Geodesy Fundamentals

MN Society of Professional Surveyors

68th Annual Meeting

Dave Zenk PE LS
NGS Northern Plains Regional Advisor
February 19-21, 2020
Duluth MN
SPCS2022 Webinar

- Webinar on State Plane Coordinates on March 8, 2018 by Michael Dennis

https://geodesy.noaa.gov/web/science_edu/webinar_series/Webinars.shtml
Class Outline

• Review the basics
  – Astronomy
  – Geodetic Systems
  – Datums
  – Map Projections
  – GPS
Astronomy

- Earth, Sun, Stars System
  - Earth Orbit around Sun
  - Geometry and Vocabulary
- Practical Astronomy and Celestial Coordinates
  - Altitude-Azimuth System
  - Right Ascension-Declination System
  - Sidereal Hour Angle – Local Hour Angle System
  - PZS Triangle
  - Methods for finding latitude, longitude, and azimuth.

https://www.blm.gov/cadastral/Manual/73man/id34.htm
Sections 2-53 to 73

Elgin, Knowles & Senne, Inc. (Archer-Elgin)
310 E. 6th St., Rolla, MO 65401
http://www.rollanet.org/~eksi/2008EPHEM.pdf
Astronomy

• Earth’s Elliptical Orbit Around the Sun (Northern Hemisphere)
  – 1) Winter Solstice (December 21)
    • Perihelion (January 3)
  – 2) Spring Equinox (March 21)
  – 3) Summer Solstice (June 21)
    • Aphelion (July 3)
  – 4) Autumn Equinox (September 21)

23.5 degree axis tilt from ecliptic plane

Perihelion = closest to sun on January 3 (147 M km)
Semi-dia = 0-16’-15.9”

Aphelion = furthest from sun on July 3 (152 M km)
Semi-dia = 0-15’-43.9”

Image by NASA
Astronomy

• Intersection of Orbital Plane and Equatorial Plane forms a line which points to First Point of Aries, \( \gamma \) (vernal equinox) and First Point of Libra, \( \Omega \) (autumnal equinox)
• Astronomers prefer the vernal equinox and so it is the reference point in the sky.

2 planes intersect to form a line which points to \( \gamma \)
Astronomy

- Moon’s orbit is tilted 5 degrees from ecliptic plane

*Image by NOAA/NASA SciJinks*
Astronomy

- Celestial Sphere (infinite radius)
- Rotates east-to-west
Astronomy

- Right Ascension - Declination (Sidereal Hour Angle) System
Astronomy

- Altitude-Azimuth System (based on local horizon of observer)
Astronomy

- PZS Triangle
Astronomy

• PZS Triangle

- Sun or Star
- Zenith
- Pole
- LHA
- Parallactic Angle
- Declination
- Latitude
- Altitude

“Azimuth”
Astronomy

• Spherical Trigonometry
  – 3 sides and 3 angles – all are expressed in angular units of measure!
  – If one knows any 3 of the 6 parts, one can solve for the other 3 parts.

• Spherical Law of Sines

\[
\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}
\]

• Spherical Law of Cosines

\[
\cos A = \frac{\cos b \cos c - \cos a}{\sin b \sin c}
\]
Astronomy

• What do we know and what do we measure?
  – Latitude
  – Altitude
  – Declination
  – Azimuth
  – Local Hour Angle (LHA)
  – Parallactic Angle

• Choice and circumstance dictate!

• Traditionally know: *Latitude, Longitude, & Declination*
  – Measure altitude, then use “Altitude Method” to solve for azimuth
  – Measure time, then use “Hour Angle Method” to solve for azimuth
Astronomy

• Derive practical equations for both methods (Caltrans)
  – See Unit 4, page 4

• Altitude Method
  – Uses Latitude, Declination, and Altitude

\[
Z = \cos^{-1} \frac{\sin \text{ DEC} - \sin \text{ LAT} \sin \text{ ALT}}{\cos \text{ LAT} \cos \text{ ALT}}
\]

• Hour Angle Method
  – Uses Latitude and Declination, then Longitude, Time, and Right Ascension (to compute LHA)

\[
Z = \tan^{-1} \frac{\sin \text{ LHA}}{\sin \text{ LAT} \cos \text{ LHA} - \cos \text{ LAT} \tan \text{ DEC}}
\]
Astronomy

• Refraction Correction – Altitude Method
  – One of the difficulties of the Altitude Method is refraction due to the atmosphere.
Astronomy

Altitude angle error due to refraction by atmosphere
Astronomy

- Refraction Correction – Altitude Method
  - Angular refraction error is a function of altitude
  - Also semi-diameter correction

<table>
<thead>
<tr>
<th>Altitude</th>
<th>Approximate Correction</th>
</tr>
</thead>
<tbody>
<tr>
<td>90°</td>
<td>0°</td>
</tr>
<tr>
<td>70°</td>
<td>- 0°00’18”</td>
</tr>
<tr>
<td>50°</td>
<td>- 0°00’48”</td>
</tr>
<tr>
<td>30°</td>
<td>- 0°01’36”</td>
</tr>
<tr>
<td>10°</td>
<td>- 0°05’18”</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Date</th>
<th>Semi-diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>January 3</td>
<td>0-16’-15.9”</td>
</tr>
<tr>
<td>July 3</td>
<td>0-15’-43.9”</td>
</tr>
</tbody>
</table>

*Affected by atmospheric temperature and pressure, which can only be measured at position of observer.*
Astronomy

• The dynamic relationships of Earth, Sun, & Stars are important concepts and so is the vocabulary and geometry to describe it.

• The best way to **master** practical astronomy is to go outside and do the observations.

• Learn to cast solar shadows of the telescope’s crosshair on a white card – do not look through telescope!

You can see why measurements are usually made to the tangent **trailing** edge of the moving SUN
Astronomy

- NGS datasheets often contain a BOX SCORE showing the measured AZIMUTH to nearby objects.
- These can be a good resource for a field exercise.

<table>
<thead>
<tr>
<th>PID</th>
<th>Reference Object</th>
<th>Distance</th>
<th>Geod. Az</th>
</tr>
</thead>
<tbody>
<tr>
<td>LH1243</td>
<td>CN8485 80 3 RM 1</td>
<td>10.898 Meters</td>
<td>13358</td>
</tr>
<tr>
<td>LH1243</td>
<td>CN8486 80 3 RM 2</td>
<td>19.983 Meters</td>
<td>23512</td>
</tr>
<tr>
<td>LH1243</td>
<td>CN8484 80 3 AZ MK</td>
<td>2493549.5</td>
<td></td>
</tr>
<tr>
<td>LH1243</td>
<td>LH1239 WOOD RIVER MUNICIPAL TANK</td>
<td>APPROX. 6.7 KM 3175921.5</td>
<td></td>
</tr>
</tbody>
</table>
Astronomy

• Q. Why Is Astronomy Important in Geodesy?
• Because it is the FOUNDATION of all geodetic observations.
• The entire United States was surveyed by astronomic observations to set out the passive control marks we have today.
Astronomy

Q. Why Is Astronomy Important in Geodesy?

Sometimes in *retracing* an old description, it will reference TRUE NORTH.

