Sneezing, Pixel Spacing, and Geometric Optics

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A nice observation printed in this journal[1] described how holding a transparent sphere in front of a white area on a color monitor made the red, green, and blue pixels of the luminescent layer become visible. Many people working with color screens must have observed similar effects; we draw attention here to one such effect that can be pursued a little further as a classroom or homework investigation.

I discovered (from experience, I must admit) that should you sneeze in front of a color monitor, some droplets may end up on it. Where this happens, white areas change into color-spotted ones (if it is not cold season, an atomizer used to water flowers will do). The explanation for these color effects for both the transparent sphere and the sneeze droplets is, of course, their magnifying effect. This qualitative observation of a literally colorful effect of everyday physics can be turned into quantitative reasoning; in fact, it turns out that the pixel spacing can be measured with nothing more than a ruler and a protractor.

Measuring the Pixel Spacing

The basic idea is that liquid droplets on a flat support are lenses to which the lens equation can be applied. Moreover, the thin-lens approximation will be assumed (i.e., the focal distances on either side are equal and object and image distances are measured from the center of the lens and not from its principal points), which is accurate enough because we are aiming at simplified estimates in the following. Writing the lens equation in the form

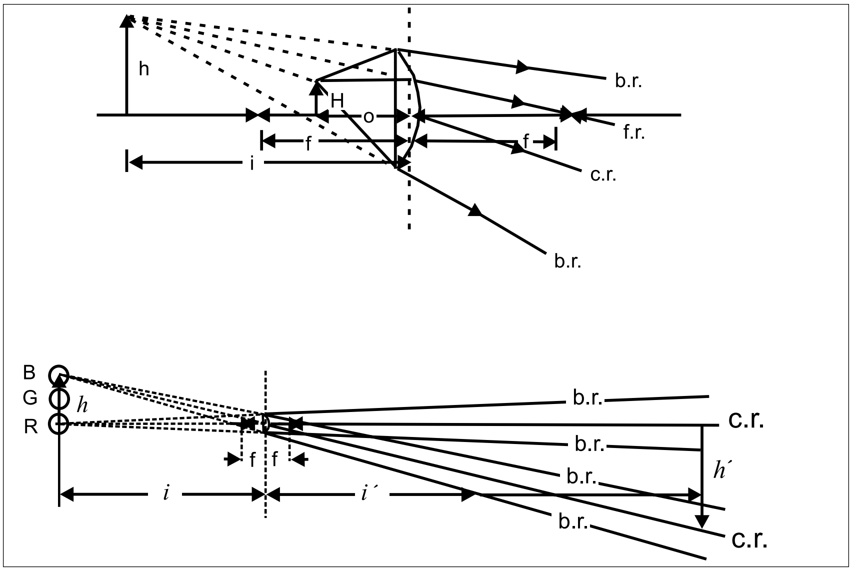

Fig. 1. Geometry of principal rays and observed image in the droplet lens. c.r. = center ray, f.r. = focus ray, b.r. = border ray; B = blue, G = green, R =red.
Construction of the image is best explained in two steps: magnifying effect (upper part) and pinhole effect (lower part). In a last step (not shown), the diverging light bundles coming from different points of the virtual image and bounded by the finite size of the lens (border rays) are focused by the eye. To understand the pinhole effect, consider the image of three adjacent pixels in the lower part of the figure: the light bundles emerging from the pixels and bounded by the border rays do not overlap and we have to move the eye laterally in order to see one pixel after the other (even with non-overlapping light bundles, we still could see all pixels at a time if the eye were large enough, but for the geometry given this is not the case).

$$H = \frac{n}{i}\, o \qquad (1)$$

(where H, o and h, i are the size and distance of the object and image, respectively), two factors $\frac{h}{i}$ and o show up, which turn out to be measurable by easy means ($\frac{h}{i}$ is nothing but the tangent of the angle of aperture).

