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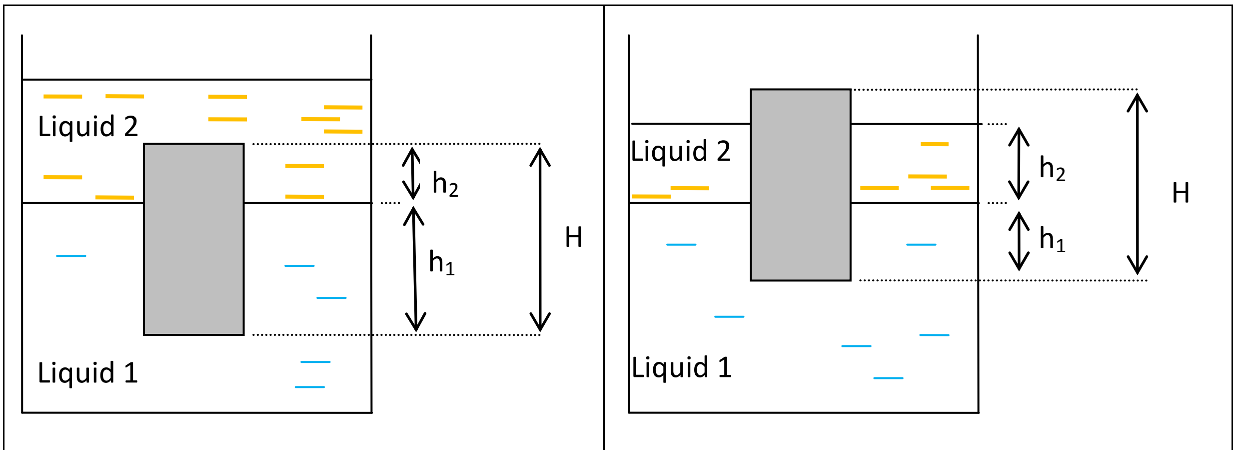
Floating between two liquids[1,2]

A very classical exercise is the analysis of the hydrostatic equilibrium of a cylindrical body –height H, area of the base S, mean density ρ_s - situated in a recipient with two immiscible liquids of density, respectively ρ_1 and ρ_2, such that $\rho_1 > \rho_s > \rho_2$. The body floats in the first one alone ($\rho_1 > \rho_s$) and sinks in the other one alone ($\rho_s > \rho_2$). The hydrostatic equilibrium of the solid "between two liquids" may be like one of the two cases (*a* or *b*) represented in Figure 1.



a The floating body is covered by liquid 2.	*b* The floating body is not covered by liquid 2 Then, the body is in contact with the air.

Figure 1. Two cases for a cylindrical solid at hydrostatic equilibrium "between" two liquids.

Materials

The materials needed to implement this experiment are very simple, for example.

- A transparent cylindrical glass filled with water (possibly with a colorant).

- Groundnut oil, usually about 50 cm^3 are sufficient.

-A plastic egg (toys, e.g. Kinder) or any empty tube of pills, to be partly filled with dense materials (e.g. coins, shot) until the floating in water (alone) is ensured. It is in a stable vertical position, the top of the solid emerging by about the third or the quarter of its height H out the free surface of water .

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1 More details in: VIENNOT, L. (2011). Floating between two liquids, http://education.epsdivisions.org/muse/twoliquid.pdf

2 The MUSE group (G. Planinsic, E. Sassi, L. Viennot) takes responsibility for the content of this paper (July 2011). The intellectual property remains with the authors.

Classical solution

For any position of the solid, the relationship of fluid statics $\Delta p = - \rho g\Delta z$ can be used for each part of the cylinder immersed in each liquid, of respective heights h_1 and h_2. Therefore, with an upward axis, the differences in pressure between lower and upper horizontal sections of the cylinder immersed in, respectively, liquid 1 and 2 are given by:

$\Delta p_1 = \rho_1 gh_1$ $\quad\quad$ $\Delta p_2 = \rho_2 gh_2$

The possible contribution of the air (case *b* in Fig. 1) can be neglected with respect to the two others, given that the density of the air is typically a thousand times smaller than those of the liquids.

