P&C Severity Modeling

IABA Conference

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Claims Actuarial Research

Travelers Insurance

Agenda

- Introduction
  - What is GB2?
  - Why GB2?
- Relationships with Other Distributions
- Interpretation of GB2 Parameters
- Practical Applications of GB2 to GL Data
- Final Thoughts
- Q&A

Introduction

Modeling the payout tail in long-tail lines, such as General Liability is critically important in Pricing, Reserving, Reinsurance Pricing, Solvency Testing and a host of other applications.

- Sophisticated models have been developed for estimating claim frequency and total expected payout, less attention has been devoted to understand the loss severity distributions.
- Most of the theoretical models have been developed based on the assumption that loss severities are gamma distributed, which is not optimal for long tail lines.
- Need to adopt a flexible distribution to model both heavy-tailed and light-tailed claim severity.
- GB2 is an extremely flexible distribution that has been shown to have excellent modeling capabilities in a wide range of applications, including security price returns and claims reserving.

Introduction

The GB2 probability density function (pdf) is defined by

$$GB2(y;a,b,p,q) = \frac{|a|\, y^{ap-1}}{b^{ap} B(p,q)(1+(y/b)^{a})^{(p+q)}}$$

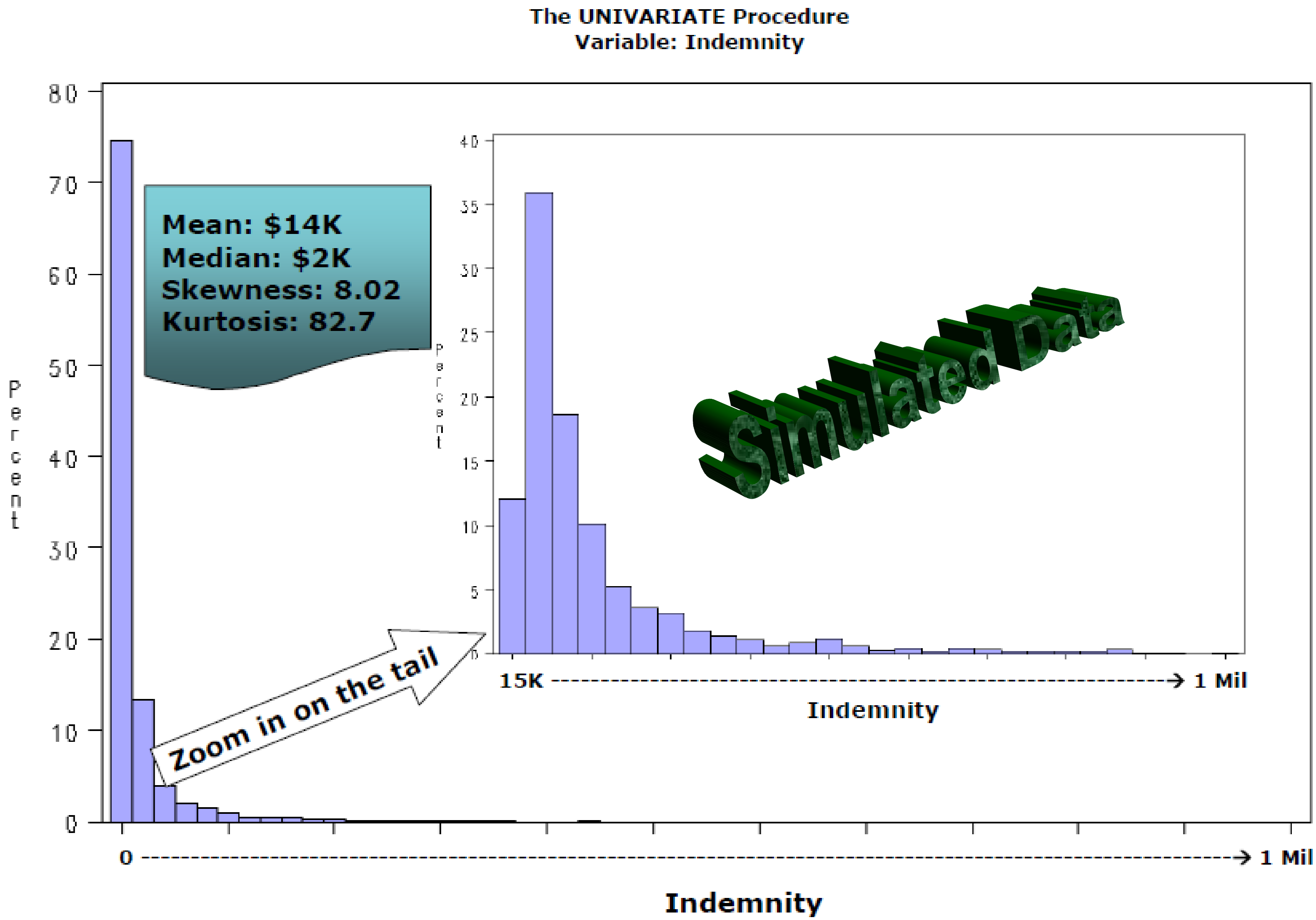
for y>= 0, with b, p, and q positive, where B(p,q) denotes the beta function.

$$B(p,q) = \int_0^1 t^{p-1}(1-t)^{q-1}dt = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$$

Benefits of GB2:

- ✓ Raise the flexibility: commonly-used distributions assume one or more parameters in GB2 as special/limited cases (when p=1 Burr 12 & when q=1 Burr 3);
- ✓ Jointly estimate both location and shape parameters, while most others (such as Exponential, Logistic, Normal and etc.) usually focus on location only;
- ✓ Therefore, better capture the long right tail to achieve better performance without compromising the performance on the left side;
- ✓ By incorporating regressors, possible to get clearer idea on claims with different characteristics, and therefore;
- ✓ Achieve better triage strategy such as segmentations of CWP/Modified payment/Large Payment.

Why GB2?

