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Colour phenomena and selective absorption: the role of thickness of absorbing medium[1]

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Abstract
This experiment (*ADT*: absorption with different thicknesses) is focused on selective absorption by solutions or other media constituted of successive transparent layers. It is designed in order to make students consider not only the spectral composition and intensity of light but also the multiplicative status of absorption rate by a filter or a solution. Thorough understanding of this aspect, indeed, is essential to reconcile classical school knowledge about colour and observation of various phenomena, such as colour of the sun or the moon at different heights, or else dichromatism of some solutions. Still more widely, this experiment provides an opportunity to illustrate, step by step, what an exponential dependence is.

Key words: colour, partial and selective absorption, exponential dependence, dichromatism.

Why this experiment?

It has been observed that colour phenomena are often dealt with as if absorption was a case of "all or nothing" process, for instance: "Black objects do not reflect any light", or, speaking of a subtractive synthesis: "with magenta plus cyan, you get blue". Preceding investigation (Viennot & de Hosson 2012) and MUSE papers[2] were focused on an experiment highlighting the role of intensity of light: A red laser beam is cast on various pigments, evidencing in particular that being "black" is not enough to guarantee total absorption.

The *"ADT*" experiment presented in this paper is centered on absorption by series of filtering layers, be they solid, liquid or gaseous. It is intended to help students,

- pass from an 'all or nothing' way of reasoning to another involving 'more or less' terms,
- comprehend the multiplicative role of a coefficient of absorption: then the idea of subtraction, suggested by the expression "subtractive synthesis", must be reinterpreted.

Both aspects are involved when it comes to grasp the meaning of a curve of absorption. They turn out to be difficult to students, as observed in the investigation just quoted.

[1] The MUSE group (Andreas Mueller, Gorazd Planinsic, Elena Sassi and Laurence Viennot) takes responsibility for the content of this paper (February 2012). The intellectual property remains with the author.
[2] See on the MUSE website, the paper: *Colour phenomena and partial absorption* by Viennot & de Hosson (2012) and the activity sheets of the MUSE workshop on the topic in Istanbul (ICPE 2012).

Concerning absorption, it is worth noting that “absorbance”, that is what a spectrophotometer indicates concerning a given solution, is often hardly defined in textbooks or teaching documents. It might be mistaken for a coefficient of absorption (a multiplying factor) whereas it refers to the logarithm of such a coefficient (see below). This may encourage surreptitiously an additive view of the cumulated contributions of successive layers, whereas multiplication is at the core of this story.

The conceptual tools introduced with this experiment are crucial to understand various phenomena, such as colours of the sun or the moon at different heights, or else of some solutions in recipients of different thicknesses (for instance the dichromatism of pumpkin seeds oil or bromophenol blue[3]).

Prerequisite and targeted conceptual steps

According to the audience, it may be necessary to recall or to introduce
-what is a curve of absorption of a solution, which implies the notion of *selective* absorption
-the multiplicative role of the coefficient - say α_λ - of absorption (or reflection) of an object, for a radiation “near” λ .
To this end, it is possible to use the experiment (*CPA)* described in the MUSE paper: colour phenomena and partial absorption (see note 2). But this experiment says nothing about the role played by the thickness of absorbing medium.

In case of transmission, absorption depends in particular on the thickness of the object.

Let $dI(\lambda)= i_\lambda\, d\lambda$ the intensity of a radiation between λ and $\lambda+d\lambda$. We call i_λ “spectral density”.
$i_\lambda(0)$ and $i_\lambda(x)$ are respectively the values of this quantity at the entrance and at a given position (x, in case of a one dimensional problem) in the medium.

$$i_\lambda(x) = \alpha_\lambda(x)\ \ i_\lambda(0) \qquad (1)$$

Let us now examine more closely the role of the thickness.
Between x and x+dx, one has

$$di_\lambda = - \beta_\lambda\, i_\lambda(x)\, dx \qquad (2)$$

with $\beta_\lambda = \varepsilon_\lambda\, c$ where ε_λ is a coefficient characteristic of the dissolved substance and c is the concentration (Lambert-Beer’s law).
Hence, given (2):