A state of equilibrium occurs when Archimedes' up-thrusts due to each liquid and the weight of the body balance out: $(\rho_1 h_1 + \rho_2 h_2)\, Sg - \rho_s HS\, g = 0$, or else

$\rho_1 h_1 + \rho_2 h_2 = \rho_s H$

Solving for, say, h_1 gives

$\boldsymbol{h_1 = (\rho_s H - \rho_2 h_2)/\rho_1}$ (1)

h_1 and h_2 being the heights of each part of the cylinder immersed in each liquid **in case the cylinder actually floats between the liquids.**

In particular, relationship (1) gives the value of $\boldsymbol{h_1}$ that is **necessary** for the body to float in liquid 1 alone (then $h_2 = 0$), or to be in a state of equilibrium of the type "floating between two liquids" with a given value of h_2. Would the actual volume of liquid 1 be too small to ensure this condition on h_1, then the body would rest on the bottom of the recipient.

The statements of this section, and in particular relationship (1), hold for case *a* and case *b* in Figure 1; with, in case *b*, the approximation that Archimedes' up-thrust due to air is neglected with respects to the two other contributions.

In case *a*, when the body is covered by fluid 2, $h_1 + h_2 = H$ and relationship (1) leads to
$h_1 = h_2\, (\rho_s - \rho_2) / (\rho_1 - \rho_s)$ (2) or else
$\boldsymbol{h_1 = H\, (\rho_s - \rho_2) / (\rho_1 - \rho_2)}$ (3)

Stability of the equilibrium "between two fluids": For the cylindrical solid to stay in a stable vertical position, it is necessary that the center of mass of the solid be lower than the center of mass of a fluid cylinder of same section filled up with the two liquids respectively by a height h_1 and h_2. The insertion in the solid cylinder of a dense material, e.g. coins or shot, can solve this problem.

Practical detail: when the immersed cylinder is just resting (with nearly zero interaction) on the bottom of the recipient, it is particularly striking to see it "taking off" when the oil is added.

This situation can be used to

- address some students' difficulties about: pressure in statics of fluids, role of atmospheric pressure, Archimedes' up-thrust, **thinking this setting as a system**.

- overcome some of these difficulties, especially the last one, by using graphical representations, thereby addressing the interpretation of abstract representations as Cartesian graphs.

Evidencing possible difficulties[3]

Question: A cylindrical body (mean density ρ_s) floats on a liquid 1 (density ρ_1). Another liquid (2: $\rho_{2<}\ \rho_s$) is poured on top of liquid 1. The two liquids are not miscible. What will happen with the cylinder? Choose the right answer and explain: The new equilibrium position of the cylinder will be

A. same as before
B. higher than before
C. lower than before
D. at the bottom
E. more information is needed

(In Bennhold & Feldmann ‘s wording, it is said that “an object floats in water with ¾ of it’s volume submerged, and “oil is poured on top of the water”; and the questions are slightly different.)

A correct answer

Adding liquid 2 results in a part of the solid being then immersed in this liquid, with a corresponding contribution to Archimedes’ up-thrust. Relationship (1) shows that a non-zero value of h_2 always contributes to a smaller value of h_1, comparing the equilibrium “between two fluids” to the situation when the solid floats in liquid 1 only. The solid will then move up.

However, h_2 being always smaller than H (or equal to H), and ρ_2 being smaller than ρ_s (the body is said not to float in liquid 2) , relationship (1) also entails that h_1 cannot be zero. A part of the body will stay in liquid 1.

Possible origins of common difficulties

Common answers may be due to

-the idea that a liquid in which the solid cannot float ($\rho_2 < \rho_s$) cannot facilitate the floating of this solid - floating which is ensured only by the first liquid (item A).