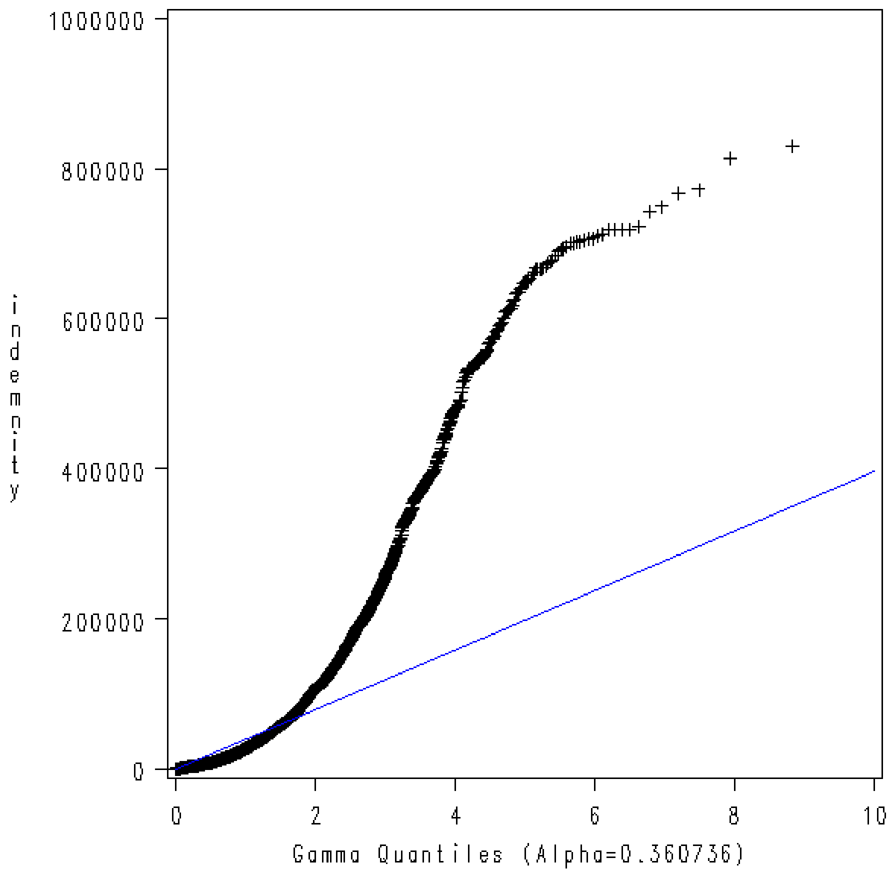


The UNIVARIATE Procedure
Variable: Indemnity
Mean: $14K
Median: $2K
Skewness: 8.02
Kurtosis: 82.7
Zoom in on the tail
Simulated Data
Percent
15K --→ 1 Mil
Indemnity
0 --→ 1 Mil
Indemnity

Why GB2?

Fitting the GL Severity data using the traditional Gamma Distribution

Gamma Severity Q-Q Plot



Observations:

- If the quantiles of the theoretical and data distributions agree, the plotted points fall on or near the line.
- Gamma distribution fails to pick up the long tail and skewness of the data.
- What next?

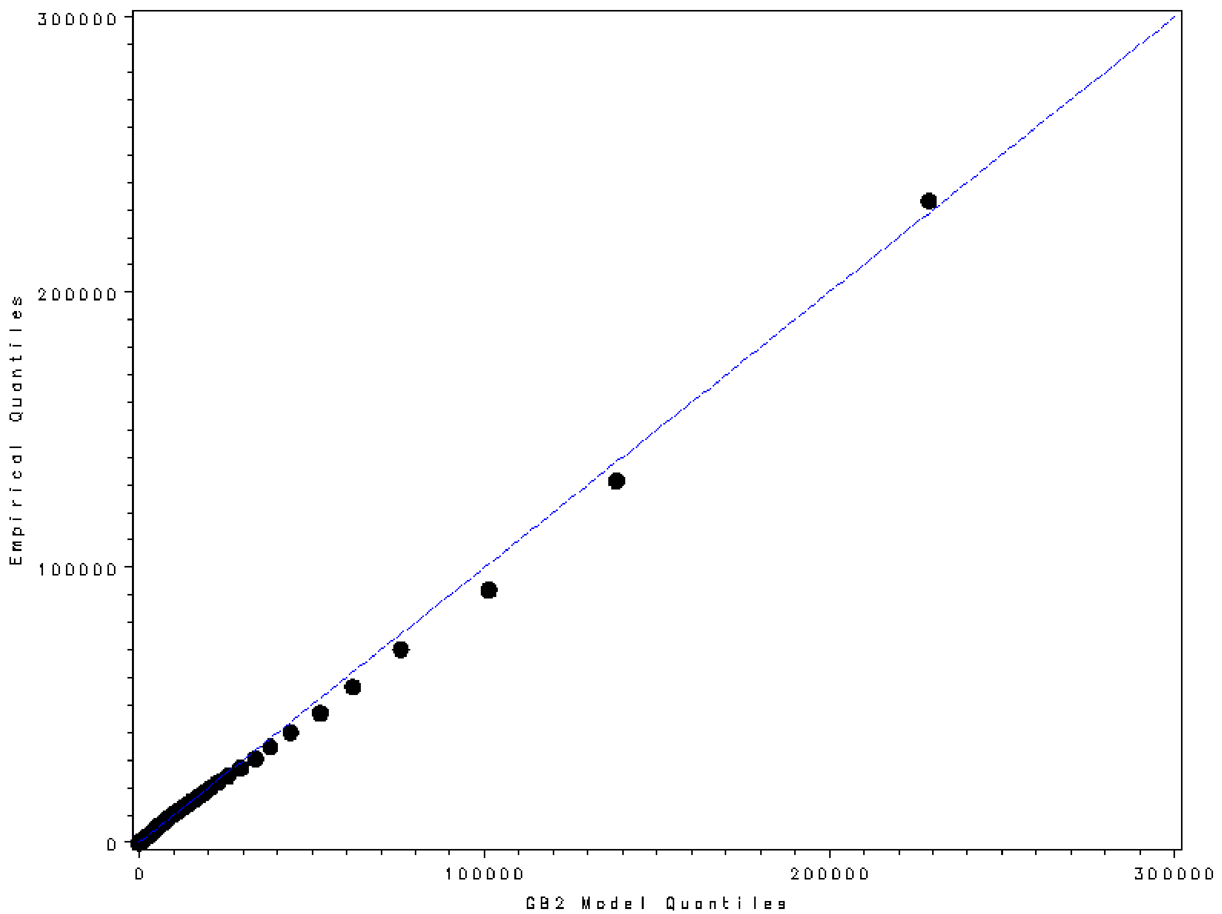
Goodness-of-Fit Tests for Gamma Distribution

Test		p Value
Kolmogorov-Smirnov	Pr > D	<0.001
Cramer-von Mises	Pr > W-Sq	<0.001
Anderson-Darling	Pr > A-Sq	<0.001

Why GB2?

