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Holes in a bottle filled with water: which water-jet has the largest range?

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ABSTRACT

In some textbooks, a simple experiment is often presented about a jet of fluid streaming out of holes made in a vessel full of water. In this paper, we have chosen to discuss this topic partly because of erroneous assumptions that are prevalent is such treatments. Textbooks commonly ignore that the range of a water jet depends not only on the exit velocity but also on the time of flight until the jet reaches a horizontal surface. In addition, the connections made between this experiment and the dependence of hydrostatic pressure on depth often lead to erroneous explanations in both words and drawings. These errors seem to recur over extensive periods of time possibly through copying from one textbook to another. The authors' intention seems to be to illustrate that, in a fluid at rest, pressure increases with depth, whereas in this case the fluid is clearly not at rest. We clarify crucial aspects of this situation by applying the Bernoulli theorem in the case of water as an almost non viscous liquid and calculating the predicted trajectory of the jet as a projectile. Emphasis is placed on the fact that, in order to calculate the range of a given jet, the exit velocity is only one of the relevant factors. The duration of the free fall outside the recipient also needs to be taken into account. We also note that this analysis leads to the equivalence with water free fall and not with the hydrostatic situation. Our presentation here is restricted to the case of non-viscous incompressible fluids, jet streams from a container with a stationary free surface and geometrically perfect exit holes. For instance, hot and cold water behave differently due to the corresponding change in viscosity. All of these parameters present additional complexities some of which will be analyzed in a future paper.

Key words: water-jets from a bottle

As pointed out by Atkin (1988; see also, quoted by Slisko 2009: Biser 1966, Paldy 1963, Grimvall, 1987), it is surprisingly common to find a drawing such as the one shown in Figure 1, including in textbooks treating hydrostatic pressure or encouraging students to try out some experiments for themselves. This drawing is intended to represent the water-jets emerging from three holes punched into an open bottle filled with water. The intent is to demonstrate that hydrostatic pressure increases with depth, a correct idea. However, the sketch of experimental outcome shown in Figure 1 is simply imaginary. When actually performed, the experiment shows a different outcome, more like Figure 2.

Given that the level of the bottle's stand and that of the stand on which the jets impact are the same, the maximum range is obtained for a hole punched at half height (*H/2*) of the water column, and two holes punched at heights adding up to *H* (i.e. equidistant holes from the middle hole) yield water streams with the same range (Figure 2).

Simple reasoning is sufficient to suspect an error in the prediction represented in Figure 1: a hole drilled just at the level of the stand, that is at the bottom of the bottle, could not result in a far reaching jet.

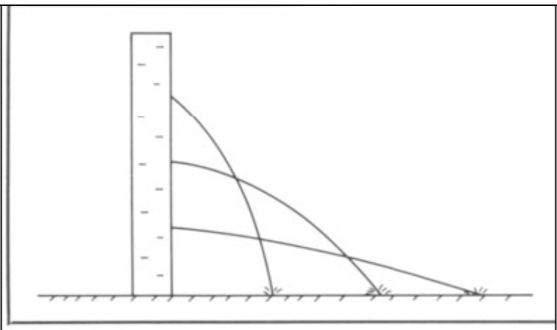


Figure 1 Typical diagram found

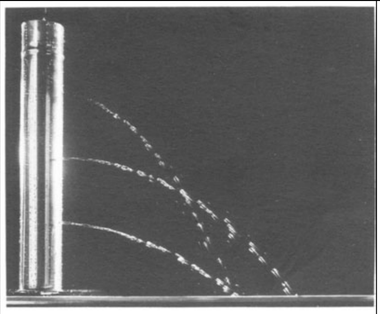
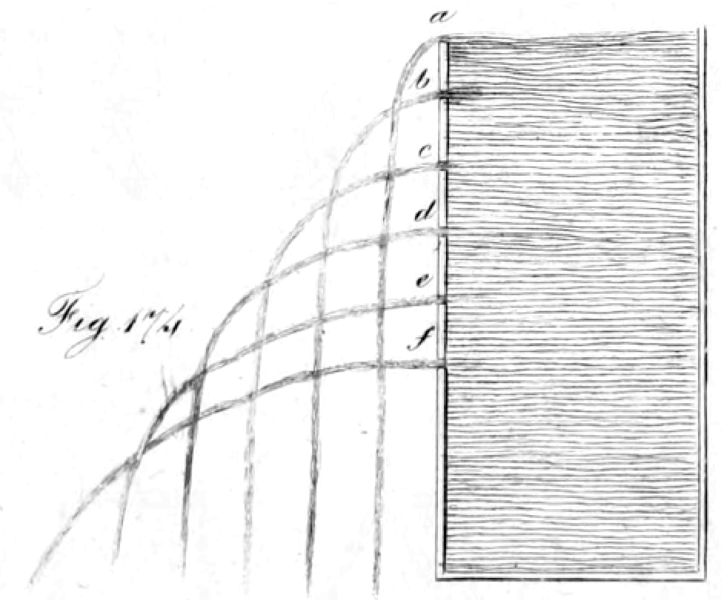


Figure 2 Apparatus in action

Two figures from Atkin's paper (1988)

As shown by Slisko (GIREP-EPEC 2009, see also Slisko & Corona Cruz 1997), a similar misleading drawing was included by Leonardo da Vinci in his book *Del moto e misura dell' acqua.*



DEL
MOTO E MISURA
DELL' ACQUA
DI
LEONARDO DA VINCI

BOLOGNA
A SPESE DI FRANCESCO CARDINALI
1828.