Ratio of image size to image distance ($\frac{h}{i}$)

Keep your head at a given observation distance i' from the screen and fix on a given droplet. Then move your head slowly laterally (parallel to the screen). You will see how pixels of different color move through the droplet in question. By counting the number of appearing pixels (k) and measuring the distance of lateral movement of your head (L), you can calculate the observed pixel spacing: $h' = L/(k-1)$. (Why $k-1$ and not k? Look at an array of equally spaced dots!) A very easy way to do this measurement is to move your head in a way that the tip of your nose touches two obstacles of given distance, which is L (this can be achieved by using a frame cut from cardboard).

Note that the observation through the water droplet is different from the familiar use of a magnifying glass, where usually the object is in the focal plane [whereas here we cannot choose the object distance because it is fixed by the screen thickness; (see Eq. (3)], and the magnifying glass is much larger than the image (whereas here the diameter of the droplet lens is roughly of the size of the virtual image of a single pixel).

The main consequence of this is that in the droplet lens we can hardly see the entire image of one pixel at a time, but by moving the head laterally we see one pixel after the other.

The whole effect of the droplet lens is best explained in two steps. First, it acts as a magnifying glass producing an enlarged virtual image; see upper part of Fig. 1. Second, this virtual image is observed as through a pinhole of the size of the droplet; see lower part of Fig. 1. In these figures, h' and i' are the pixel spacing and the screen distance *where the observation takes place*, whereas h and i are the image size and distance *occurring in the lens equation*. In order to avoid possible confusion, attention should be drawn to the fact that these quantities have a different meaning; the *ratios* $\frac{h}{i}$ and $\frac{h'}{i'}$, however, have the same value; that is,

$$\frac{h}{i} = \frac{h'}{i'} \qquad (2)$$

This is due to the law of direct proportions; see lower part of Fig. 1. With numerical values $h' \approx 0.5$ cm, $i' \approx 20$ cm, from Eq. (2) we get $\frac{h}{i} \approx 0.025$. Since Eq. (2) is exactly true only for the center rays and since we have to move our heads in order to see different parts of the image, we have to make sure that every part is looked at well centered within the ray bundle; that is, within the borders of the lens itself. This is the major source of imprecision in the measurement.

Measuring the Object Distance (o)

This is a bit more tricky (if you want to do it without destroying the screen). Assuming that the luminescent layer is attached directly on the back side of the glass screen (and making furthermore the thin-lens approximation), the object distance o (the distance between the droplet lens and

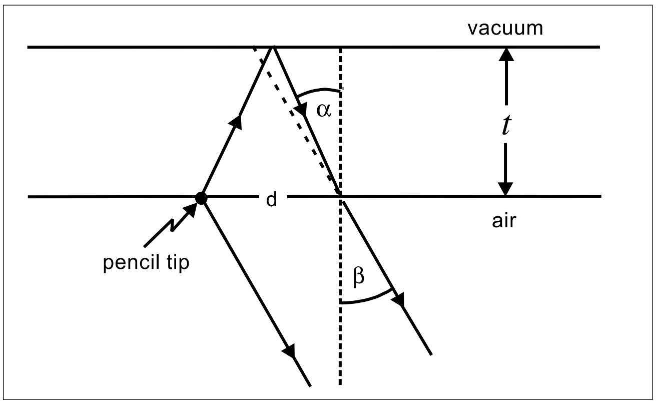

Fig. 2. Backside reflection and thickness of a television screen.

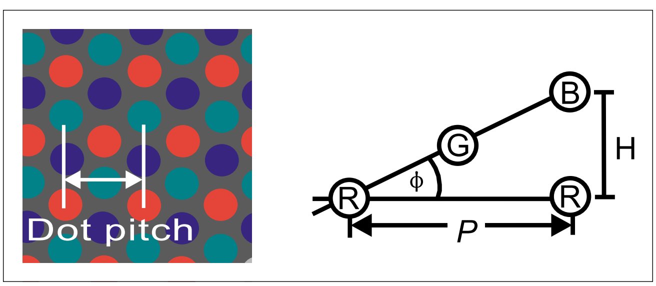

Fig. 3. Dot pitch *P* and next-neighbor spacing *H*.