Fitting the GL Severity data using GB2 Distribution

GB2 Severity Q-Q Plot



Observations:

- The linearity of the points on the Q-Q plot suggests that the GB2 is an appropriate candidate for the severity distribution.
- Furthermore, the Goodness of Fit tests fail to reject the null hypothesis that the GL severity data fit the GB2 distribution.

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Relationships with Other Distributions

Parameter Estimates

Parameter	Estimate	Standard Error	DF	t Value	Pr > |t|	Alpha	Lower	Upper	Gradient
eta_a	-1.5938	0.1643	45E3	-9.70	<.0001	0.05	-1.9157	-1.2719	1.056606
eta_b	6.1659	0.5259	45E3	11.72	<.0001	0.05	5.1350	7.1967	-0.83887
eta_p	2.7512	0.3398	45E3	8.10	<.0001	0.05	2.0851	3.4173	-4.2301
eta_q	2.4481	0.3020	45E3	8.11	<.0001	0.05	1.8562	3.0399	4.253954

**** Log Transformation was applied to the underlying severity data.***

Additional Estimates

Label	Estimate	Standard Error	DF	t Value	Pr > |t|	Alpha	Lower	Upper
a	0.2032	0.03337	45E3	6.09	<.0001	0.05	0.1377	0.2686
b	476.21	250.45	45E3	1.90	0.0573	0.05	-14.6777	967.09
p	15.6611	5.3223	45E3	2.94	0.0033	0.05	5.2293	26.0930
q	11.5661	3.4926	45E3	3.31	0.0009	0.05	4.7207	18.4116
mean	17065	626.69	45E3	27.23	<.0001	0.05	15837	18294
mode	1526.66	399.24	45E3	3.82	0.0001	0.05	744.16	2309.17

Relationships with Other Distributions

Contrasts				
Label	*Num DF*	*Den DF*	*F Value*	*Pr > F*
Beta2: a=1	1	45E3	570.26	<.0001
Singh-Maddala B12: p=1	1	45E3	7.59	0.0059
Dagum B3: q=1	1	45E3	9.15	0.0025
Generalized Gamma: q->large	1	45E3	80095.4	<.0001
log-t: a->0	1	45E3	8.977E8	<.0001
Inverse Lomax a=q=1	2	45E3	28500.8	<.0001
Fisk (Log-Logistic): p=q=1	2	45E3	5.26	0.0052
Lomax (Pareto type II) a=p=1	2	45E3	20498.3	<.0001