Figure 3: A similarly misleading drawing from Del moto e misura dell' acqua by Leonardo da Vinci, digitised copy from Harvard College Library, Google books.

Slisko (1997, 2009) has also demonstrated that, a century after Leonardo da Vinci's drawing, the issue was correctly resolved by Torricelli around 1640 and was correctly dealt with by most textbook authors during the 19th century. However, the erroneous prediction and the misguided association of this measurement with hydrostatic pressure resurfaced again in textbooks of the 19th century and have remained with us since then (see for instance Santamaria 2007).

How does this error emerge?

Assuming that the Bernoulli theorem is valid in this situation (in particular, the liquid is not viscous), and given that the pressure is the same at the free surface and at the exit hole, the square speed v^2_h is proportional to the difference, $(H-h)$, between the surface altitude H and that of the hole, h,

$$v^2_h = 2g\,(H-h)$$

It is interesting to note that this formula is the same as that obtained with the model of free fall. The relationship $\Delta p = \rho g(H-h)$, where Δp is the difference in pressure between the altitudes h and H, cannot be used because it assumes hydrostatic conditions which clearly do not hold in the case of an accelerating fluid stream.

At this point, a one variable approach would lead to the common erroneous drawing. Indeed, were the range of a jet, d, only dependent on the horizontal velocity v_h of the water at the exit point, the situation would be correctly depicted by the drawing in figure 1.
But another quantity affects the range of the water-jet: the time t_{ff} of the water's free fall. The square of this time, t_{ff}^2, is proportional to the altitude h of the hole measured from the stand on which the jet is impacting $t_{ff}^2 = 2h/g$
Assuming the water jet is horizontal at the exit hole, the range of the water jet is obtained by multiplying velocity v_h at the exit hole by the time of free fall, t_{ff}.

$$d = v_h\, t_{ff} = 2[h(H-h)]^{1/2}$$

It is easy to see that when two holes have altitudes adding up to H, the value of this product, $v_h\, t_{ff}$, is the same for the corresponding jets. Therefore, their range should be identical, as verified also by experiment (see Figure 2).

The experimental confirmation of this simple analysis is not as easy as might be expected. With a stabilized level of water in the bottle, it is not difficult to obtain the predicted order in the observed water jet ranges, as well as a maximum range at the middle hole, as in Figure 2. However, each of these horizontal ranges may differ significantly from the predicted ones. The following practical advice is helpful in performing a successful demonstration:

- When making holes make sure to hold the tool perpendicular to the bottle wall and to make clean edges of the holes. Small imperfections or bits of material that remain from drilling can have significant effects on the corresponding water jet and could change the qualitative outcome of the experiment.

- Holes with smaller size will result in a slower descend of the water surface, thus providing more time to observe the experiment. On the other hand, smaller holes tend to produce water jets that break into droplets and may give trajectories that significantly deviate from the theoretical ones due to larger resistance to water flow. You will notice this if you want to compare measured and calculated ranges; for the purpose of making qualitative comparisons and for showing that the range of the middle stream is longest and the ranges from equidistant holes from the middle one approximately equal, small holes should still work well.

- Making holes with heated objects proved to produce cleaner edges though it is more difficult to control the diameter of the hole than by drilling. Getting equal diameters of the holes is not a critical factor in achieving a successful outcome. An alternative way is to punch the holes with push pins, but this gives rather small holes and consequently the problems associated with small size.

- Starting the experiment is normally done in one of the following ways.
  - Cut a piece of self adhesive tape (one for electrical insulation works best) a few centimetres longer than the distance between the bottom and the top hole. Stick the tape vertically on the bottle to cover all the holes. Fill the bottle with water, put it on a flat surface and quickly pull the tape off the bottle.
  - Obtain push pins (of as large a diameter as possible) and simply push them in the wall of the plastic bottle at the places where you want the holes. Fill the bottle with water and place it on a flat surface. Quickly remove the push pins starting from the top.
- In order to avoid any interference between the water jets, make holes slightly shifted from each other in a horizontal direction (3 mm is enough).

Bottle standing at a different level

The case discussed up to now is when the impacts are observed on the same horizontal plane as that on which the bottle stands. The simple expression $d= v_h\, t_{ff} = 2[h(H-h)]^{1/2}$ obtained in this case is due to the fact that the horizontal exit velocity, v_h and the time of free fall, t_{ff}, are proportional to the square root of, respectively, H-h and h, which also explains the symmetry of the expression for d with respect to H/2. This example is very appropriate to encourage students to reason with several variables, given that, in this case, the two relevant variables have a symmetrical effect.