That was determined by astronomy – or by magnetic bearing corrected for magnetic declination, which was itself measured by astronomy.

A good surveyor knows how it was done then and could still retrace it by the *same methods*. 
Geodesy

• How does Geodesy differ from Astronomy?
  – Geodesy seeks to describe the size, shape, and other physical characteristics of the Earth.
  – Deeply related to gravity, geology, geometry, and physics.
  – Astronomical observations are all conducted relative to the vertical defined by the direction of local gravity
  – Geodetic calculations are all computed on the surface of an ellipsoid chosen to represent the Earth
  – On a perfect Earth with a perfect choice of ellipsoid there’d be no differences.
  – Unfortunately, nothing’s perfect.
We Must Choose a Reference Surface

• We need a reference surface from which to base measurements.
• NGS has chosen the equipotential surface that most closely matches the mean sea surface on a global basis as the reference surface.
• This surface is called the GEOID.
• Then selected an ellipsoid that is a “best fit” to the geoid and whose center is located at Earth’s center of mass.
• Compared to a smooth mathematical surface, the GEOID is lumpy.
Some Trial Geoid Shapes to Consider

[Images of different geoid shapes, with one labeled "least bad choice"]
What the Geoid Looks Like

• The “geoid” represents the equipotential (level) surface that best approximates the mean sea surface (which is NOT a level surface)

https://earthobservatory.nasa.gov/Features/GRACE/page3.php
Geodesy

• Many astronomic and geodetic concepts are related, but need to be modified and refined for the reality of the Earth.
  – Astronomic vs Geodetic
    • Vertical
    • North
    • Azimuth
    • Latitude
    • Longitude
    • Other terms
Angular difference is a few arc-seconds and leads to Laplace correction:

geodetic azimuth = astronomic azimuth + Laplace correction
Geodesy

• Laplace Correction

Setup perpendicular to gravity
Geodesy

- Laplace Correction

Laplace correction = angular difference in azimuth
\[ \text{geodetic azimuth} = \text{astronomic azimuth} + \text{Laplace correction} \]
Geodesy

- Laplace Correction is listed on appropriate NGS datasheets

<table>
<thead>
<tr>
<th>1</th>
<th>National Geodetic Survey, Retrieval Date = OCTOBER 4, 2016</th>
</tr>
</thead>
<tbody>
<tr>
<td>LH1243</td>
<td>**********************************************************************</td>
</tr>
<tr>
<td>LH1243</td>
<td>DESIGNATION - 80 3</td>
</tr>
<tr>
<td>LH1243</td>
<td>PID - LH1243</td>
</tr>
<tr>
<td>LH1243</td>
<td>STATE/COUNTY- NE/HALL</td>
</tr>
<tr>
<td>LH1243</td>
<td>COUNTRY - US</td>
</tr>
<tr>
<td>LH1243</td>
<td>USGS QUAD - WOOD RIVER (1993)</td>
</tr>
<tr>
<td>LH1243</td>
<td>*CURRENT SURVEY CONTROL</td>
</tr>
<tr>
<td>LH1243</td>
<td>**********************************************************************</td>
</tr>
<tr>
<td>LH1243</td>
<td>NAVD 88 ORTHO HEIGHT - 592.0 (meters) 1942. (feet) VERTCON</td>
</tr>
<tr>
<td>LH1243</td>
<td>**********************************************************************</td>
</tr>
<tr>
<td>LH1243</td>
<td>GEOID HEIGHT - 24.911 (meters) GEOID12B</td>
</tr>
<tr>
<td>LH1243</td>
<td>LAPLACE CORR - 2.14 (seconds) DEFLEC12B</td>
</tr>
<tr>
<td>LH1243</td>
<td>HORZ ORDER - SECOND</td>
</tr>
</tbody>
</table>

| LH1243 | PID | Reference Object | Distance | Geod. Az | |
|-------|-----|-----------------|----------|----------| |
| LH1243 | CN8485 80 3 RM 1 | 10.898 METERS | 13358 | |
| LH1243 | CN8486 80 3 RM 2 | 19.983 METERS | 23512 | |
| LH1243 | CN8484 80 3 AZ MK | 2493549.5 | | |
| LH1243 | LH1239 WOOD RIVER MUNICIPAL TANK | APPROX. 6.7 KM | 3175921.5 | |
Geodesy

• The **Prime Vertical** is a great circle passing through the geodetic zenith from east-to-west.
• The prime vertical component of deflection of the vertical leads to Laplace correction.
• Meridional component does not.
Geodesy

• Astronomic Latitude is the angle from the equator up to the observer’s astronomic zenith.
• Geodetic Latitude is the angle from the equator up to the observer’s geodetic zenith.
• Affected by meridional component of deflection of the vertical
Geodesy

- Latitude affected by Meridional component of deflection of the vertical
Geodesy

- Longitude affected by Prime Vertical component of deflection of the vertical
Geodesy

• Datums
  – Horizontal (NAD27, NAD83)
  – Vertical (NGVD29, NAVD88)

• Types
  – Geometric
  – Geopotential
  – 3D (NATRF2022 – which includes NAPGD2022)
Geodesy

• 8 Horizontal Datum Definition Requirements
  – Size and Shape of Reference Ellipsoid (2 elements)
  – 3 Translations (*relative to a chosen point on or in the Earth*)
  – 3 Rotations (*relative to a chosen point on or in the Earth*)
• Specifically, the 8 Defining Elements of Horizontal Datum include:
  1. Semi-major Axis length
  2. Flattening Ratio
  3. Latitude of Initial Point
  4. Longitude of Initial Point
  5. Ellipsoid Height of Initial Point
  6. Prime Meridian Deflection of the Vertical at Initial Point
  7. Meridional Deflection of the Vertical at Initial Point
  8. Geodetic Azimuth of a Line at Initial Point
Among the infinite choices for Reference Ellipsoid are the following commonly used ellipsoids with their names:

<table>
<thead>
<tr>
<th>Ellipsoid Name</th>
<th>Semi-major Axis</th>
<th>Flattening Ratio (approx.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Airy</td>
<td>6377563.396</td>
<td>299.32</td>
</tr>
<tr>
<td>Australian National</td>
<td>6378160.000</td>
<td>298.25</td>
</tr>
<tr>
<td>Bessel 1841</td>
<td>6377397.155</td>
<td>299.15</td>
</tr>
<tr>
<td>Clarke 1866</td>
<td>6378206.400</td>
<td>294.9786982</td>
</tr>
<tr>
<td>Clarke 1880</td>
<td>6378249.145</td>
<td>293.47</td>
</tr>
<tr>
<td>Everest</td>
<td>6377276.345</td>
<td>300.80</td>
</tr>
<tr>
<td>Fischer 1960 (Mercury)</td>
<td>6378166.000</td>
<td>298.30</td>
</tr>
<tr>
<td>Fischer 1968</td>
<td>6378150.000</td>
<td>298.30</td>
</tr>
<tr>
<td>GRS 1967</td>
<td>6378160.000</td>
<td>298.25</td>
</tr>
<tr>
<td>GRS 1980</td>
<td>6378137.000</td>
<td>298.257222101</td>
</tr>
<tr>
<td>Helmert 1906</td>
<td>6378200.000</td>
<td>298.30</td>
</tr>
<tr>
<td>Hough</td>
<td>6378270.000</td>
<td>297.00</td>
</tr>
<tr>
<td>International</td>
<td>6378388.000</td>
<td>297.00</td>
</tr>
<tr>
<td>Krassovksy</td>
<td>6378245.000</td>
<td>298.30</td>
</tr>
<tr>
<td>WGS 60</td>
<td>6378165.000</td>
<td>298.30</td>
</tr>
<tr>
<td>WGS 66</td>
<td>6378145.000</td>
<td>298.25</td>
</tr>
<tr>
<td>WGS-72</td>
<td>6378135.000</td>
<td>298.26</td>
</tr>
<tr>
<td>WGS-84</td>
<td>6378137.000</td>
<td>298.257223563</td>
</tr>
</tbody>
</table>
Geodesy

- NAD27
  - used Clarke 1866 ellipsoid and oriented at Meades Ranch and Waldo.
- NAD83
  - used GRS80 ellipsoid and tried to orient at mass center of Earth
- WGS84
  - used WGS84 ellipsoid and tried to orient at mass center of Earth
Geodesy

• Datum-to-Datum Conversion Methods are needed to convert coordinates to/from-between datums.

• Several Methods with often confusing names
  – 3 parameter (NADCON)
  – 7 parameter (Helmert)
  – 7 parameter (Bursa-Wolfe)
  – 7 parameter (Molodensky)
  – 10 parameter (Molodensky-Badekas)
  – 14 parameter (scientific community)

• Exactness of geometry is often sacrificed for simplicity
Geodesy

• 3 parameter (NADCON) cannot model more than a limited patch on the Earth.
• DX, DY, DZ here not same as there

Geodesy

• 7 & 10 parameter are more rigorous and cover a larger area.

• 14 parameter also includes velocities for the 7-parameters.

• Note that in the long-term, the velocities are also changing, so maybe we should consider a 21 parameter solution someday!
Geodesy

• Datum Conversion “mileage chart problem”
• Any datum should be convertible to any other datum.
• Since large numbers of datums (N) exist, there are (N^2) conversions needed.
• So, a common datum is used as an intermediary datum.
• Scientific community cooperates by creating International Terrestrial Reference Frame (ITRF).
• Only need 2N conversions:  \( X \rightarrow \text{ITRF} \) and \( \text{ITRF} \rightarrow Y \)
  – But, only need to tabulate N conversions - (then treat as \textit{to} or \textit{from})
• Also makes it easier to add a new datum if needed.
Geodesy

- Datum Conversion Table and Parameters
  - [https://www.ngs.noaa.gov/CORS/coords.shtml#Col1Exp]
Geodesy

• 14 Parameters from NGS
  – 3 translations, 3 rotations, 1 scale factor
  – and 7 Velocities

\[ \begin{align*}
  T_x(t_0) &= 0.99343 \text{ m}; \\
  T_y(t_0) &= -1.90331 \text{ m}; \\
  T_z(t_0) &= -0.52655 \text{ m} \\
  \varepsilon_x(t_0) &= 25.91467 \text{ mas}; \\
  \varepsilon_y(t_0) &= 9.42645 \text{ mas}; \\
  \varepsilon_z(t_0) &= 11.59935 \text{ mas} \\
  s(t_0) &= 1.71504 \cdot 10^{-9} \text{ (unitless)} \\
  \dot{T}_x &= 0.00079 \text{ m \cdot year}^{-1}; \\
  \dot{T}_y &= -0.00060 \text{ m \cdot year}^{-1}; \\
  \dot{T}_z &= -0.00134 \text{ m \cdot year}^{-1} \\
  \dot{\varepsilon}_x &= 0.06667 \text{ mas \cdot year}^{-1}; \\
  \dot{\varepsilon}_y &= -0.75744 \text{ mas \cdot year}^{-1}; \\
  \dot{\varepsilon}_z &= -0.05133 \text{ mas \cdot year}^{-1} \\
  \dot{s} &= -0.10201 \cdot 10^{-9} \text{ year}^{-1}
\end{align*} \]
### Geodesy

- **Datum Conversion Chart for direct conversions**

<table>
<thead>
<tr>
<th>DATUM NAME</th>
<th>NAD27</th>
<th>NAD83</th>
<th>NATRF2022</th>
<th>AGD84</th>
</tr>
</thead>
<tbody>
<tr>
<td>NAD27</td>
<td>Null</td>
<td>s dx, dy, dz, ω, φ, κ</td>
<td>s dx, dy, dz, ω, φ, κ</td>
<td>s dx, dy, dz, ω, φ, κ</td>
</tr>
<tr>
<td>NAD83</td>
<td>s dx, dy, dz, ω, φ, κ</td>
<td>Null</td>
<td>s dx, dy, dz, ω, φ, κ</td>
<td>s dx, dy, dz, ω, φ, κ</td>
</tr>
<tr>
<td>NATRF2022</td>
<td>s dx, dy, dz, ω, φ, κ</td>
<td>s dx, dy, dz, ω, φ, κ</td>
<td>Null</td>
<td>s dx, dy, dz, ω, φ, κ</td>
</tr>
<tr>
<td>AGD84</td>
<td>s dx, dy, dz, ω, φ, κ</td>
<td>s dx, dy, dz, ω, φ, κ</td>
<td>s dx, dy, dz, ω, φ, κ</td>
<td>Null</td>
</tr>
</tbody>
</table>

N\(^2\) entries

\(\frac{1}{2} (N^2 - N)\) entries

Not needed – these are negatives of the others

AGD84 = Australian Geodetic Datum 1984
Geodesy

- Datum Conversion Chart using intermediary conversions

<table>
<thead>
<tr>
<th>DATUM NAME</th>
<th>ITRF</th>
</tr>
</thead>
<tbody>
<tr>
<td>NAD27</td>
<td>s, dx, dy, dz, ω, φ, κ</td>
</tr>
<tr>
<td>NAD83</td>
<td>s, dx, dy, dz, ω, φ, κ</td>
</tr>
<tr>
<td>NATRF2022</td>
<td>s, dx, dy, dz, ω, φ, κ</td>
</tr>
<tr>
<td>AGD84</td>
<td>s, dx, dy, dz, ω, φ, κ</td>
</tr>
</tbody>
</table>