-the idea of a load added on top of the cylinder, pushing downwards (items C and D). It is likely that this idea will be particularly strong when it is stated that the solid, before being released from its initial position, is covered by liquid 2. This common idea contradicts a well-known fact: If your add water in a recipient with an object floating in water, this object will float to the top. But seeing a contradiction is not enough to reach a satisfying comprehension.

In both cases, the change in the setting due to the addition of liquid 2 is not envisaged in a systemic way, but **very locally**. Instead of taking into account that the whole system experiences changes (here in pressure), many students seem to consider that only a local change occurs. This suggests the following staging of the situation.

Staging the situation in order to stress the systemic aspect

[3] This questionnaire has been inspired by Bennhold, C. & Feldmann, G. 2005. Instructor Notes On Conceptual Test Questions, In *Giancoli Physics- Principle with applications* , 6th Edition, Pearson, Prentice Hall, 290-291.

A graph can be constructed, showing pressure against altitude without (black line in fig. 2) and with (coloured line in fig. 2) the second liquid.

The approximation done previously about the role of the air - i.e. a negligible contribution to Archimedes' up-thrust - has a graphical equivalent: a constant value of atmospheric pressure near the recipient.

What counts to evaluate the up-thrust on the cylinder is the difference between the pressure forces exerted on the lower and upper horizontal surfaces of this body, situated at a given altitude interval H. For **any hydrostatic equilibrium**, this difference in pressure, Δp_{eq}, is such that

$S\, \Delta p_{eq} = g\, (\rho_s\, S\, H)$ or else $\boldsymbol{\Delta p_{eq} = g\, \rho_s\, H}$

It can be seen that, at the initial equilibrium position, the difference in pressure Δp between the lower and the upper horizontal surface limiting the cylinder is larger when the second liquid is added. Hence the resulting increased up-thrust. The body then moves upwards and reaches a new equilibrium position, where Δp will retrieve its initial value (fig. 3).

An important point to be stressed is that the effect of adding the second fluid on top of the first one is a change of the *whole* field of pressure: a systemic view.

As regards Figure 2, some possibly useful questions to ask the students may be:
- explain, in your words, why, concerning a part of the liquid, the coloured and black lines are parallel;
- explain the physical meaning of their distance when these lines are parallel;
- explain what happens when the atmospheric pressure increases;
- explain what happens if the two liquids have the same density (the case when liquid 1 = liquid 2.

All the various cases (not enough water to ensure floating only in liquid 1, case *a* and *b* in Figure 1, density ρ_2 lower than ρ_1 and larger than ρ_s) can be discussed in terms of the difference in pressure Δp between lower an upper sections of the solid.

Performing all the corresponding experiments after justified predictions will feed all the discussions about these various cases.

Practical suggestion: the set square

Finding any equilibrium position sums up in fitting:

-the field of pressure, graphically represented, and characteristic of **the fluid**.

-two characteristics of **the body**: height H and Δp_{eq} ($\Delta p_{eq} = g\, \rho_s\, H$)

This can be highlighted by using a cardboard set-square and fitting it on a graph drawn on the blackboard, like in fig. 3. Many different situations can be solved with this technique.

Using the set square also makes it possible to predict qualitatively the force exerted by the fluids on the cylinder when this body is pushed downward by a distance d from its equilibrium position.