What would be the relative values of the ranges in case the bottle stands at a distance a above the horizontal level at which the impacts are observed. In this case, the time of free fall, t_{ff}, is larger than before, but proportionally less affected by the distance h between the exit hole and the stand of the bottle. The contribution of h to t_{ff} is, proportionally, lower. We then expect a larger influence of the first variable, v_h, itself linked to H-h. For a sufficient of value of a, we should observe ranges increasing with H-h, as if only the distance between the hole and the free surface of the water was relevant.
This is indeed confirmed by the following calculation, still assuming a non viscous fluid (see also Avison, 1988).

$$v_h = [2g(H-h)]^{1/2}$$

$$t_{ff} = [2\,(h+a)/g]^{1/2}$$
$$d = v_h\, t_{ff} = 2\,[(H-h)(h+a)]^{1/2}$$

Figure 4 illustrates two cases: The standard picture and the more generalized case, where the standard jets may change the ranking of the ranges with respect to the red line.

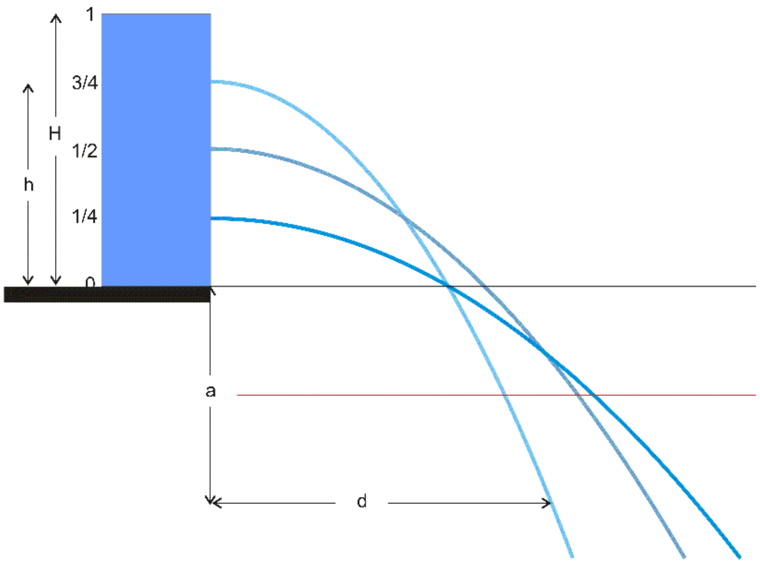


Figure 4. The generalized solution to the water jets problem. Two situations can be observed: (a) the standard picture, where the three jets impact the horizontal surface shown in black, and (b) the more generalized case, where the water jets may change the ranking of their ranges (eg with respect to the horizontal surface shown in red).

In the case represented in Figure 4, when the ranges increase with the distance from the hole to the free surface, there is a risk of misunderstanding. The outcome of the experiment may reinforce two misleading intuitions. One is that the pressure at the exit hole increases with this distance, as in a static case, whereas the Bernoulli theorem has been used considering that this pressure was the atmospheric pressure. The other possible wrong idea is that the range of the jets only depends on the exit velocity, whereas it still also depends on the time of free fall.

An effort to compare detailed observations with numerical modeling of this effect reveals additional complexities with respect to the viscosity of the liquid and the size and shape of the fluid jet hole. Figure 5 presents theoretical predictions (red curves) superimposed on a photograph of the jet streams. Likewise, hot and cold water behave differently due to the corresponding change in viscosity. A detailed analysis of these complexities will follow in a separate paper in the future.

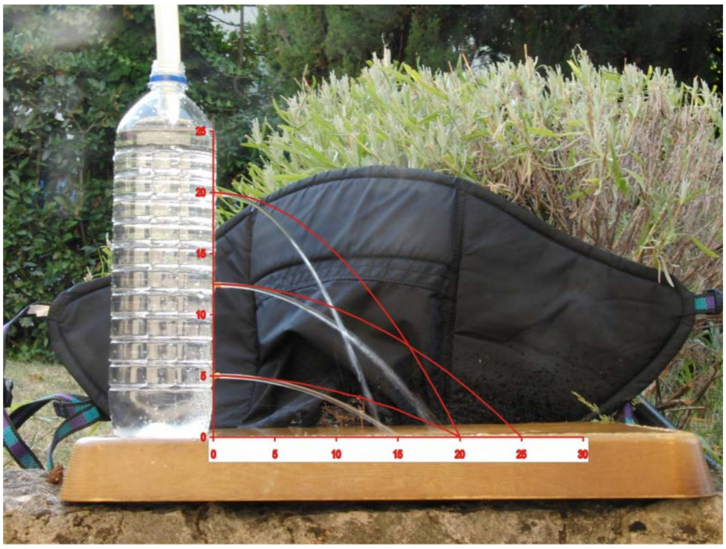


Figure 5. Bottle with short nozzles (length 15mm, diameter 3 mm). Water level was kept stationary by constant inflow from the hose.

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