Only N entries are needed when using an intermediary datum
HEIGHTS
Geodesy

• Heights above or below a reference surface
  – Mean Sea Level Heights
  – Ellipsoid Heights
  – Orthometric Heights
  – Geoid Heights
  – Vertical Datums
    • 1912
    • NGVD29
    • NAVD88
  – Dynamic Heights
Geodesy

Height above Mean Sea Level

Positive is UP, negative is down

terrain

mean sea level
Geodesy

Height above Geoid (Orthometric)

Positive is UP, negative is down

Geoid

Terrain

H
Geodesy

Geoid Height above ellipsoid

ellipsoid

geoid

Positive is UP, negative is down
Geodesy

Height above Mean Sea Level
Height above Ellipsoid
Height above Geoid (Orthometric)
Geoid Height

Positive is UP, negative is down
Geodesy

Height above Mean Sea Level
Height above Ellipsoid
Height above Geoid (Orthometric)
Geoid Height

\[ H = h - N \]

Positive is UP, negative is down

Note that \( N \) is negative in most of CONUS.
Note that \( h \) may be negative near coastlines.
Could \( H \) be negative?
Geodesy

• Vertical Datums in the United States
  – There have been several vertical datums in use across the United States. The earliest ones were “local”, since the leveling did not form an interconnected network.
  – The earliest unified datums were the 1912 Datum (4th General Adjustment).
  – Followed by the NGVD29 adjustment.
  – Followed by the NAVD88 adjustment.
GA1912 in USA

https://geodesy.noaa.gov/library/pdfs/Special_Publication_No_18.pdf
GA1912 in MN
Geodesy

• NGVD29 consists of a leveling network across the United States.
  – Leveling was adjusted to fit tide gauges which forced the NGVD29 datum surface to be an approximation of the mean sea surface.
  – Unfortunately, the sea surface is NOT level (fails to conform to gravitational potential energy due to salinity, temperature, and currents.
  – It seemed like a good idea at the time.
  – Resulted in the severely misleading term “Mean Sea Level”.
    • Preferred term “mean sea surface”.

February 19, 2019
2020 MSPS Annual Meeting, Duluth MN
• NAVD88 consists of a leveling network on the North American Continent, ranging from Alaska, through Canada, across the United States, affixed to a single origin point on the continent.
  – Leveling was adjusted “internally” and tied to only 1 point – Pointe au Pere (Father’s Point) in Quebec
  • Tide Station & Location = Pointe-au-Pere, Rimouski, Quebec, Canada
  • Located in corner of a school building at Rue St Germaine and Goulet Avenue
  • PID = TY5255
  • Bench Mark = 1250 G
  • Height above LMSL = 6.271 meters
As if we weren’t confused already, let’s add the datum surfaces to the diagram.

- Height above Mean Sea Level
- Height above Ellipsoid
- Height above Geoid (Orthometric)
- Geoid Height

The datums are NOT parallel, nor smooth.
Geodesy

• Vertical Datum Conversions
  – There is NO SINGLE VALUED conversion between 1912, NGVD29, nor NAVD88.
    • Datums are not parallel.
    • Datums are not smooth.
    • Underlying leveling is not error-free.
  • Must research the relationship among the known published datum heights for specific passive marks in the area of interest.
• Devise a diagram for your local area.
  – See “Digging for Datums” by David Zenk (2012)
  – Sample next slide - - - >
Draw a DATUM PROFILE

Elevation above datum = Depth to datum

St. Paul, MN (PBM 67) to Dresbach, MN (PBM 182)

<table>
<thead>
<tr>
<th>Datum</th>
<th>St. Paul, MN</th>
<th>Dresbach, MN</th>
</tr>
</thead>
<tbody>
<tr>
<td>NAVD88</td>
<td>0.046 m</td>
<td>0.013 m</td>
</tr>
<tr>
<td>NGVD29</td>
<td>0.098 m</td>
<td>0.071 m</td>
</tr>
<tr>
<td>Mean Gulf 1912</td>
<td>0.003 m</td>
<td>0.078 m</td>
</tr>
<tr>
<td>Memphis</td>
<td>2.222 m</td>
<td>2.148 m</td>
</tr>
</tbody>
</table>

NAVD88 1912 NGVD29 Memphis
Geodesy

• **Dynamic Heights**
  
  – A dynamic height is directly related to the amount of potential energy at a given point.
  
  – As an object is raised it acquires gravitational potential energy.
  
  – Potential energy = Work Done in raising a weight through a Distance
  
  – $PE = D \times F$

$$D = Distance \ (m)$$
$$D = 1 \ m$$

$$F = Weight$$

$$F = Mass \times Gravity$$

$$F = 1 \ kg \times 9.8 \ m/sec^2$$

$$PE = 1 \ m \times 1 \ kg \times 9.8 \ m/sec^2$$

$$PE = 9.8 \ kg \ m^2/sec^2$$
Geodesy

- Note that gravity varies over distances, so equal potential energy surfaces are not parallel.

\[
\text{stronger gravity zone}
\]

\[
D = \text{Distance (m)}
\]
\[
D = 1 \text{ m}
\]

\[
F = \text{Weight}
\]
\[
F = \text{Mass} \times \text{Gravity}
\]
\[
F = 1 \text{ kg} \times 9.8 \text{ m/sec}^2
\]
\[
PE = 1 \text{ m} \times 1 \text{ kg} \times 9.8 \text{ m/sec}^2
\]
\[
PE = 9.8 \text{ kg m}^2/\text{sec}^2
\]

\[
\text{weaker gravity zone}
\]

\[
D = \text{Distance (m)}
\]
\[
D = 1.01 \text{ m}
\]

\[
F = \text{Weight}
\]
\[
F = \text{Mass} \times \text{Gravity}
\]
\[
F = 1 \text{ kg} \times 9.7 \text{ m/sec}^2
\]
\[
PE = 1.01 \text{ m} \times 1 \text{ kg} \times 9.7 \text{ m/sec}^2
\]
\[
PE = 9.8 \text{ kg m}^2/\text{sec}^2
\]
Geodesy

- Equal PE but not equal distances.
- Geopotential surfaces are not parallel, but ARE level.

\[
\text{PE} = 9.8 \, \text{kg m}^2/\text{sec}^2
\]

\[
\text{D} = 1.01 \, \text{m}
\]

\[
\text{PE} = 0 \, \text{kg m}^2/\text{sec}^2
\]
Geodesy

• Every geopotential surface could be computed and labeled with its Geopotential Number.
• All such geopotential surfaces directly denote level surfaces.
• Water does not flow along these level surfaces.
• A set of Geopotential Numbers can be divided by a constant amount and all resulting numbers are still in proportion and still represent a series of level surfaces.
• So choose to divide by normal gravity at 45° latitude (9.806199 m/sec$^2$) and by 1 kg.
• Result is a Dynamic Height whose unit is meters, but is not an actual height – it is merely a label for a level surface.
• Here’s what NGS has to say about DYNAMIC HEIGHTS near the Great Lakes