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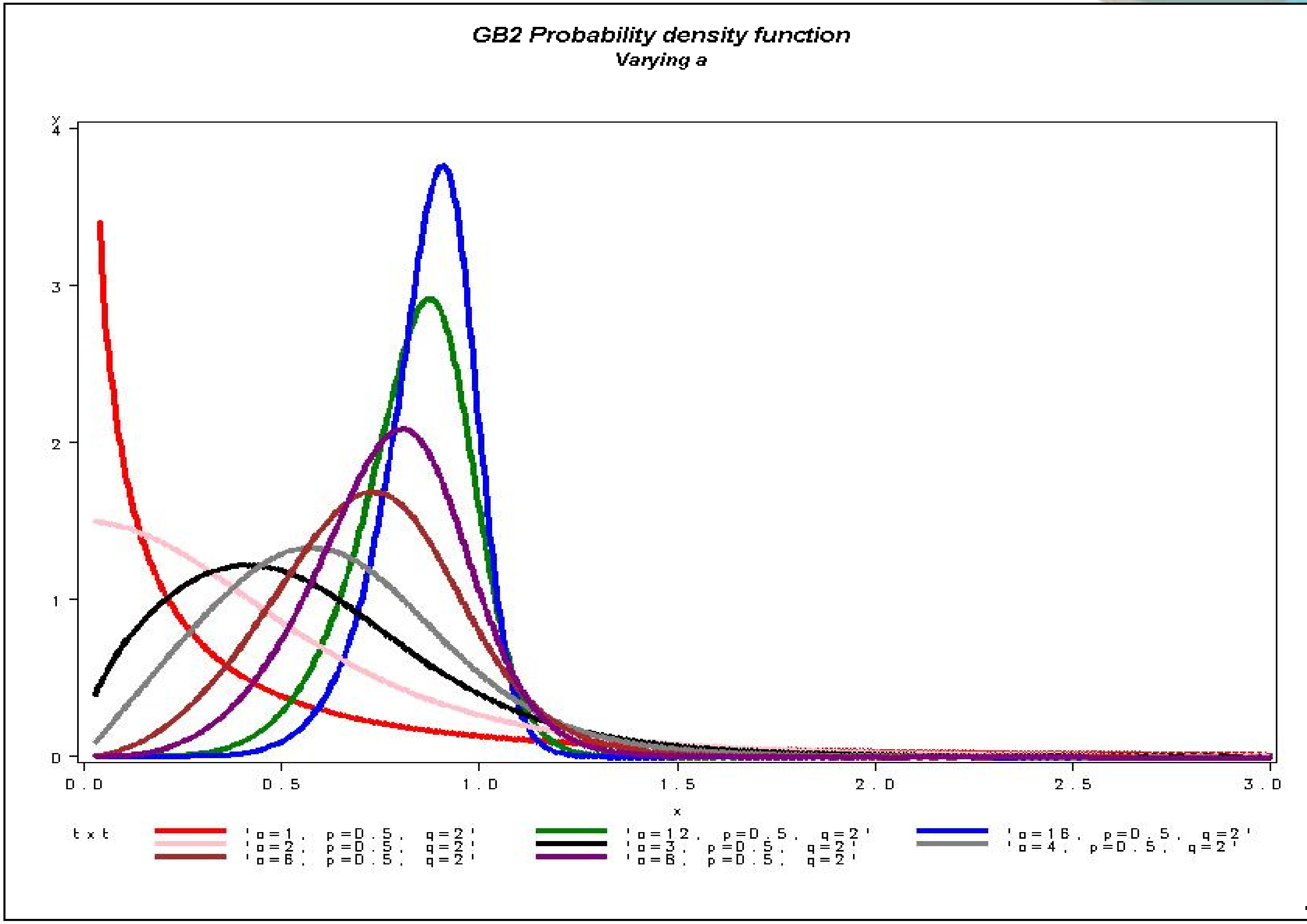
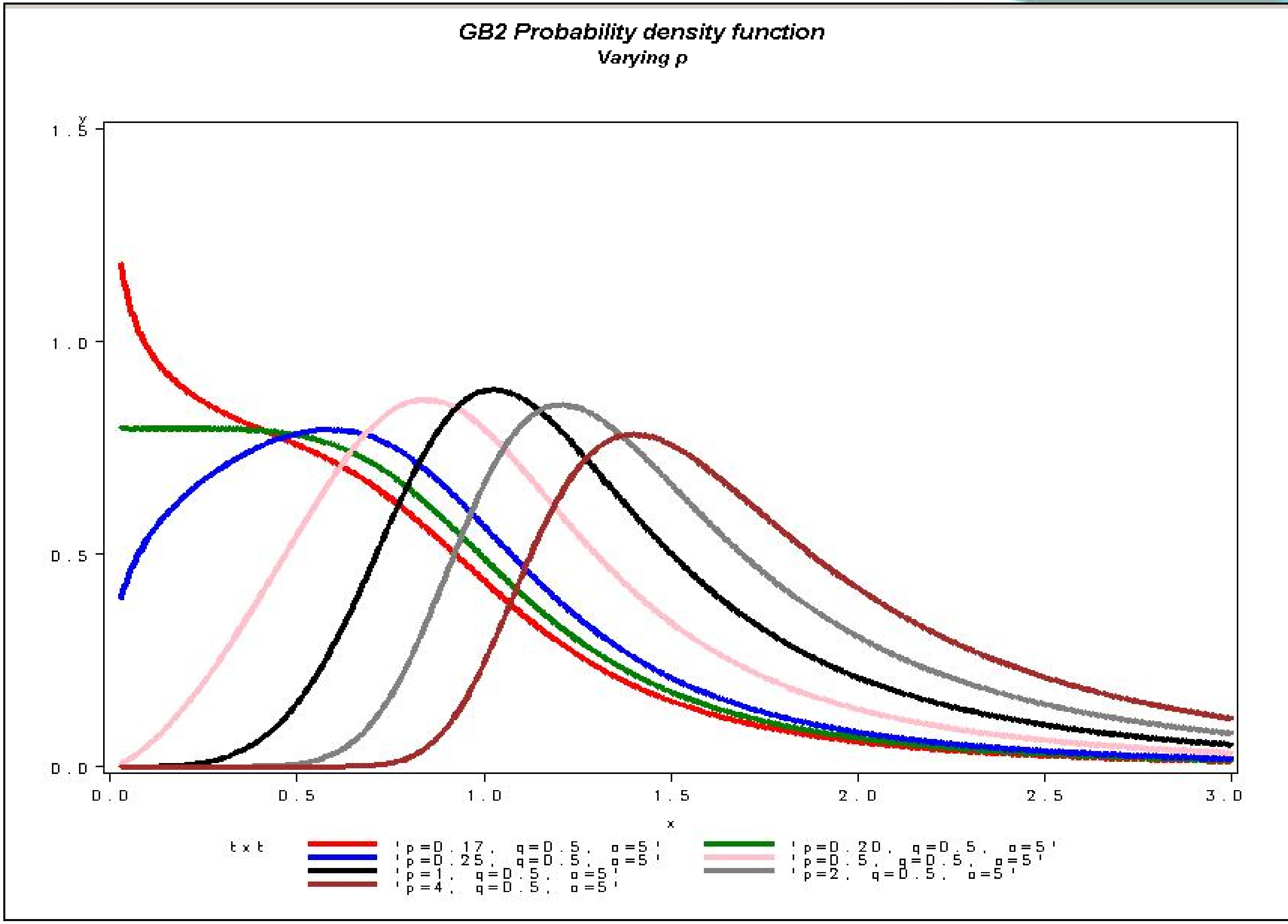
Interpretation of GB2 Parameters

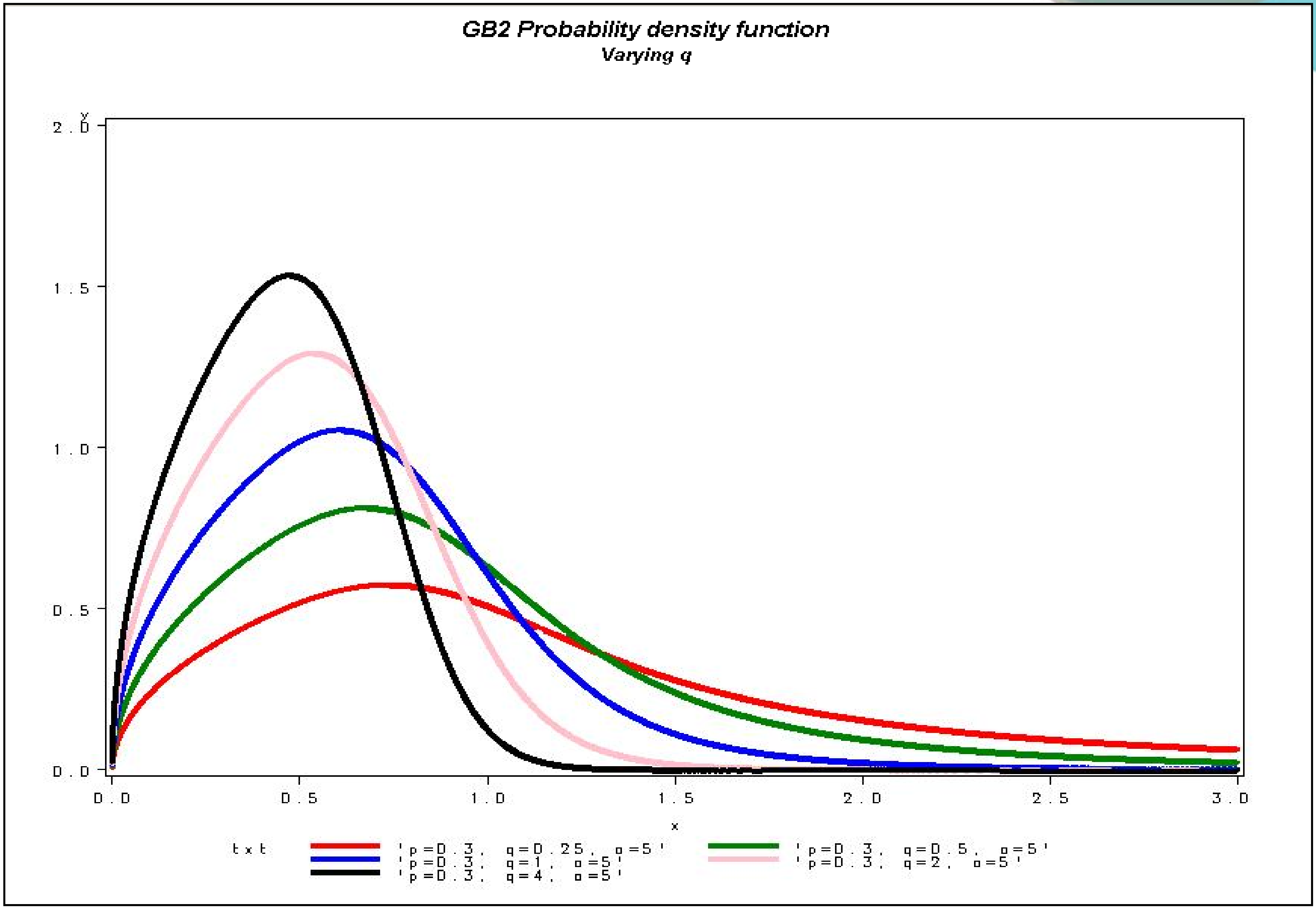
$$GB2(y;a,b,p,q) = \frac{|a|\, y^{ap-1}}{b^{ap} B(p,q)(1+(y/b)^{a})^{(p+q)}}$$