the pixels on the luminescent layer) is just the thickness of the glass t:

$$o \approx t \qquad (3)$$

Now we need a way of measuring the thickness of a pane of glass when we have access to only one side of it. One way to achieve this is provided by the observation that a glass pane shows reflection also on its backward side. This can be verified for a monitor screen (which is best left black for this purpose; you also have to experiment a little for the best illumination). Figure 2 shows how the backside reflection can be used to determine the glass thickness and thus the object distance. If you put the tip of a pencil on the screen (with index of refraction n) and look at it at an angle β (measured between the line of sight and the normal to the screen), you will observe a lateral displacement d between the tip and its image (reflected at the backward side), which you can measure by holding a ruler tightly to the surface of the glass pane. Looking at an angle β there will be a certain mark on the ruler seen precisely in line with the image of the tip; the distance from this mark to the position of the tip itself is d in Fig. 2, which is directly related to the glass thickness t we are looking for:

$$t = \frac{d}{2 \tan\left[\arcsin\left(\frac{1}{n} \sin\beta\right)\right]}$$

$$\approx \frac{n}{2 \sin \beta} d \qquad (4)$$

The first line follows from Snell's law and elementary trigonometry, the second from the approximation $\tan(\arcsin x) \approx x$, which holds within 15% accuracy up to $x \leq 0.5$. With numerical values $d \approx 0.5$ cm, $\beta \approx 30^o$, and $n \approx 1.5$ (glass) and Eqs. (3) and (4), we get $o \approx 0.75$ cm.

Comparison with Technical Data

On the one hand, we now can infer the distance of neighboring pixels on the luminescent screen from our measurements. According to Eq. (1) and with the numerical values given at the end of the preceding subsections, we obtain

$$H \approx 0.19 \text{ mm} \qquad (5)$$

On the other hand, we can look into data sheets of monitors. The so-called dot pitch, P, given there is not quite the same as the H inferred from measurement. Rather, P is the distance of next neighbors of the same color, whereas H is the distance of adjacent pixels (which have different colors).

The relationship between these two quantities is shown in Fig. 3 and given by

$$H = \tan(\phi)P = \frac{1}{\sqrt{3}}\, P \qquad (6)$$

where $\phi = 30^o$ follows from the geometry of the equilateral triangle formed by the color triples (left side of Fig. 3); other information about screen technology can be found on the Internet.[2] With a dot pitch of 0.26 mm for my monitor, Eq. (6) yields $H = 0.15$ mm. (Besides the screens with circular pixels for which the above considerations apply, there are also screens with rod-like pixels; the droplet lens will tell you what type of screen you are working with).

The values inferred from measurement and taken from the data sheet thus agree within 25%, which is quite satisfactory in view of the simplicity of the experimental approach and of the quite rough approximations made.

Conclusion

Turning qualitative into quantitative observation and explanation is a central issue in science, and a very important intermediate step for this is approximate first estimates, obtained by simple measurements and calculations. The importance of this step is well supported. In science, approximate and even order-of-magnitude estimates are often indispensable as a guide for further experimental and theoretical work. In science education, this part of the scientific method can be presented in class or given as homework, due to the very fact that a simplified approach is taken.

The example treated here belongs to a type of measurement considered an important application of the mathematical-scientific approach to problems of everyday life,[3] and thus as an interesting issue of science education, namely the size and distance measurement of inaccessible objects. This task occurs in a whole range of settings beginning with astronomy or geodesy (where the objects are too far away or too large), down to the microscopic realm (where the objects are too small), not forgetting the frequent case where some obstacle prevents direct measurement. Measurement of pixel spacing and screen thickness of a color monitor are a mixture of the two.

For those who do not consider sneezing an appropriate step in a scientific investigation, the necessary water droplets can also be produced in a quite controlled manner with the help of a syringe. For all your further investigations of the issue: Bless you!

References

1. J. L. King, *Phys. Teach.* **33**, 459 (1995).
2. Dot pitch and geometry of the luminescent layer: www.apocalypse.org/pub/u/milktree/dot-pitch.html Screen and other hardware technology in general (in German): http://i31www.ira.uka.de/saar/mmd2/Kap2a/ppframe.htm
3. Garfunkel and L. A Steen, *For All Practical Purposes: Introduction to Contemporary Mathematics* (W. H. Freeman, New York, 1988), Chap. 16.