Special Note on IGLD 85 Heights

Dynamic heights are relevant near large bodies of water such as reservoirs or very large lakes, so the Great Lakes are an ideal candidate for dynamic heights. However, the Great Lakes are referenced to the International Great Lakes Datum of 1985 (IGLD 85) dynamic heights. The difference between IGLD 85 dynamic heights and NAVD 88 dynamic heights is known as a hydraulic corrector. An application which includes these hydraulic correctors is available for users who wish to compute IGLD 85 heights directly.
<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>FBN</td>
<td>Federal Base Network Control Station.</td>
<td>This is a Federal Base Network Control Station.</td>
</tr>
<tr>
<td>WATER LEVEL</td>
<td>This is a Water Level Survey Control Monument.</td>
<td></td>
</tr>
<tr>
<td>DESIGNATION</td>
<td>MARAIS RESET</td>
<td></td>
</tr>
<tr>
<td>PID</td>
<td>AA2869</td>
<td></td>
</tr>
<tr>
<td>STATE/COUNTY</td>
<td>MN/COOK</td>
<td></td>
</tr>
<tr>
<td>COUNTRY</td>
<td>US</td>
<td></td>
</tr>
<tr>
<td>USGS QUAD</td>
<td>GOOD HARBOR BAY (1986)</td>
<td></td>
</tr>
</tbody>
</table>

### CURRENT SURVEY CONTROL

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>NAD 83(2011) POSITION</td>
<td>47 44 42.56480(N) 090 20 04.66233(W) ADJUSTED</td>
</tr>
<tr>
<td>ELLIP HT</td>
<td>156.068 (meters) (06/27/12) ADJUSTED</td>
</tr>
<tr>
<td>EPOCH</td>
<td>2010.00</td>
</tr>
<tr>
<td>NAVD 88 ORTHO HEIGHT</td>
<td>186.961 (meters) 613.39 (feet) ADJUSTED</td>
</tr>
<tr>
<td>GEOID HEIGHT</td>
<td>-30.892 (meters) GEOID12B</td>
</tr>
<tr>
<td>NAD 83(2011) X</td>
<td>-25,094.971 (meters) COMP</td>
</tr>
<tr>
<td>Y</td>
<td>-4,296,764.579 (meters) COMP</td>
</tr>
<tr>
<td>Z</td>
<td>4,697,984.958 (meters) COMP</td>
</tr>
<tr>
<td>LAPLACE CORR</td>
<td>-15.88 (seconds) DEFLEC12B</td>
</tr>
<tr>
<td>DYNAMIC HEIGHT</td>
<td>187.007 (meters) 613.54 (feet) COMP</td>
</tr>
<tr>
<td>MODELED GRAVITY</td>
<td>980,852.0 (mgal) NAVD 88</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>VERT ORDER</th>
<th>SECOND</th>
<th>CLASS I</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Map Projections
Geodesy

- A 3-dimensional curved surface cannot be reduced to a 2-dimensional flat surface (map) without some form of “distortion” being applied.
- A set of mathematical rules must be developed that can be published and adopted by users.
- Results of conversion must meet the intended use of the 2-D “map”.
- Lots of choices developed over the years to meet many needs.
- Global coverage vs. local coverage.
Geodesy

• Surveyors use “conformal” projections
  – Preserves scale in all directions at a given point and preserves angular relationships between lines at a given point.
  – Distances will be modified by a scale factor.
• Common Projections
  – Lambert Conformal
  – Transverse Mercator
  – Oblique Mercator
• State Plane and Local
Geodesy

• State Plane Coordinate Myths
  – Computations are too difficult.
  – Scale factors are too much trouble.
  – Surveyors don’t have time.

• State Plane Coordinate Facts
  – Computations are too difficult.
    • If it ever was true, modern computers have no problem with coordinate conversion or coordinate geometry. This includes data collectors.
  – Scale factors are too much trouble.
    • All too often the concept of distortion (scale factor) in a map projection system is interpreted as an error of the system. However, map projection systems provide for a rigorous mathematical conversions – scale factors are reversible and knowable.
    • Data collectors can now be programmed to apply a scale factor – both on measurements and on layouts.
  – Surveyors don’t have time.
    • There’s never enough time to do it right the first time, but always enough time to do it over!
Geodesy

- Lambert Conformal Projection
  - Used in regions that are longer east-west extent
  - State Plane Coordinates Zones in ND, SD, NE, IA, MN

NOAA Manual NOS NGS 5

State Plane Coordinate System of 1983

James E. Stem

Rockville, MD
January 1989

Reprinted with minor corrections
March 1990
Geometry of Lambert Projection

- Defining parameters for a Lambert Projection
  - Ellipsoid shape
    - Semi-major axis (a) and the inverse flattening ratio (1/f)
  - Latitude of northerly standard parallel (φn)
  - Latitude of southerly standard parallel (φs)
  - Latitude of Grid Origin (φb)
  - Longitude of Central Meridian (λo)
  - Easting (Eb) and Northing (Nb) at Grid Origin
Geometry of Lambert Projection
Unroll the Cone of Lat/Lon Grid

Slit the cone down the back and “unroll”.

This becomes a flat sheet of paper with a LAT/LON grid projected on it!
Flat Sheet of Paper Can Be Rolled into a Cone

This is a flat sheet of paper with a Northing/Easting grid!
Flat Sheet of Paper Can Be Rolled into a Cone
2 Flat Sheets of Paper Combined
2 Flat Sheets of Paper Combined

\[ \phi_b, \phi_s, \phi_n, \lambda_0, N, E \]
2 Flat Sheets of Paper Combined
## Compare Projection Parameters

### Colorado North Zone

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SPCS 27</th>
<th>SPCS 83</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ellipsoid</td>
<td>Clarke 1866</td>
<td>GRS80</td>
</tr>
<tr>
<td>a</td>
<td>6,378,206.4 meters</td>
<td>6,378,137 meters</td>
</tr>
<tr>
<td>1/f</td>
<td>294.978698214</td>
<td>298.2572222101</td>
</tr>
<tr>
<td>∅n</td>
<td>40°47’</td>
<td>40°47’</td>
</tr>
<tr>
<td>∅s</td>
<td>39°43’</td>
<td>39°43’</td>
</tr>
<tr>
<td>∅b</td>
<td>39°20’</td>
<td>39°20’</td>
</tr>
<tr>
<td>∅o</td>
<td>40°15’</td>
<td>40°15’02.56123”</td>
</tr>
<tr>
<td>λo</td>
<td>105°30’</td>
<td>105°30’</td>
</tr>
<tr>
<td>Nb</td>
<td>0 feet</td>
<td>304,800.6096 meters</td>
</tr>
<tr>
<td>Eb</td>
<td>2,000,000 feet</td>
<td>914,401.8289 meters</td>
</tr>
</tbody>
</table>

