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This chapter presents two examples that focus on the dynamic analysis of steel frame structures:

1. A 12-story steel frame building in Stockton, California. The highly irregular structure is analyzed using three techniques: equivalent lateral force analysis, modal response spectrum analysis and modal response history analysis. In each case, the structure is modeled in three dimensions and only linear elastic response is considered. The results from each of the analyses are compared and the accuracy and relative merits of the different analytical approaches are discussed.

2. A six-story steel frame building in Seattle, Washington. This regular structure is analyzed using both linear and nonlinear techniques. Due to the regular configuration of the structural system, the analyses are performed for only two dimensions. For the nonlinear analysis, two approaches are used: static pushover analysis and nonlinear response history analysis. The relative merits of pushover analysis versus response history analysis are discussed.

Although the Seattle building, as originally designed, responds reasonably well under the design ground motions, a second set of response history analyses is presented for the structure augmented with added viscous fluid damping devices. As shown, the devices have the desired effect of reducing the deformation demands in the critical regions of the structure.

In addition to the Standard, the following documents are referenced:

<table>
<thead>
<tr>
<th>Reference</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASCE 41</td>
<td>American Society of Civil Engineers. 2006. <em>Seismic Rehabilitation of Existing Buildings.</em></td>
</tr>
<tr>
<td>ASCE 7-10</td>
<td>American Society of Civil Engineers. 2010. <em>Minimum Design Loads for Buildings and Other Structures</em></td>
</tr>
</tbody>
</table>
4.1 IRREGULAR 12-STORY STEEL FRAME BUILDING, STOCKTON, CALIFORNIA

4.1.1 Introduction

This example presents the analysis of a 12-story steel frame building under seismic effects acting alone. Gravity forces due to dead and live load are not computed. For this reason, member stress checks, member design and detailing are not discussed. Load combinations that include gravity effects are considered, however. For detailed examples of the seismic-resistant design of structural steel buildings, see Chapter 6 of this volume of design examples.

The analysis of the structure, shown in Figures 4.1-1 through 4.1-3, is performed using three methods:

1. The Equivalent Lateral Force (ELF) procedure based on the requirements of Standard Section 12.8,
2. The modal response spectrum procedure based on the requirements of Standard Section 12.9 and
3. The modal response history procedure based on the requirements of Chapter 16 of ASCE 7-10. (The 2010 version of the Standard is used for this part of the example because it eliminates several omissions and inconsistencies that were present in Chapter 16 of ASCE 7-05.)

In each case, special attention is given to applying the Standard’ rules for direction of loading and for accidental torsion. All analyses were performed in three dimensions using the finite element analysis program SAP2000 (developed by Computers and Structures, Inc., Berkeley, California).

4.1.2 Description of Building and Structure

The building has 12 stories above grade and a one-story basement below grade and is laid out on a rectangular grid with a maximum of seven 30-foot-wide bays in the X direction and seven 25-foot bays in the Y direction. Both the plan and elevation of the structure are irregular with setbacks occurring at Levels 5 and 9. All stories have a height of 12.5 feet except for the first story which is 18 feet high and the basement which extends 18 feet below grade. Reinforced 1-foot-thick concrete walls form the perimeter of the basement. The total height of the building above grade is 155.5 feet.

Gravity loads are resisted by composite beams and girders that support a normal-weight concrete slab on metal deck. The slab has an average thickness of 4.0 inches at all levels except Levels G, 5 and 9. The slabs on Levels 5 and 9 have an average thickness of 6.0 inches for more effective shear transfer through the diaphragm. The slab at Level G is 6.0 inches thick to minimize pedestrian-induced vibrations and to support heavy floor loads. The low roofs at Levels 5 and 9 are used as outdoor patios and support heavier live loads than do the upper roofs or typical floors.
At the perimeter of the base of the building, the columns are embedded into pilasters cast into the basement walls, with the walls supported on reinforced concrete tie beams over drilled piers. Interior columns are supported by concrete caps over drilled piers. A grid of reinforced concrete grade beams connects all tie beams and pier caps.

The lateral load-resisting system consists of special steel moment frames at the perimeter of the building and along Grids C and F. For the frames on Grids C and F, the columns extend down to the foundation, but the lateral load-resisting girders terminate at Level 5 for Grid C and Level 9 for Grid F. Girders below these levels are simply connected. Since the moment-resisting girders terminate in Frames C and F, much of the Y direction seismic shears below Level 9 are transferred through the diaphragms to the frames on Grids A and H. Overturining moments developed in the upper levels of these frames are transferred down to the foundation by axial forces in the columns. Columns in the moment-resisting frame range in size from W24x146 at the roof to W24x229 at Level G. Girders in the moment frames vary from W30x108 at the roof to W30x132 at Level G. Members of the moment-resisting frames have a nominal yield strength of 36 ksi and floor members and interior columns that are sized strictly for gravity forces have a nominal yield strength of 50 ksi.