$$i_\lambda(x) = i_\lambda(0)\, e^{-\beta_\lambda x} \qquad (3)$$

For a given objet (solution in a recipient of thickness L), (3) writes

$$i_\lambda(L) = i_\lambda(0)\, e^{-\beta_\lambda L} \qquad (4)$$

The quantity ($\beta_\lambda L / \ln 10$) is called the absorbance γ_λ . In other terms, γ_λ is the decimal logarithm of the ratio $i_\lambda(0)\ /\ i_\lambda(L)$:

$$i_\lambda(L) = i_\lambda(0)\, 10^{-\gamma_\lambda}$$

[3] Kreft, S. & Kreft, M. 2007. Physicological basis of dichromatic colour, *Naturwissenschaften* 94, 935-939. Thanks to Gorazd Planinsic who called attention on this paper. See Appendix 2.

From (1) and (4) we get $\alpha_\lambda = e^{-\beta_\lambda L} = 10^{-\gamma_\lambda}$ (5)

The main goal of this experiment "*ADT*" is to understand what happens when successive parallel absorbing layers interact with a given radiation. In brief, it aims at fostering the idea that the coefficients (multiplicative factors) of absorption of two slices of a given medium have to be multiplied in order to characterize the absorption by the set of two layers. This is because the second layer interacts with a radiation the spectral density of which has been already diminished by a factor (which depends on λ) characterizing the first layer.

To understand cases when the colour of an object changes due to absorption by a thinner or thicker slice of a transparent medium,(i.e.: changing colours of the Sun or the Moon, the dichromatism of some liquids), it is also needed to know the rules of additive and "subtractive" colour mixing (see Appendix 1).

Materials

- If available: an optic projector for diapositives (experiment in large group/classroom), or any imaging device (in case of work in small groups/workshop)
- a grating about 600 lines/nm
-A slice (width about 1mm), possibly made with two razor blades
-Strips made of transparent and thin plastic: light yellow, light pink
-Square frames made of thin cardboard or Bristol (like the frames of old diapositives)
-if available a spectrophotometer

Staging sequence

A possible staging for a sequence using *"ADT*" experiment comprises the following main steps:

Phase "R"
Concerning the rules of additive and "subtractive" mixing and the multiplicative status of absorption, a phase of reminder can be useful for students with low expertise about colour. To this end, the following excerpt from a previous MUSE paper can be used, at least partly:

1- Ask justified predictions concerning the impact of the red laser beam (633nm) on the various zones of the sheet represented in figure 1.

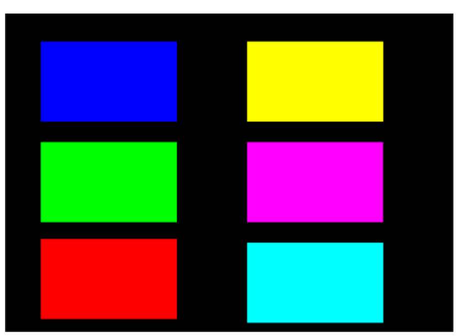


Figure 1 A sheet of paper with six coloured areas (Red, Green, Blue, Yellow, Magenta, Cyan) , from the graphic palette of a computer, on a black background.

2-Ask students to perform the experiment. This should be done in dim light, so that the colours on the sheet be visible; given that too much ambient light would attenuate the targeted effect of contrast (described in the next item).

3-Ask students to situate their predictions with respect to what they observe. Once the experiment is performed, it is possible to observe a bright red impact on the areas of the sheet which are, respectively, white, red, magenta and yellow. By contrast, the impact is still red but less bright on the areas of the sheet which are, respectively, black, green, blue and cyan. In each group, the brightness is similar across the different pigments, low for the latter group, high for the former.

4-Ask them to explain the surprising outcome of the experiment. The targeted comprehension is that, surprising as it may be, the outcome of this experiment can be explained provided the concept of partial absorption is used. To this end, it is necessary to consider that there is a brighter impact on the pigments that are classically declared not to absorb a red light – in fact they absorb a small proportion of the incident red light; and there is a dim impact on the pigments that are classically said to absorb the red light – but in fact absorb only a high proportion of such a light. To this end, some curves of reflectance (see MUSE document quoted in note 2) are provided at this moment. The meaning of these curves should be duly explained to students, who, most of the time, are destabilized by this confrontation. What should be particularly considered in the reflectance curves is the part of each curve which reaches the wavelength value of a red laser beam (λ = 633 nm). As said above, a very low value of percentage of diffuse reflection may result in a perceptible effect. In other words, the attention paid to the feet of the curves indicates that a blue or green pigment can diffuse red light depending on the intensity of the incident flux: 1% of "a lot of" incident light provides a non negligible amount of diffused light.