- The parameters a, b, p and q flexibly define the shape and location of the density.
- The parameter b is merely a scale parameter and depends upon the units of measurement.
- The larger the value of a or q, the "thinner" the tails of the density function.
- The parameters p and q determine the shape of the density function. The relative values of p and q determine the magnitude of skewness.
- The values of p and q also allow positive or negative skewness – this is a flexible advantage over distributions such as lognormal, which are always positively skewed.
- The parameter a can be either positive or negative. It is interesting to note: GB2(y; -a,b,p,q) = GB2(y; a,b,q,p)


GB2 Probability density function
Varying a
y
0
1
2
3
4
0.0
0.5
1.0
1.5
2.0
2.5
3.0
x
txt
'a=1, p=0.5, q=2'
'a=2, p=0.5, q=2'
'a=6, p=0.5, q=2'
'a=12, p=0.5, q=2'
'a=3, p=0.5, q=2'
'a=8, p=0.5, q=2'
'a=16, p=0.5, q=2'
'a=4, p=0.5, q=2'



GB2 Probability density function
Varying p
y
1.5
1.0
0.5
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
x
txt
'p=0.17, q=0.5, a=5'
'p=0.25, q=0.5, a=5'
'p=1, q=0.5, a=5'
'p=4, q=0.5, a=5'
'p=0.20, q=0.5, a=5'
'p=0.5, q=0.5, a=5'
'p=2, q=0.5, a=5'


GB2 Probability density function
Varying q
y
2.0
1.5
1.0
0.5
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
x
txt
'p=0.3, q=0.25, a=5'
'p=0.3, q=1, a=5'
'p=0.3, q=4, a=5'
'p=0.3, q=0.5, a=5'
'p=0.3, q=2, a=5'

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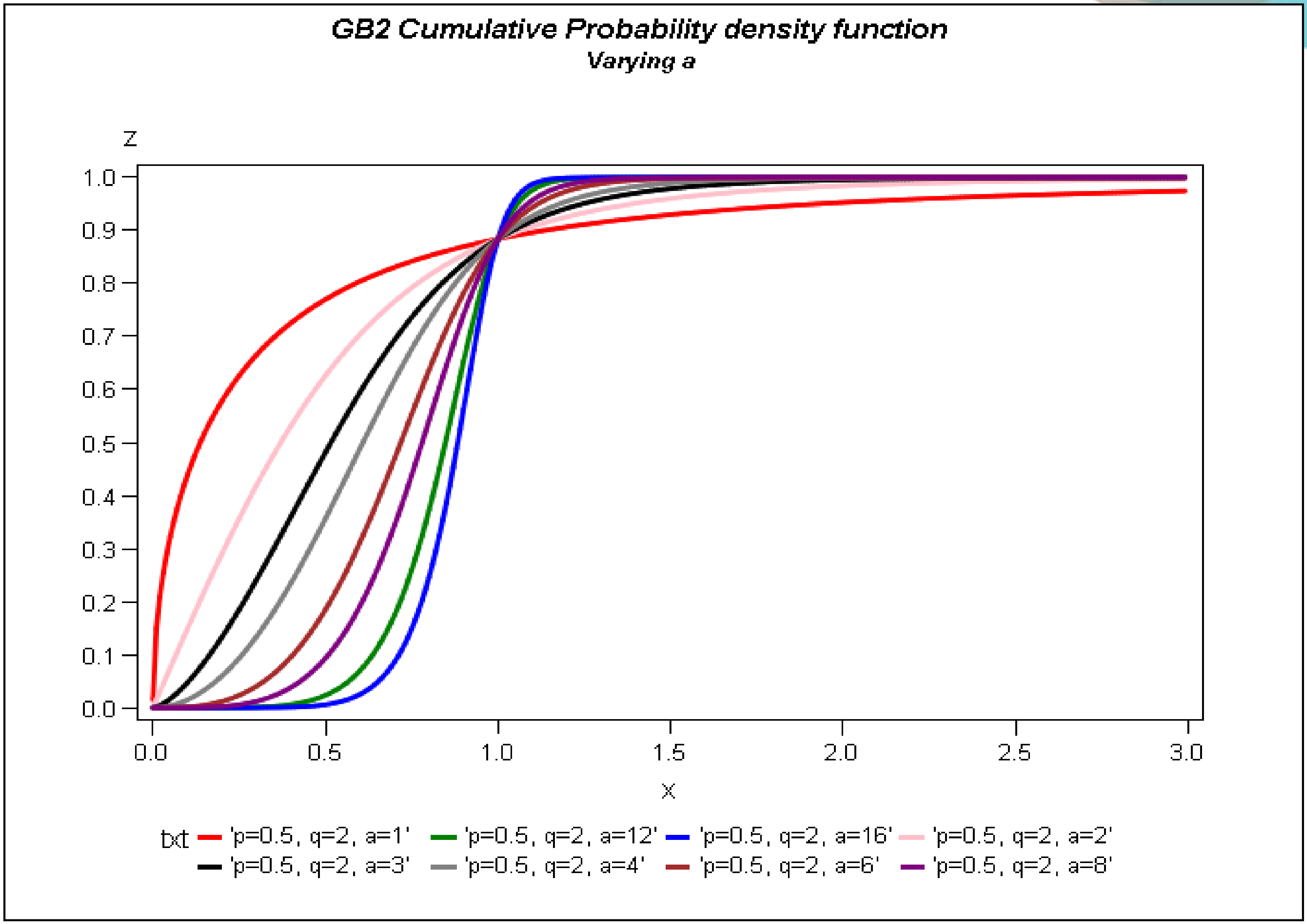
GB2 Cumulative Probability density function
Varying a
z
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
x
txt
'p=0.5, q=2, a=1'
'p=0.5, q=2, a=12'
'p=0.5, q=2, a=16'
'p=0.5, q=2, a=2'
'p=0.5, q=2, a=3'
'p=0.5, q=2, a=4'
'p=0.5, q=2, a=6'
'p=0.5, q=2, a=8'