1,000,000 feet
3,000,000 feet
### Compare Projection Parameters

**Colorado Central Zone**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SPCS 27</th>
<th>SPCS 83</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ellipsoid</td>
<td>Clarke 1866</td>
<td>GRS80</td>
</tr>
<tr>
<td>a</td>
<td>6,378,206.4 meters</td>
<td>6,378,137 meters</td>
</tr>
<tr>
<td>1/f</td>
<td>294.978698214</td>
<td>298.257222101</td>
</tr>
<tr>
<td>φn</td>
<td>39°45’</td>
<td>39°45’</td>
</tr>
<tr>
<td>φs</td>
<td>38°27’</td>
<td>38°27’</td>
</tr>
<tr>
<td>φb</td>
<td>37°50’</td>
<td>37°50’</td>
</tr>
<tr>
<td>φo</td>
<td>39°06’</td>
<td>39°06’03.65404”</td>
</tr>
<tr>
<td>λo</td>
<td>105°30’</td>
<td>105°30’</td>
</tr>
<tr>
<td>Nb</td>
<td>0 feet</td>
<td>304,800.6096 meters</td>
</tr>
<tr>
<td>Eb</td>
<td>2,000,000 feet</td>
<td>914,401.8289 meters</td>
</tr>
</tbody>
</table>

1,000,000 feet

3,000,000 feet
## Compare Projection Parameters

### Colorado South Zone

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SPCS 27</th>
<th>SPCS 83</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ellipsoid</td>
<td>Clarke 1866</td>
<td>GRS80</td>
</tr>
<tr>
<td>a</td>
<td>6,378,206.4 meters</td>
<td>6,378,137 meters</td>
</tr>
<tr>
<td>1/f</td>
<td>294.978698214</td>
<td>298.257222101</td>
</tr>
<tr>
<td>φn</td>
<td>38°26’</td>
<td>38°26’</td>
</tr>
<tr>
<td>φs</td>
<td>37°14’</td>
<td>37°14’</td>
</tr>
<tr>
<td>φb</td>
<td>36°40’</td>
<td>36°40’</td>
</tr>
<tr>
<td>φo</td>
<td>37°50’</td>
<td>37°50’02.76973”</td>
</tr>
<tr>
<td>λo</td>
<td>105°30’</td>
<td>105°30’</td>
</tr>
<tr>
<td>Nb</td>
<td>0 feet</td>
<td>304,800.6096 meters</td>
</tr>
<tr>
<td>Eb</td>
<td>2,000,000 feet</td>
<td>914,401.8289 meters</td>
</tr>
</tbody>
</table>

1,000,000 feet
3,000,000 feet
Sample Calculation

• Two non-familiar calculations are required
  – Lat/Lon to Northing/Easting
  – Northing/Easting to Lat/Long
• All other calculations are very similar to any other cartesian grid coordinate system.
Sample Calculation

• Outline of Steps
  – Design traverse to include one or more NGS control marks (or use OPUS-S).
  – Field measure all angles and distances as usual, record elevations of each traverse point.
  – Convert Ground Distances to Grid Distances
  – Convert Geodetic Azimuth to Grid Azimuth
  – Convert the Control Lat/Lon to Control N/E
  – Perform all calculations using grid azimuth and grid distances. Apply compass rule, etc, whatever.
  – Convert the grid N/E for all new points to Lat/Lon
Sample Calculation

• Lambert Conformal Systems are called “conformal” because the scale factors are uniform in all directions AT a given point
• That means shapes are not distorted and
• Angles on the ground equal angles on the grid
• So, no need to convert angles!
  – Except for “long lines” (more later)
Sample Calculation

• Distance Conversions

  – Ground distances in Colorado are measured at high elevations. Because the Ellipsoid is beneath you, the ground distance must be reduced to ellipsoid (the so-called “sea level” scale factor).

  – The SPCS grid is not coincident with the ellipsoid, so a second scale factor must be applied (the “grid scale factor”)

  – If the entire survey is at a similar elevation and is in a confined region, the 2 scale factors can be combined and used for all lines in the survey.
Sample Calculation

• Sea level scale factor
  – Use orthometric heights for SPCS 27
  – Use ellipsoid heights for SPCS 83
• \( S = D \times \left( \frac{R}{R+N+H} \right) \)
• \( R \approx 20,906,000 \) feet (verify in CO)
Sample Calculation

• Grid Scale Factor
  – Possible to compute (need computer program)
  – Easier to look up in the Projection Table
  – Tabulated by latitude

<table>
<thead>
<tr>
<th>Lat</th>
<th>R (meters)</th>
<th>tab diff.</th>
<th>k</th>
</tr>
</thead>
<tbody>
<tr>
<td>40° 40'</td>
<td>7498007.099</td>
<td>30.84608</td>
<td>0.99998315</td>
</tr>
<tr>
<td>40 41</td>
<td>7496156.335</td>
<td>30.84623</td>
<td>0.99998530</td>
</tr>
<tr>
<td>40 42</td>
<td>7494305.561</td>
<td>30.84639</td>
<td>0.99998754</td>
</tr>
<tr>
<td>40 43</td>
<td>7492454.777</td>
<td>30.84656</td>
<td>0.99998986</td>
</tr>
<tr>
<td>40 44</td>
<td>7490603.984</td>
<td>30.84672</td>
<td>0.99999227</td>
</tr>
</tbody>
</table>
Sample Calculation

• Example: Compute the grid length \( (L) \) of a line between two marks whose average ellipsoid height is 8300 feet and are at average latitude of 40°41’ and horizontal surface distance (D) is 3,956.548 feet

Apply Sea level Factor
\[
S = 3956.548 \times \frac{20906000}{20906000 + 8300}
\]
\[
S = 3954.978 \text{ feet}
\]

Apply Grid Scale factor
\[
L = 3954.978 \times 0.99998530
\]

Result
\[
L = 3954.920 \text{ feet}
\]
Sample Calculation

• Where is NORTH?
  – Astronomic North
  – Geodetic North
  – Grid North
Sample Calculation

• Grid North’s are all parallel with the SPCS grid
Sample Calculation

• Geodetic North’s all converge toward the Pole
Sample Calculation

• At any known longitude, the angular difference ($\gamma$ - gamma) between Geodetic North and Grid North is constant and is called the “mapping angle” or “meridian convergence”.