5-Conduct a final discussion about the difference between the classical experiments such as the one depicted in Appendix 1 – when the classical rules (understood in an "all or nothing" register) seem to apply properly, on the one hand, and the *CPA* experiment, which apparently invalidate these rules, on the other hand. The decisive factor – the power of the light sources – should be identified, and justified.

6-Lead students toward a conclusion: properly formulated, the classical rules are not invalidated by this experiment. Ask them to recapitulate their conceptual acquisitions and their feelings in this respect.

Phase ADT (experiments centered on Absorption by layers of Different Thicknesses of an absorbing medium)
With or without a previous phase of reminder, and/or another phase of motivation using triggering situations (see below "Links with other experiments"), we suggest the following "hard core" for this *ADT* experiment.

The students are asked to

1 Consider a sheet of light yellow or magenta filter and the spectrum of a white light having passed through it. The spectrum is first observed visually (setting described in Figure 2).

2 Predict and explain how this situation is transformed when two, three, four, … , superposed layers of filter replace the unique one previously used.

3 Perform the experiment. Explain similarities and differences with predictions.

Figure 3 shows photos of spectra at two different exposures for slits, respectively yellow and magenta, equipped with transversal strips of increasing thickness (one, two, three, …layers superposed).

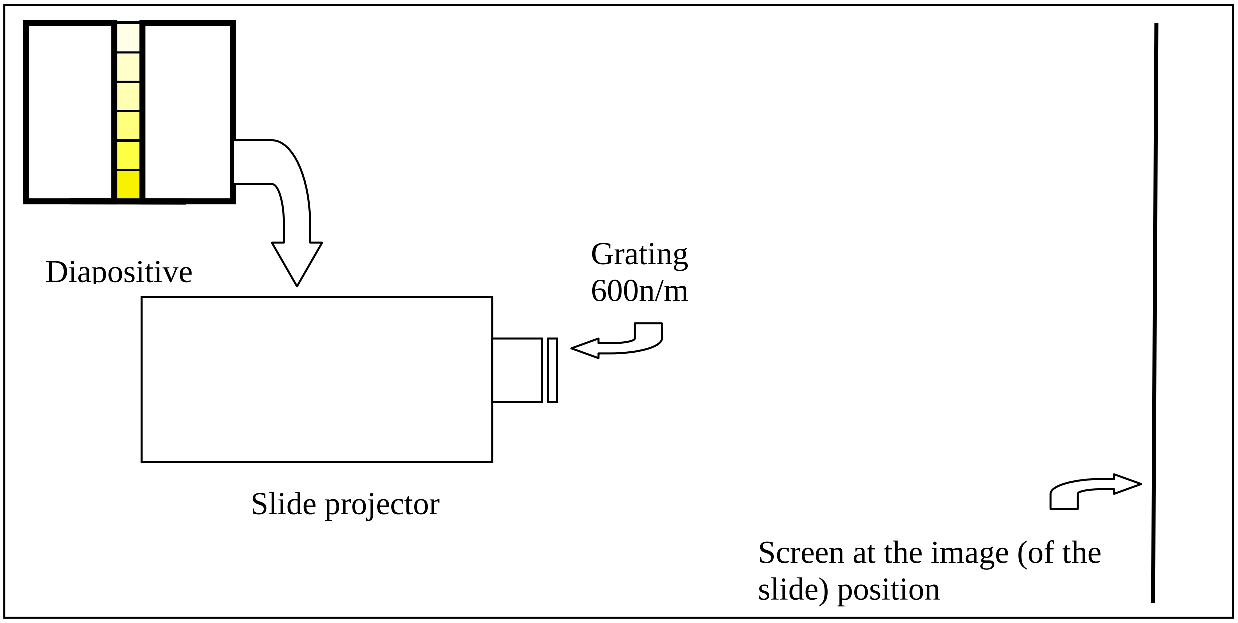


Figure 2. Sketch of the setting used to make the spectra shown in Figure 3

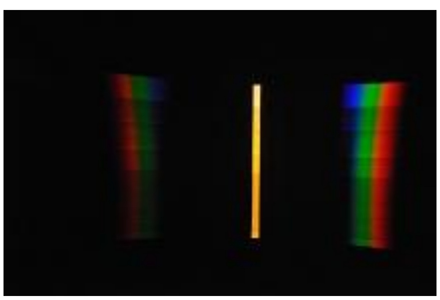
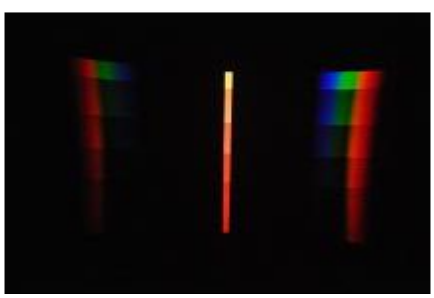
	Lower exposure	Higher exposure
Yellow		
Magenta		

Figure 3. Photos at two different exposures of spectra obtained for two slits equipped with transversal strips (resp. yellow and magenta) of increasing thickness: one, two, three, … layers superposed. (Grating 600/mm)

Phase CUR (discussions centered on thickness of an absorbing medium, using CURves)

Consider one layer of a – say – yellow filter, of thickness *l*. Let α_λ the ratio between emergent and incident density of light at λ (with the notations introduced above, $\alpha_\lambda = 10^{-\gamma_\lambda l}$. A likely shape of a curve of absorption giving α_λ versus λ is sketched for the first case (one sheet of yellow filter) is discussed and sketched qualitatively on a sheet of paper (a wide bump covering red and green third of spectrum, with soft "feet". At this step, we just need that students understand the meaning of α_λ.

The target of the discussion is to have the students understand and appropriately use in their explanations the following points:

-In terms of absorption, each layer results in the multiplication of the (spectral density of) light impacting on it by a factor smaller than 1: α_λ