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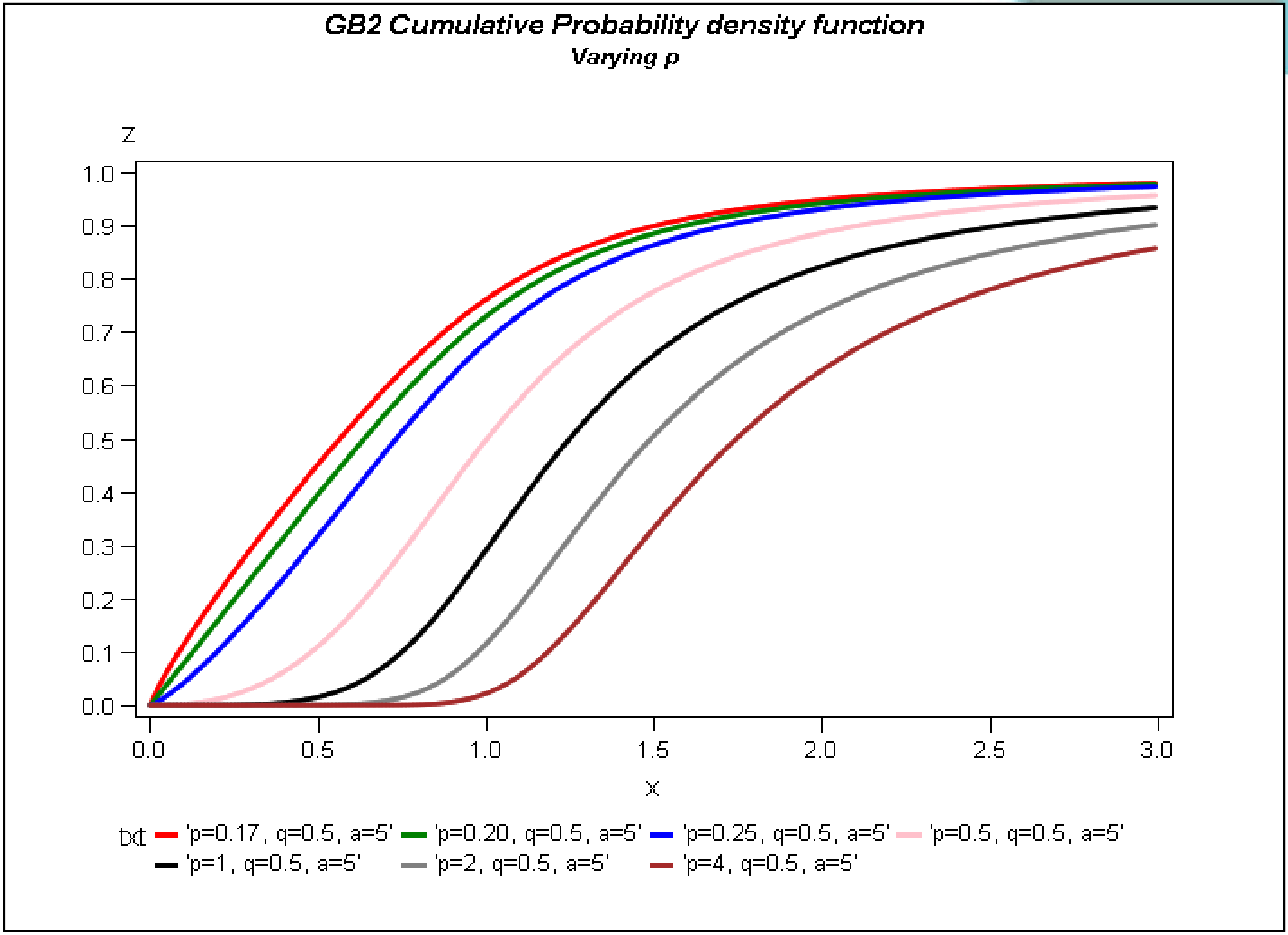
GB2 Cumulative Probability density function
Varying p
Z
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
X
txt
'p=0.17, q=0.5, a=5'
'p=0.20, q=0.5, a=5'
'p=0.25, q=0.5, a=5'
'p=0.5, q=0.5, a=5'
'p=1, q=0.5, a=5'
'p=2, q=0.5, a=5'
'p=4, q=0.5, a=5'