Systemic approach via graphs

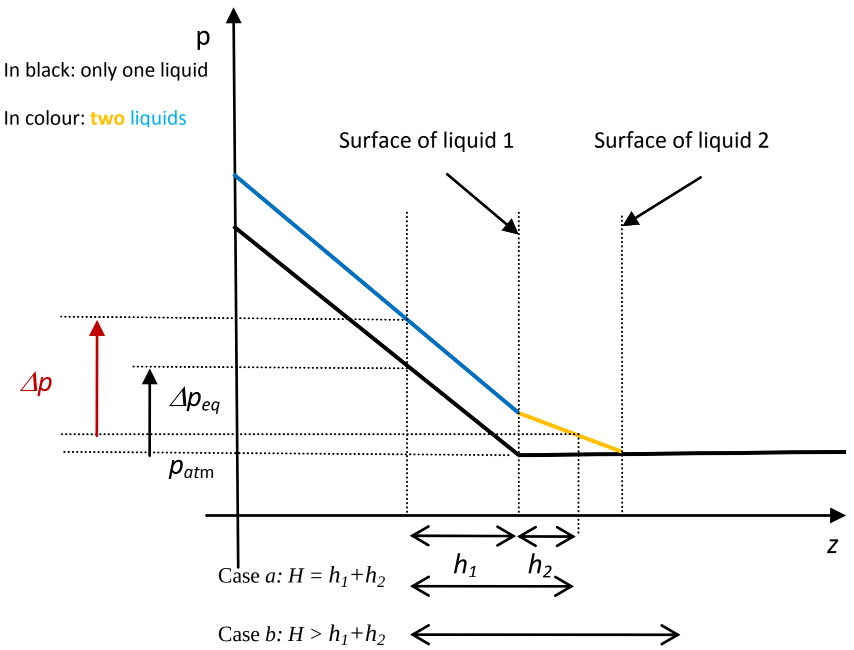

Fig. 2 Pressure versus altitude in two situations: only liquid1 (black), liquid 2 added on top of liquid 1 (coloured). When the second liquid is added the solid will move upwards (due to increased Δp), reaching a new equilibrium: see Fig. 3.

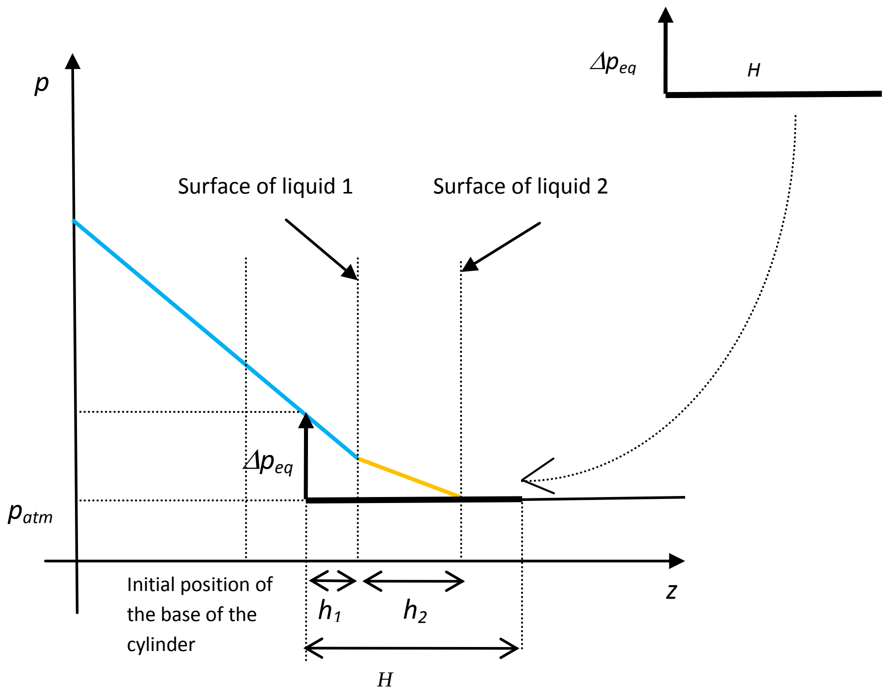

Fig. 3 Looking for the new equilibrium. For given values of H and Δp_{eq} , the “set square” in the top right angle should be moved and “stuck” against the coloured line representing the field of pressure. This gives at the same time the new position of the body and the new values of the heights of the immersed parts.

A set square for the blackboard: do it yourself