-Two layers will "multiply" the intensity (spectral density) of the incoming light by α_λ^2 ; three layers by α_λ^3, four layers by α_λ^4. Hence a first access to what an exponential is.

-For some wavelengths, α_λ is high (say 0,9). In this spectral zone, three layers of filter will let emerge a light damped by a factor about 0,73 (0,9 times 0,9 times 0,9).). But in case α_λ is low, say 0,2, then only less than one hundredth of the incoming light will emerge from these same three layers.

-(The same process can be performed and discussed concerning the red filter).

-Consequently, the shape of a curve giving spectral density versus wavelength may be strongly distorted when light travels in a thick medium.

Links with other experiments:

ADT experiments may be used as a model situation concerning objects that change colour according to the thickness of medium crossed by light on its path from source to receiver.

Such situations may be used to trigger motivation and previous discussion, thus serving to introduce the *ADT* experiment. They may also appear as an extension, after the core of the experiment has been performed.

The Sun (or the Moon) changes colour when down on the horizon

The simplest (although…) case is that of the Sun or the Moon, the colour of which changes with the thickness of atmosphere the light incident light went through.

S*olutions*

Other experiments, including that described in Appendix 2, concerns the various colours that can be observed when the light received crosses thicker or thinner reservoirs of some liquids.

These examples permit to underline that the preceding analysis holds for three states of matter: solid, liquid and gas.

Conceptual generalisation:

These examples permit to underline that the preceding analysis holds for three states of matter: solid, liquid and gas. In all these cases, this analysis may constitute a first approach of what an exponential dependence is (see section "prerequisite", and/or Appendix 3). Particularly with the introductive situation – solid filters with increasing numbers of layers – it is possible to illustrate, step by step, the fact that each layer operates on (multiplies) what is "left" (the intensity of light) by the preceding one.

Another classical example of exponential dependence, radioactive decay, is analogous provided slices of space are replaced by time intervals.

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Appendix 1

With this type of presentation, white light is understood as comprising three components, red, green and blue; or, in more academic terms, three thirds of spectrum of relatively large, medium or short wavelengths in the interval 0,7µ -0,4µ. From there on, in order to analyse various situations of colour mixing, a given "third" may be considered as being "there" or not, regardless of the intensity of the corresponding component.

However, concerning additive mixing, it should be clear that if a very bright red beam of light converges on a screen with a very faint green beam, the impact zone will not be yellow but red.

