac{|\mathbf{Es}|}{|\mathbf{Z_T}|}\angle\textbf{ positive}$$

Equation 3

$$\mathbf{V_{ca}} = \mathbf{IRs} = \left(\mathbf{Rs}\right)\left(|\mathbf{I}|\angle\textbf{ positive}\right)$$

Equation 4

$$|\mathbf{I}| = \frac{|\mathbf{V_{ca}}|}{|\mathbf{R_s}|} = \frac{|\mathbf{V_{cb}}|}{|\mathbf{Z_U}|}$$

Equation 5

$$\frac{1}{\mathbf{Z_U}} = \frac{1}{\mathbf{R_U}} + \frac{1}{\mathbf{X_c}\angle -90}$$

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Equation 6
$$\frac{1}{|\mathbf{Z_U}|} = \sqrt{\left(\frac{1}{|\mathbf{R_U}|}^2 + \frac{1}{|\mathbf{X_C}|}^2\right)} = \frac{|\mathbf{V_{ca}}|}{|\mathbf{R_s}||\mathbf{V_{cb}}|}$$

Equation 7
$$\left(\frac{1}{|\mathbf{R_U}|}^2 + \frac{1}{|\mathbf{X_C}|}^2\right) = \left(\frac{|\mathbf{V_{ca}}|^2}{|\mathbf{R_s}|^2|\mathbf{V_{cb}}|^2}\right)$$

Equation 8
$$|\mathbf{X_C}| = \sqrt{\frac{1}{\left(\dfrac{|\mathbf{V_{ca}}|^2}{|\mathbf{R_s}|^2|\mathbf{V_{cb}}|^2} - \dfrac{1}{|\mathbf{R_U}|^2}\right)}}$$

Equation 9
$$C = \frac{1}{2\pi f_{45} X_c}$$

Equation 10
$$X_C = \frac{1}{2\pi fC}$$

Equation 11
$$|\mathbf{I}|' = \frac{|\mathbf{V_{ca}}|'}{|\mathbf{R_s}|} = \frac{|\mathbf{V_{cb}}|'}{|\mathbf{Z_U}|'}$$

Equation 12
$$|\mathbf{R_U}| \cong |\mathbf{Z_U}|' = \frac{|\mathbf{V_{cb}}|'|\mathbf{R_s}|}{|\mathbf{V_{ca}}|'}$$

Equation 13
$$\mathbf{Z_U} = R_U - jX_C$$

Equation 14
$$|\mathbf{Z_U}| = \sqrt{|R_U|^2 + |\mathbf{X_C}|^2}$$

Equation 15
$$|\mathbf{I}| = \frac{|\mathbf{V_{ca}}|}{|\mathbf{R_s}|} = \frac{|\mathbf{V_{cb}}|}{|\mathbf{Z_U}|}$$

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Equation 16
$$|\mathbf{Z_U}| = \frac{|\mathbf{V_{cb}}||\mathbf{R_s}|}{|\mathbf{V_{ca}}|} = \sqrt{|\mathbf{R_U}|^2 + |\mathbf{X_c}|^2}$$

Equation 17
$$|\mathbf{X_c}| = \sqrt{\frac{|\mathbf{V_{cb}}|^2|\mathbf{R_s}|^2}{|\mathbf{V_{ca}}|^2} - |\mathbf{R_U}|^2}$$

Equation 18
$$C = \frac{1}{2\pi f_{45} X_c}$$

Equation 19
$$X_L = 2\pi fL$$

Equation 20
$$|\mathbf{I}|' = \frac{|\mathbf{V_{ca}}|'}{|\mathbf{R_s}|} = \frac{|\mathbf{V_{cb}}|'}{|\mathbf{Z_U}|'}$$

Equation 21
$$|\mathbf{X_L}|' \cong |\mathbf{Z_U}|' = \frac{|\mathbf{V_{cb}}|'|\mathbf{R_s}|}{|\mathbf{V_{ca}}|'}$$

Equation 22
$$L = \frac{X_L{}'}{2\pi f_{100}}$$

Equation 23
$$\mathbf{Z_U} = R_U + jX_L$$

Equation 24
$$|\mathbf{Z_U}| = \sqrt{|R_U|^2 + |\mathbf{X_L}|^2}$$

Equation 25
$$|\mathbf{I}| = \frac{|\mathbf{V_{ca}}|}{|\mathbf{R_s}|} = \frac{|\mathbf{V_{cb}}|}{|\mathbf{Z_U}|}$$

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Equation 26
$$|\mathbf{Z_U}| = \frac{|\mathbf{V_{cb}}||\mathbf{R_s}|}{|\mathbf{V_{ca}}|} = \sqrt{|\mathbf{R_U}|^2 + |\mathbf{X_L}|^2}$$

Equation 27
$$|\mathbf{R_U}| = \sqrt{\frac{|\mathbf{V_{cb}}|^2|\mathbf{R_s}|^2}{|\mathbf{V_{ca}}|^2} - |\mathbf{X_L}|^2}$$

Equation 28
$$|\mathbf{I}|' = \frac{|\mathbf{V_{ca}}|'}{|\mathbf{R_s}|'} = \frac{|\mathbf{V_{cb}}|'}{|\mathbf{Z_U}|'}$$

Equation 29
$$|\mathbf{R_U}| \cong |\mathbf{Z_U}|' = \frac{|\mathbf{V_{cb}}|'|\mathbf{R_s}|'}{|\mathbf{V_{ca}}|'}$$

Equation 30
$$\frac{1}{\mathbf{Z_U}} = \frac{1}{\mathbf{R_U}} + \frac{1}{\mathbf{X_L}\angle + 90}$$

Equation 31
$$\frac{1}{|\mathbf{Z_U}|} = \sqrt{\left(\frac{1}{|\mathbf{R_U}|^2} + \frac{1}{|\mathbf{X_L}|^2}\right)} = \frac{|\mathbf{V_{ca}}|}{|\mathbf{R_s}||\mathbf{V_{cb}}|}$$

Equation 32
$$\left(\frac{1}{|\mathbf{R_U}|^2} + \frac{1}{|\mathbf{X_L}|^2}\right) = \left(\frac{|\mathbf{V_{ca}}|^2}{|\mathbf{R_s}|^2|\mathbf{V_{cb}}|^2}\right)$$

Equation 33
$$|\mathbf{X_L}| = \sqrt{\frac{1}{\left(\dfrac{|\mathbf{V_{ca}}|^2}{|\mathbf{R_s}|^2|\mathbf{V_{cb}}|^2} - \dfrac{1}{|\mathbf{R_U}|^2}\right)}}$$

Equation 34
$$L = \frac{X_L}{2\pi f_{45}}$$