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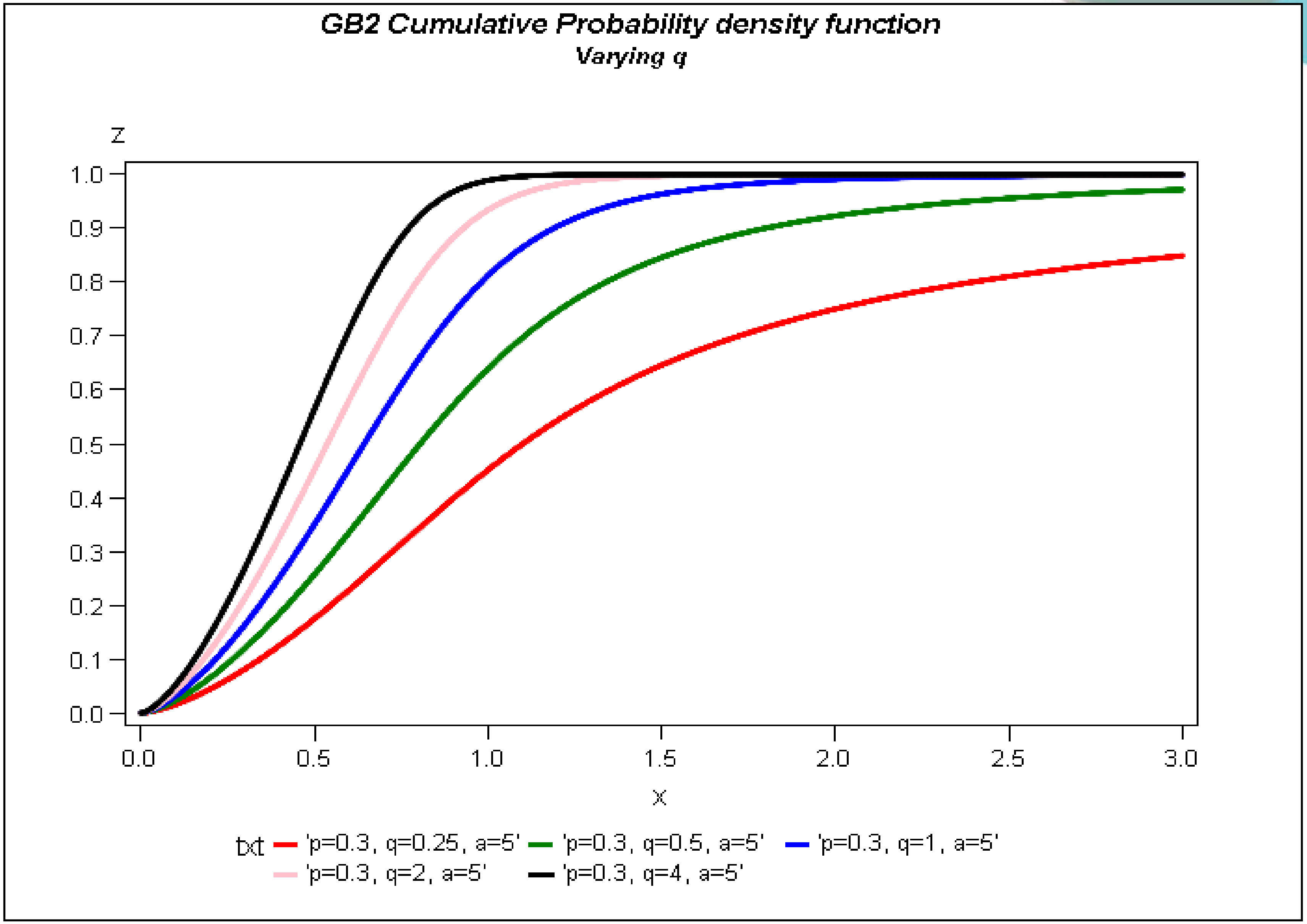

GB2 Cumulative Probability density function
Varying q
z
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
x
txt
'p=0.3, q=0.25, a=5'
'p=0.3, q=0.5, a=5'
'p=0.3, q=1, a=5'
'p=0.3, q=2, a=5'
'p=0.3, q=4, a=5'

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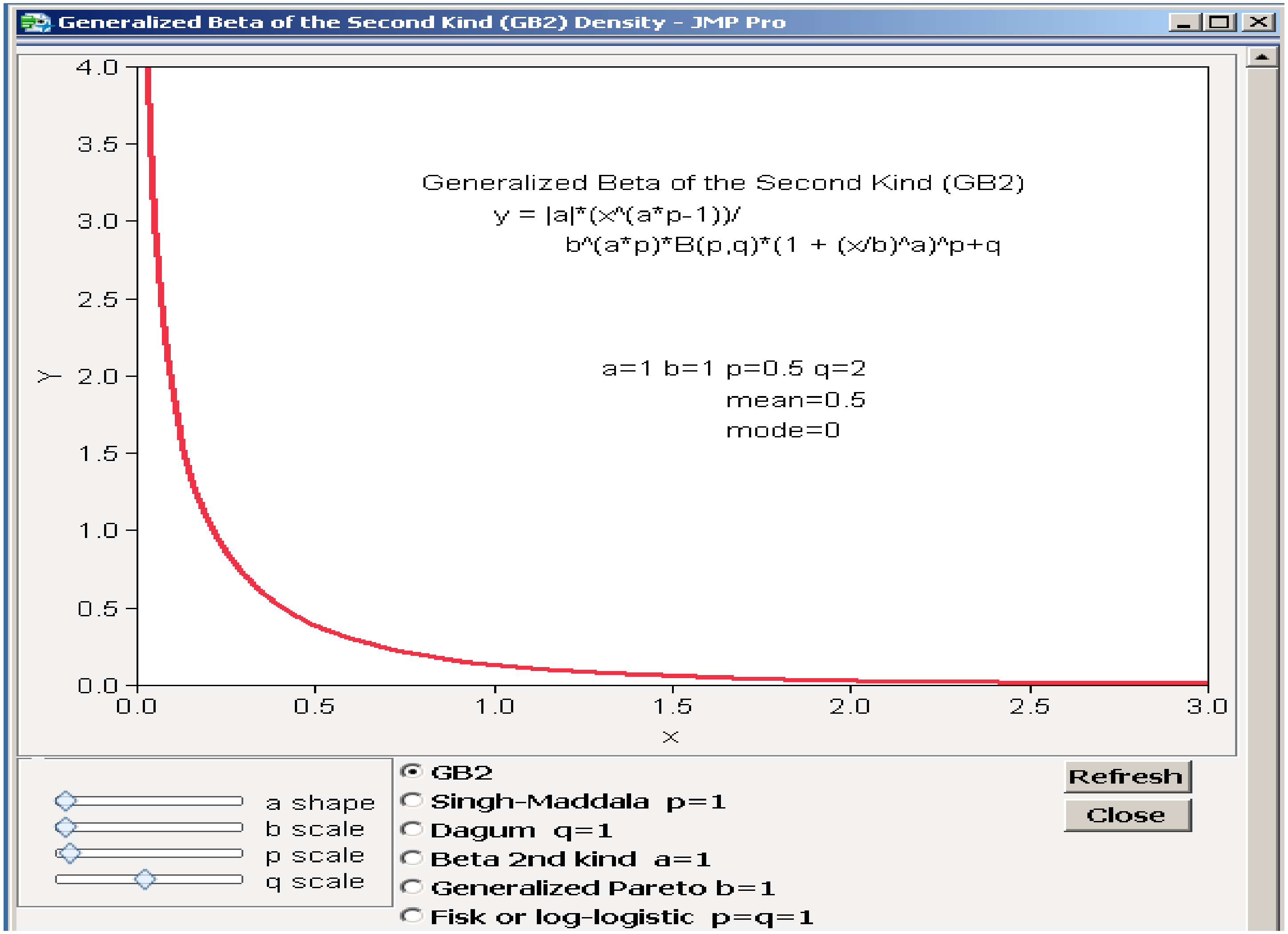
Live Demo Using JMP



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Practical Applications of the GB2 to GL Data

- How do we incorporate GB2 into the multivariate regression model framework?
- How do we identify key GL claim attributes, which materially impact the indemnity payout?
- How to predict ultimate GL indemnity payout at an individual claim level?
- How to detect different development patterns among segments, such as injury type, injury complexity level, pricing track and etc?