Similarly, concerning subtractive mixing, the impact zone of a very intense beam of, say, red light on a given pigment, say green, is red, in practice, because the absorbing power of the pigment is not 100%. By contrast, with an "all or nothing" perspective, the impact zone of a red light on a green pigment, for instance, would be predicted as black.

The same remarks hold for a black area. It is often said that "black absorbs all the light", whereas the minimum reflecting power reached for a black material is about 0,045%[4].

In fact, in order to guide usefully our interpretation of daily life phenomena, the classical rules should be understood as indicating the outcome of

-additive mixing of coloured lights of *appropriately balanced intensities*;

-subtractive process with enough absorption for a given light to become visually undetectable; keeping in mind that a statement like "a green pigment absorbs *mainly* red and blue lights" holds in any case.

[4] *Sciences et Avenir,* March 2008, p 23, announced this record held by a team of Rensselaer Polytechnic Institute (USA)

The classical rules of additive and subtractive mixing.

Additive and subtractive mixing: colours associated with "thirds of the spectrum".

Additive colour mixing

The spectrum of visible white light ranges from λ=400 nm to λ=700 nm.

(λ: wavelength; measured in a vacuum; 1 nm = 10^{-9} m).

The spectrum is here schematically divided into three thirds.

Coloured lights with a spectrum corresponding to a third of the spectrum of white light

in long wavelengths appears red

in intermediate wavelengths appears green

in short wavelengths appears blue

Combining these three lights, in various proportions, results in a large range of colours and can also give white light.

Combining two of these lights in the correct proportion respectively gives a light that is

- yellow, if green light and red light are added
- cyan, if green light and blue light are added
- magenta, if red light and blue light are added

Subtractive colour mixing: A filter (or a pigment) absorbs a part of the spectrum of white light

- a yellow filter (or a pigment) absorbs blue light (a third) and transmits (or diffusively reflects) green and red lights.
- a cyan filter (or a pigment) absorbs red light (a third) and transmits (or diffusively reflects) green and blue lights.
- a magenta filter (or a pigment) absorbs green light (a third) and transmits (or diffusively reflects) red and blue lights.

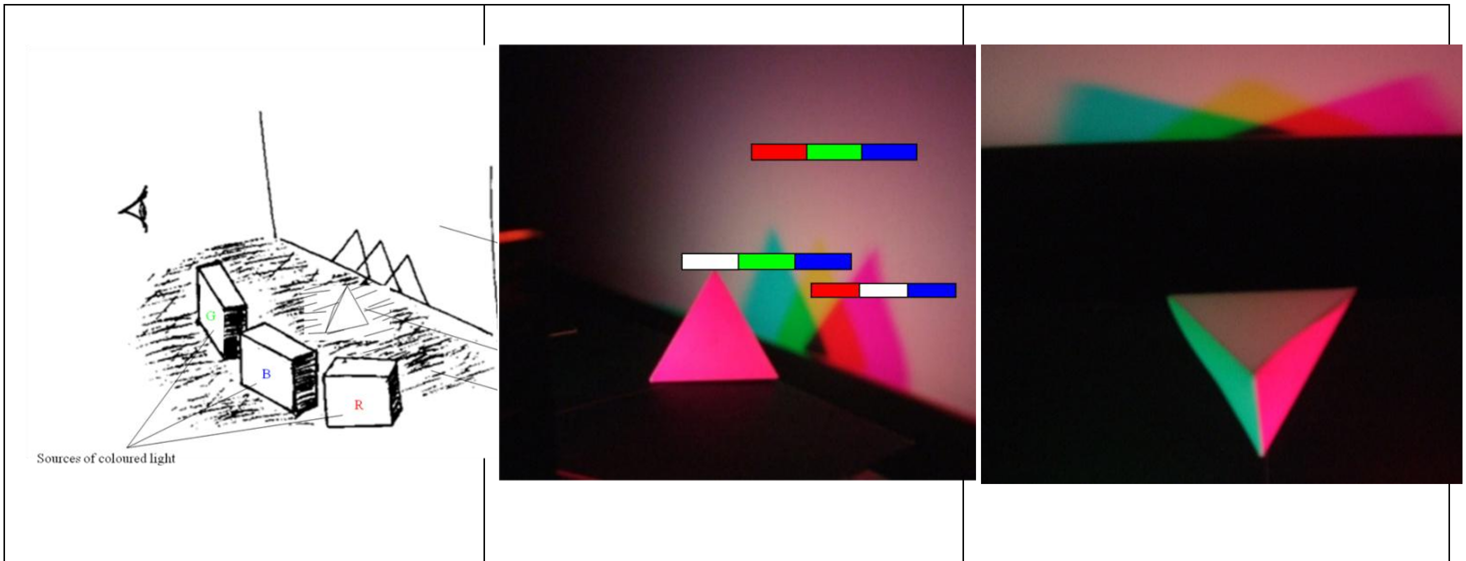
When illuminated in white light, two superimposed filters or two blended paints send back (transmit or diffusely reflect) to the eye only the part of the spectrum that they have in common:

yellow + magenta filters or pigments => red light

cyan + yellow filters or pigments => green light

magenta + cyan filters or pigments => blue light

If necessary, it is possible to use various set-up of "coloured shadows" type (Olivieri *et al.* 1998, Chauvet 1996, 1999: see Figure 1, Planinsic 2004, MUSE paper: Planinsic & Viennot 2010,) to remind students with the classical rules of additive or subtractive colour mixing.



a	b	c

Figure 1. Coloured shadows:

a) A setting to demonstrate the classical rules of colour phenomena (Chauvet, 1996)
b) A photo of what can then be observed on the screen and on the tetrahedron, with symbols suggesting, in three areas of the screen, the spectral composition of the impacting light (each coloured box indicates the fact that the corresponding coloured light reaches the considered area).
c) View of the same situation from above (credits: Pascal Sauvage)

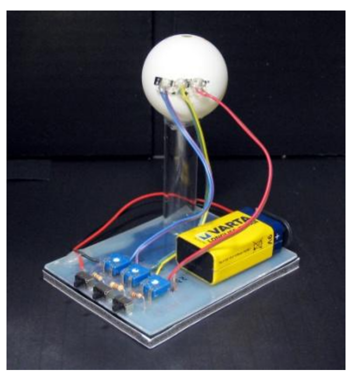


Figure 2. A colour mixer designed by G. Planinsic (2004). Color Light mixer for every student, *The Physics Teacher*, 42,138-142

Note that with coloured shadows, the presence of an obstacle ensures a complete blockage of some beams of coloured light in some directions. For instance, a cyan area can be seen as resulting from the impact of white light (see the white background on the screen, Fig. 1b) "minus" the "red" third of spectrum, given that the red beam has been blocked by the tetrahedron. In other terms, this setting is an "all or nothing" arrangement: a given beam reaches (additive process), or not (subtractive process), a given area. The experiment described in the MUSE paper (see note 2) may contribute to counterbalance this limiting aspect.

Appendix 2

If we put a cupel of small curvature on a retro projector, fill it with pumpkin seeds oil and make the image on the screen, we can see a range of colours from yellow to red.

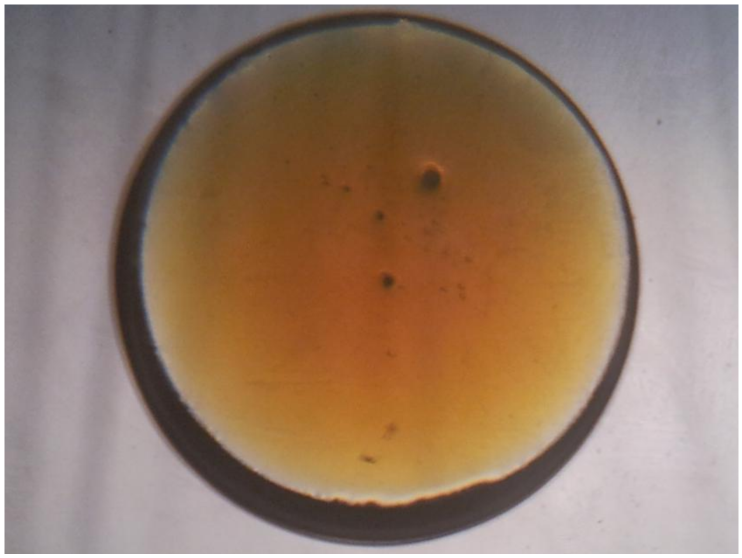


Figure 1. Pumpkin seeds oil in a cupel (thickness of the layer from zero to 5 mm).