• Simple equation:

$$\gamma = \ell \times (\lambda_o - \lambda)$$

Where:

$\ell = \text{constant for projection (see Tables)}$

$\lambda_o = \text{longitude of central meridian (see Tables)}$

$\lambda = \text{longitude at a point}$
Sample Calculation

• Example: What is the SPCS 83 mapping angle at a point in the Colorado North Zone whose longitude is 104°00’?

\[ l = \text{constant for projection} = 0.646133456811 \]
\[ \lambda_0 = \text{longitude of central meridian} = 105°30’ \]
\[ \lambda = \text{longitude at the point} = 104°00’ \]

\[ \gamma = l \times (\lambda_0 - \lambda) \]
\[ \gamma = 0.646133456811 \times (105°30’ - 104°00’) \]
\[ \gamma = +0°58’09.12067” \]
Sample Calculation

Grid Az = Geod Az - γ
Sample Calculation

• Astronomic North to Geodetic North
  – Astronomic North is that which we observe from viewing the stars or Sun.
  – Geodetic North is from the point of observation to the pole of the ellipsoid.

• In a perfect world, they’d be the same
  – Gravity variations cause the geodetic vertical to be different from the gravitational vertical, and causes an azimuth difference.

• Could be significant in Colorado or Nebraska!
  – LaPlace Correction
Sample Calculation

- Coast and Geodetic Survey Special Publication No 82, by John Hayford, 1909, page 57 states:

An unusually rapid change of the prime vertical component of the topographic deflection between adjacent stations is illustrated by the azimuth stations Gunnison Colorado No 56 and Mount Ouray Colorado No 59. Although these stations are only 63 kilometers apart, the prime vertical components of the topographic deflections differ by 21.56” being +10.31” at the former and -11.25” at the latter. These stations are in a region of high steep mountains.
Sample Calculation

Azimuth Difference caused by Deflection of the Vertical
Sample Calculation

• Second Term Azimuth Correction
  – Because the ellipsoidal trace of the line of sight from point A to point B follows a curve on the grid, a minor correction must be applied for LONG LINES.
  – The magnitude of the correction increases away the Central Parallel and with length of line.

• Most surveys are too imprecise to need this correction. See table, next slide.
Sample Calculation

- NOAA Manual NOS NGS 5, page 51:

From a practical perspective in many survey operations the \((t-T)\) correction is negligible, observed angles are used with grid azimuths, and survey computations are done on a plane. In a precise survey it is necessary to evaluate the magnitude of \((t-T)\). Table 4.3a provides an approximation.

Table 4.3a.—Approximate size of \((t-T)\) in seconds of arc for Lambert or transverse Mercator projection (see note 1)

<table>
<thead>
<tr>
<th>(\Delta E) or (\Delta N) (See note 2) (km)</th>
<th>Perpendicular distance from central axis to midpoint of the line (see note 3) (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.3 0.5 0.8 1.0 1.3</td>
</tr>
<tr>
<td>100</td>
<td>0.6 1.3 1.9 2.5 3.2</td>
</tr>
<tr>
<td>150</td>
<td>1.3 2.5 3.8 5.1 6.4</td>
</tr>
<tr>
<td>200</td>
<td>2.5 5.1 7.6 10.2 12.7</td>
</tr>
<tr>
<td>250</td>
<td></td>
</tr>
</tbody>
</table>
Sample Calculation

- Second Term Corrections affect Field Measured Angles
Sample Calculation

• Convert Geodetic Azimuth to Grid Azimuth
  • See NOAA Manual NOS NGS 5, page 18

\[ \text{Grid Az} = \text{Geod Az} - \gamma \]

\[ t = \alpha - \gamma + \delta \]
Sample Calculation

• Convert the Control Lat/Lon to N/E

Lat = 40°41’00”
Lon = 104°00’
Sample Calculation

Convert Using Colorado State Plane Coordinate Projection Tables

---

Lambert conformal conic projection with two standard parallels
Plane coordinate projection tables

DATUM: NAD 83
The projection is COLORADO NORTH

Ellipsoidal constants

\[ \begin{align*}
a & = 6378137 \text{ m} \\
f & = 1/298.25722210 
\end{align*} \]

Defining constants

\[ \begin{align*}
\phi_0 & = 39'20'' \quad \text{(latitude of grid origin)} \\
\lambda_0 & = 105.30 \quad \text{(longitude of origin and Central Meridian, CM)} \\
\phi_s & = 39 43 \quad \text{(southern standard parallel)} \\
\phi_n & = 40 47 \quad \text{(northern standard parallel)} \\
E_e & = 914401.8289 \text{ m} \quad \text{(easting coordinate of origin)} \\
N_n & = 304800.6096 \text{ m} \quad \text{(northing coordinate of origin)}
\end{align*} \]

Derived constants

\[ \begin{align*}
\xi & = 0.646133456811 = \sin(\phi) \\
K & = 12361909.8309 \text{ m} \quad \text{(mapping radius at the equator)} \\
R_0 & = 7646051.6244 \text{ m} \quad \text{(mapping radius at grid origin)}
\end{align*} \]

Lambert coordinates \((N, E)\) from geodetic positions \((\phi, \lambda)\)

\[ \begin{align*}
\gamma & = (\lambda - \lambda_0) \sin(\phi_s) \quad \gamma \text{ is the meridional convergence} \\
E & = R \sin(\gamma) + E_e \quad \text{\(R\) from table} \\
N & = R_0 - R \cos(\gamma) + N_n
\end{align*} \]

---

https://www.ngs.noaa.gov/library/
Sample Calculation

Nebraska State Plane Coordinate Projection Tables

Lambert conformal conic projection with two standard parallels
Plane coordinate projection tables

DATUM: NAD 83
The projection is NEBRASKA

Ellipsoidal constants

\[ a = 6378137 \text{ m} \]
\[ f = 1/298.25722210 \]

Defining constants

\[ \phi_b = 39^\circ 50' \quad \text{(latitude of grid origin)} \]
\[ \lambda CM = 100 \quad 0 \quad \text{(longitude of origin and Central Meridian, CM)} \]
\[ \phi_s = 40 \quad 0 \quad \text{(southern standard parallel)} \]
\[ \phi_n = 43 \quad 0 \quad \text{(northern standard parallel)} \]
\[ E_0 = 500000.0000 \text{ m (easting coordinate of origin)} \]
\[ N_0 = 0.0000 \text{ m (northing coordinate of origin)} \]

Derived constants

\[ \xi = 0.662696910933 = \sin(\phi_s) \]
\[ K = 12205748.1618 \text{ m (mapping radius at the equator)} \]
\[ R_b = 7401530.8340 \text{ m (mapping radius at grid origin)} \]

Lambert coordinates \((N,E)\) from geodetic positions \((\phi,\lambda)\)

\[ \gamma = (\lambda CM - \lambda) \sin(\phi_s) \quad \text{(\(\gamma\) is the meridional convergence)} \]
\[ E = R \sin(\gamma) + E_0 \quad \text{(\(R\) from table)} \]
\[ N = R_b - R \cos(\gamma) + N_0 \]
Sample Calculation