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Final Thoughts

- Leverage Statistical Modeling methodologies to provide additional insight to reserving practices

- Using Multivariate Analysis, to understand key drivers behind the claim severity – ultimately reduce expenses and indemnity payouts

- Predicting ultimate at an individual claim level – Improving segmentation for Pricing and Reserving

- Incorporating this methodology into CAT modeling and Reinsurance Pricing

Appendix

Skewness and Kurtosis

- Skewness is a measure of symmetry, or more precisely, the lack of symmetry.
- Kurtosis is a measure of whether the data is peaked or flat relative to a normal distribution. Data sets with low kurtosis tend to have a flat top near the mean rather than a sharp peak. A uniform distribution would be the extreme case.

***Definition of Skewness**:*

For univariate data Y1, Y2, ..., YN, the formula for skewness is: $\text{skewness} = \frac{\sum_{i=1}^{N}(Y_i - \bar{Y})^3}{(N-1)s^3}$

where $\bar{Y}$ is the mean, s is the standard deviation, and *N* is the number of data points.

Note: The skewness for a normal distribution is zero, and any symmetric data should have a skewness near zero. Negative values for the skewness indicate data that are skewed left and positive values for the skewness indicate data that are skewed right.

Definition of Kurtosis:

For univariate data Y1, Y2, ..., YN, the formula for kurtosis is: $\text{kurtosis} = \frac{\sum_{i=1}^{N}(Y_i - \bar{Y})^4}{(N-1)s^4}$

where $\bar{Y}$ is the mean, s is the standard deviation, and N is the number of data points.

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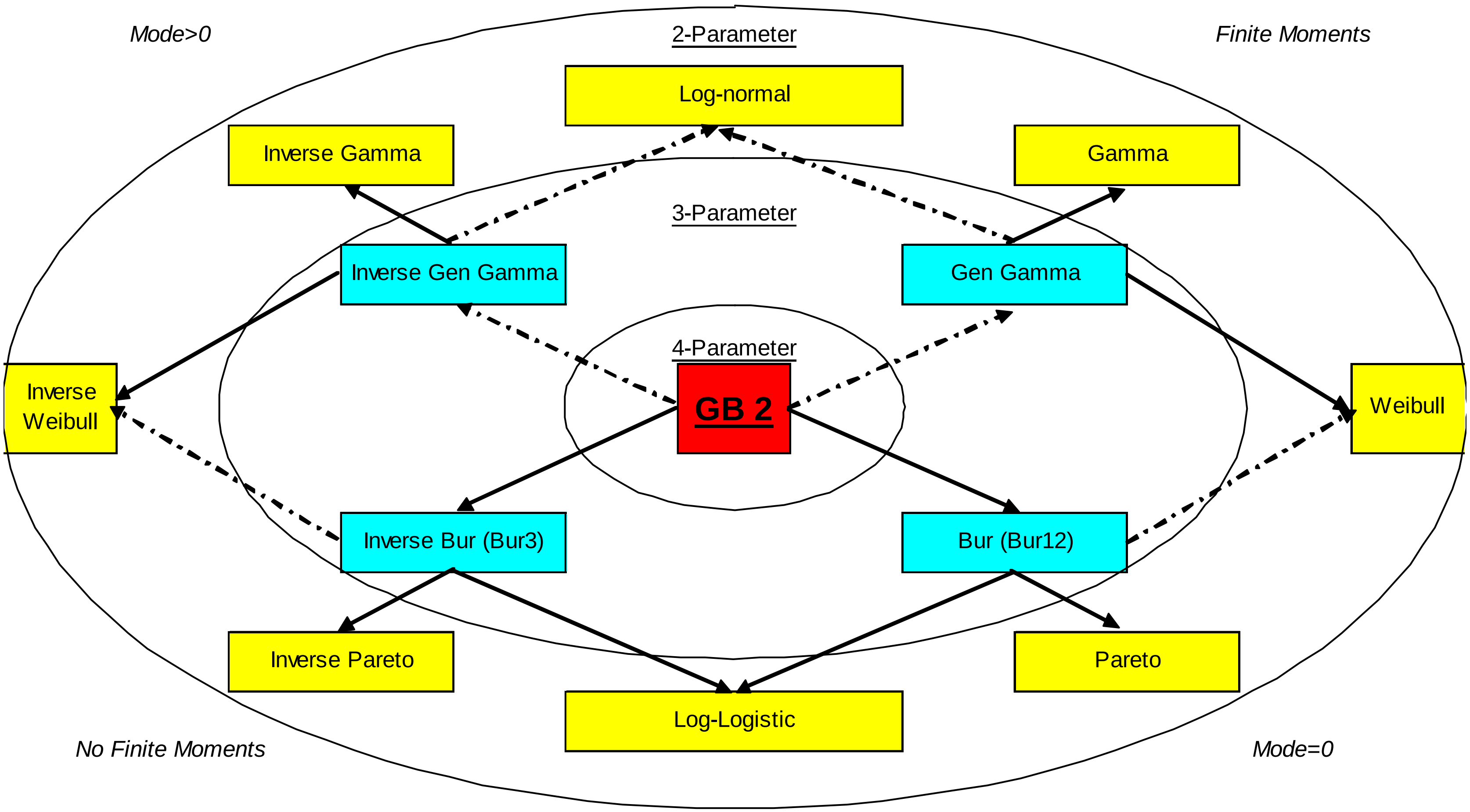
Relationships with Other Distributions



Source: Special case

Loss Model Limited case

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