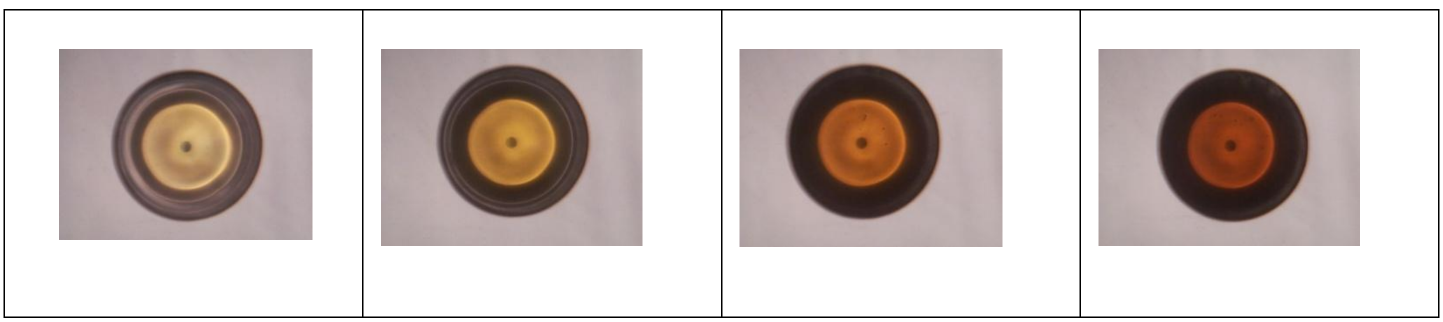


Figure 2. Pumpkin seeds oil with four different thicknesses, from 1 to 8mm, and the corresponding colour (seen via a retro-projector)

A paper describes a controlled experiment about this effect:
Kreft, S. & Kreft, M. 2007. Physicochemical and physiological basis of dichromatic colour. *Naturwissenschaften* 94, 935-939.

The paper describes and analyses the dependence of colour of emerging light on thickness for two liquids: pumpkin seeds oil and blue of bromophenol.
The main explicative elements are shown in the two following figures from this paper and their captions.

Note how difficult it may be to understand the last sentence of caption “Fig. 2”:
“A transmission spectrum for thin layer was calculated directly from the absorption spectrum, while a transmission spectrum for thick layer was calculated from the absorption spectrum after its multiplication by factor 10”.

To say the least, the analysis presented above is not apparent in this statement, when it is suggested that a constant factor (10) affects all the spectral bands in the same way.