South Dakota State Plane Coordinate Projection Tables

Lambert conformal conic projection with two standard parallels Plane coordinate projection tables

DATUM: NAD 83
The projection is SOUTH DAKOTA NORTH

Ellipsoidal constants

a = 6378137 m
f = 1/298.25722210

Defining constants

\( \phi_0 = 43^\circ 50' \) (latitude of grid origin)
\( \lambda_{CM} = 100^\circ 00' \) (longitude of origin and Central Meridian, CM)
\( \phi_s = 44^\circ 25' \) (southern standard parallel)
\( \phi_n = 45^\circ 41' \) (northern standard parallel)
\( E = 600000.0000 \) m (easting coordinate of origin)
\( N = 0.0000 \) m (northing coordinate of origin)

Derived constants

\( \ell = 0.707738135595 = \sin(\phi) \)
\( K = 11870154.6246 \) m (mapping radius at the equator)
\( R_o = 6512395.0582 \) m (mapping radius at grid origin)

Lambert coordinates \((N, E)\) from geodetic positions \((\phi, \lambda)\)

\[
\begin{align*}
\gamma &= (\lambda_{CM} - \lambda) \sin(\phi) \quad (\gamma \text{ is the meridional convergence}) \\
E &= R \sin(\gamma) + E \quad (R \text{ from table}) \\
N &= R_o - R \cos(\gamma) + N_o
\end{align*}
\]
Sample Calculation

\[ \Delta N = R \cos \gamma \]
\[ \Delta E = R \sin \gamma \]
\[ R = \text{function of latitude} \]
(see Table on slide 104)

\[ \gamma = 0^\circ 58'09.12'' \]
\[ R = 7,496,156.335 \text{ M} \]

\[ \Delta N = R \cos \gamma \]
\[ \Delta N = 7,495,083.878 \text{ M} \]

\[ \Delta E = R \sin \gamma \]
\[ \Delta E = 126,796.918 \text{ M} \]

\[ N = Rb - \Delta N + Nb \]
\[ E = \Delta E + Eb \]

\[ N = 455,768.356 \text{ M} \]
\[ E = 1,041,198.747 \text{ M} \]
Sample Calculation

• Verify Calculation at NGS Online
  http://www.ngs.noaa.gov/TOOLS/spc.shtml

| INPUT | | | | |
|-------|-------|-------|-------|
| Latitude | Longitude | Datum | Zone |
| N40°41′00″.00 | W104°00′00″.00 | NAD83 | 0501 |

<table>
<thead>
<tr>
<th>OUTPUT</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>NORTH (Y)</td>
<td>EAST (X)</td>
<td>AREA</td>
<td>CONVERGENCE</td>
<td>SCALE</td>
</tr>
<tr>
<td>455768.357</td>
<td>1041198.771</td>
<td>CO N</td>
<td>0° 58′ 09.12″</td>
<td>0.99998530</td>
</tr>
</tbody>
</table>

N = 455,768.356 M  E = 1,041,198.747 M

Diff N = 0.001 M
Diff E = 0.024 M
due to round off in γ
(should have used 5 decimal seconds)
γ = 0°58′09.12067″
Sample Calculation

• Review outline of steps
  – Design traverse to include one or more NGS control marks (or use OPUS-S).
  – Field measure all angles and distances as usual, record elevations of each traverse point.
  – Convert Ground Distances to Grid Distances
  – Convert Geodetic Azimuth to Grid Azimuth
  – Convert the Control Lat/Lon to Control N/E
  – Perform all calculations using grid azimuth and grid distances. Apply compass rule, etc, whatever.
  – Convert the grid N/E for all new points to Lat/Lon
Sample Calculation
Sample Calculation

- Due to time limitations, I will not demonstrate conversion of N/E to Lat/Lon
  - Easily done by following equations and example in Colorado State Plane Coordinate Projection Tables

\[
\tan(\gamma) = \frac{(E - E_s)}{(R_b - (N - N_b))}
\]

\[
R = \frac{(R_b - (N - N_b))}{\cos(\gamma)}
\]

\[
\lambda = \frac{\lambda_{GR}_{b} - \gamma}{\ell}
\]

\[
\phi \text{ from table using } R
\]

<table>
<thead>
<tr>
<th>Station</th>
<th>E</th>
<th>E - E_s</th>
<th>R</th>
<th>\gamma</th>
<th>Latitude</th>
<th>Longitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>sample 2</td>
<td>964401.829 m</td>
<td>50000.000 m</td>
<td>7536217.4920 m</td>
<td>40 19 21.1964</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>414800.610 m</td>
<td>7536051.624 m</td>
<td>0 22 48.50031</td>
<td>104 54 42.0160</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

WARNING: Use sufficient significant digits for trig.functions
Future

• When NGS releases NAD2022, there will be impacts on what to do with SPCS.
• As in the NAD27 to NAD83 changeover, states told NGS what they wanted to see happen.
• Nebraska and Montana wanted single zone.
• Most wanted to retain defining parameters.
• But, most wanted a shift in Nb and Eb to avoid confusion.
• Most wanted to not alter county-by-county zonal assignments.
SPCS2022 Webinar

- Webinar on State Plane Coordinates on March 8, 2018 by Michael Dennis

https://geodesy.noaa.gov/web/science_edu/webinar_series/Webinars.shtml
Future

• I strongly urge Nebraska survey leaders to carefully read
  NOAA Manual NOS NGS 5, State Plane Coordinate System of 1983, by
  James E. Stem, 1989, pages 1-12

• Discuss impacts of new datum on mapping activities,
  legislation to enable, desired changes to SPCS, and action plan
  to implement.

• Work with NGS Advisor.
References for Study

Geodesy

- Transverse Mercator Projection
  - Not used in Nebraska or nearby states.
  - It’s covered in detail in NOS NGS 5 by Stem, 1990
Global Positioning Systems (GPS)
Geodesy

• Modern surveyors need to know how the GPS system works.
• Need to be able to explain it to clients, co-workers, and to jury!
• On-Line Tutorials
    • https://www.ngs.noaa.gov/web/science_edu/online_lessons/
    • Free user account
  – US Coast Guard - NAVCEN
    • https://www.navcen.uscg.gov/?pageName=gpsmain
  – US Government
    • https://www.gps.gov/
  – Trimble GPS Tutorial
    • http://www.trimble.com/gps_tutorial/
Geodesy

• Due to time limitations, I will not cover GPS operation in detail
• But, I’d be happy to take questions about it.
Dave Zenk PE, LS
National Geodetic Survey
Northern Plains Regional Advisor
1735 Lake Drive West
Chanhassen, MN 55317-8581

763-600-6912 home office
952-368-2548 office
612-414-9522 mobile

dave.zenk@noaa.gov (federal business)
zenkx002@umn.edu (personal business)