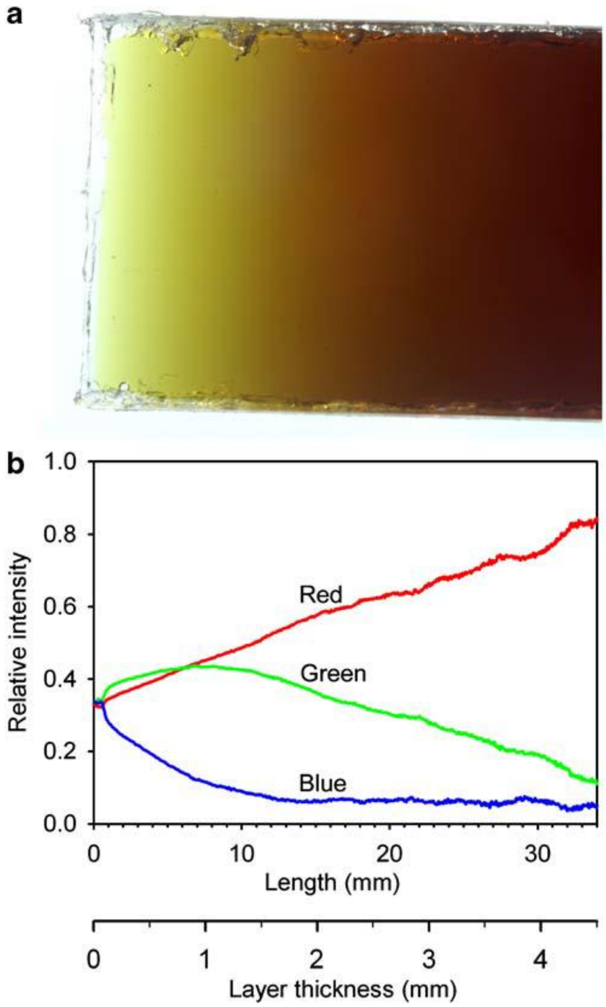


Fig. 1 A sample of pumpkin seed oil in a wedge-shaped cuvette. a Aglass cuvette, made of two standard microscope glass slides (the width is 26 mm) filled with a sample of pumpkin seed oil. The thickness of the oil layer linearly increases from 0 mm at the left to 4.5 mm at the right. b Horizontal relative intensity profile of the red, green and blue channel taken at the middle of the image (a). At the zero thickness position, the colour is white, which results in relative value 0.33 for all three channels. At the thickness from 0 to 0.7 mm, the prevailing channel is green, and beyond the thickness of 0.7 mm, the prevailing channel is red.

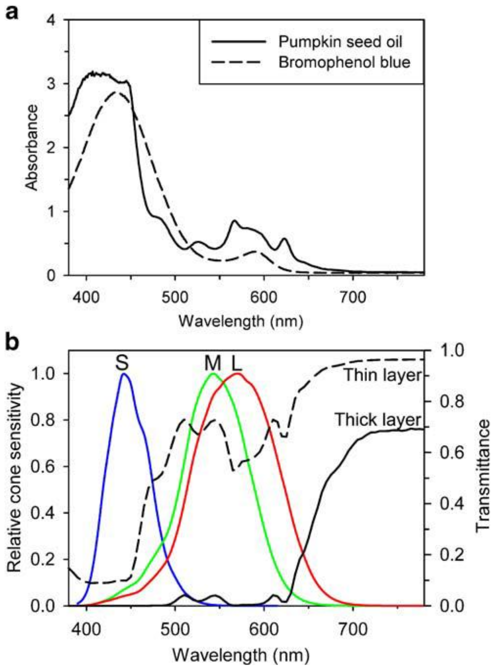


Fig. 2 Absorbance and transmission spectra of the pumpkin seed oil and the cone sensitivity functions. a Absorbance spectra of the pumpkin seed oil (solid line) and the aqueous solution of bromophenol blue (dashed line). Both substances display dichromatic properties due to one wide shallow local minimum around 540 nm and one narrow deep local minimum above 650 nm on the absorption spectra. b Transmission spectra of pumpkin seed oil in thin (dashed line) and thick layer (solid line), and the normalized long-wavelength (L), medium-wavelength (M) and short-wavelength (S) human cone sensitivity functions (three solid lines; Stockman and Sharpe 2000). Both transmission spectra were calculated from the same absorption spectrum measured in thin layer (presented in a). A transmission spectrum for thin layer was calculated directly from the absorption spectrum, while a transmission spectrum for thick layer was calculated from the absorption spectrum after its multiplication by factor 10.

Appendix 3
From multiplication to exponentiation

We know that for layers of small thickness d the following linear approximation holds

$i_\lambda(d) \approx i_{\lambda 0} - k \cdot d \cdot i_{\lambda 0}$ or

$i_\lambda(d)/ i_{\lambda 0} \approx (1 - k \cdot d)$

For larger d, this will be a bad approximation. But decomposing d in to equal layers with d/2, and given that the coefficients (multiplicative factors) of absorption of two slices of a given medium have to be multiplied in order to characterize the absorption by the set of two layers, a better approximation will be

$i_\lambda(d)/ i_{\lambda 0} \approx (1 - k \cdot d/2)^2$

Decomposing into n equal sheets of thickness d/n, we obtain a still better approximation

$i_\lambda(d)/ i_{\lambda 0} \approx (1 - k \cdot d/\mathrm{n})^{\mathrm{n}}$

This becomes the more accurate, the larger n, and in the limit of $n \to \infty$, one has the exact (up to multiple reflection and interference effects) result

$i_\lambda(d)/ i_{\lambda 0} \approx \exp(-kd)$

which is the Lambert-Beer (exponential attenuation) law.

Thus, for short, multiplicativity and a basic property of the exponential lead to the familiar Lambert-Beer law.