### 4.1.3 Seismic Ground Motion Parameters

For this example the relevant seismic ground motion parameters are as follows:

- $S_S = 1.25$
- $S_I = 0.40$
- Site Class C

From Standard Tables 11.4-1 and 11.4-2:

- $F_a = 1.0$
- $F_v = 1.4$

Using Standard Equations 11.4-1 through 11.4-4:

- $S_{MS} = F_a S_S = 1.0(1.25) = 1.25$
- $S_{MI} = F_a S_I = 1.4(0.4) = 0.56$
- $S_{DS} = \frac{2}{3} S_{MS} = \frac{2}{3}(1.25) = 0.833$
- $S_{DI} = \frac{2}{3} S_{MI} = \frac{2}{3}(0.56) = 0.373$

As the primary occupancy of the building is business offices, the Occupancy Category is II (Standard Table 1-1) and the Importance Factor ($I$) is 1.0 (Standard Table 11.5-1). According to Standard Tables 11.6-1 and 11.6-2, the Seismic Design Category (SDC) for this building is D.
Figure 4.1-1 Various floor plans of 12-story Stockton building
Figure 4.1-2 Sections through Stockton building
The lateral load-resisting system of the building is a special moment-resisting frame of structural steel. For this type of system, Standard Table 12.2-1 has a response modification coefficient ($R$) of 8 and a deflection amplification coefficient ($C_d$) of 5.5. There is no height limit for special moment frames. Section 12.2.5.5 of the standard requires that special moment frames in SDC D, where required by Table 12.1-1, be continuous to the foundation. While the girders of the interior moment frames are not present at the lower levels of the interior frames, the frames are continuous to the foundation and the columns are detailed as required for special moment frames. Additionally, there are no other structural system types below the moment frames. Therefore, in the opinion of the author, the requirement is met.

Standard Table 12.6-1 is used to determine the minimum level of analysis. Because of the setbacks, the structure clearly has a weight irregularity (Irregularity Type 2 in Standard Table 12.3-2). Thus, the minimum level of analysis required for the SDC D building is modal response spectrum analysis. However, the determination of torsional irregularities, the application of accidental torsion effects and the assessment of P-delta effects are based on ELF analysis procedures. For this reason and for comparison purposes, a complete ELF analysis is carried out and described herein.
4.1.4 Dynamic Properties

Before any analysis can be carried out, it is necessary to determine the dynamic properties of the structure. These properties include stiffness, mass and damping. The stiffness of the structure is numerically represented by the system stiffness matrix, which is computed automatically by SAP2000. The terms in this matrix are a function of several modeling choices that are made. These aspects of the analysis are described later in the example. The computer can also determine the mass properties automatically, but for this analysis they are developed by hand and are explicitly included in the computer model. Damping is represented in different ways for the different methods of analysis, as described in Section 4.1.4.2.

4.1.4.1 Seismic Weight. In the past it was often advantageous to model floor plates as rigid diaphragms because this allowed for a reduction in the total number of degrees of freedom used in the analysis and a significant reduction in analysis time. Given the speed and capacity of most personal computers, the use of rigid diaphragms is no longer necessary and the floor plates may be modeled using 4-node shell elements. The use of such elements provides an added benefit of improved accuracy because the true “semi-rigid” behavior of the diaphragms is modeled directly. Where it is not necessary to recover diaphragm stresses, a very coarse element mesh may be used for modeling the diaphragm.

Where the diaphragm is modeled using finite elements, the diaphragm mass, including contributions from structural dead weight and superimposed dead weight, is automatically represented by entering the proper density and thickness of the diaphragm elements. The density may be adjusted to represent superimposed dead loads (but the thickness and modulus are “true” values). Line mass, such as window walls and exterior cladding, are modeled with frame element line masses. While complete building masses are easily represented in this manner, the SAP2000 program does not automatically compute the locations of the centers of mass, so these must be computed separately. Center of mass locations are required for the purpose of applying lateral forces in the ELF method and for determining story drift.

Due to the various sizes and shapes of the floor plates and to the different dead weights associated with areas within the same floor plate, the computation of mass properties is not easily carried out by hand. For this reason, a special purpose computer program was used. The basic input for the program consists of the shape of the floor plate, its mass density and definitions of auxiliary masses such as line, rectangular and concentrated mass.

The uniform area and line masses (in weight units) associated with the various floor plates are given in Tables 4.1-1 and 4.1-2. The line masses are based on a cladding weight of 15.0 psf, story heights of 12.5 or 18.0 feet and parapets 4.0 feet high bordering each roof region. Figure 4.1-4 shows where each mass type occurs. The total computed floor mass, mass moment of inertia and locations of center of mass are shown in Table 4.1-3. Note that the mass moments of inertia are not required for the analysis but are provided in the table for completeness. The reference point for center of mass location is the intersection of Grids A and 8.

Table 4.1-3 includes a mass computed for Level G of the building. This mass is associated with an extremely stiff story (the basement level) and is dynamically excited by the earthquake in very high frequency modes of response. As shown later, this mass is not included in equivalent lateral force computations.
### Table 4.1-1  Area Weights Contributing to Masses on Floor Diaphragms

<table>
<thead>
<tr>
<th>Mass Type</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slab and Deck (psf)</td>
<td>50</td>
<td>75</td>
<td>50</td>
<td>75</td>
<td>75</td>
</tr>
<tr>
<td>Structure (psf)</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>50</td>
</tr>
<tr>
<td>Ceiling and Mechanical (psf)</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>Partition (psf)</td>
<td>10</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>Roofing (psf)</td>
<td>0</td>
<td>0</td>
<td>15</td>
<td>15</td>
<td>0</td>
</tr>
<tr>
<td>Special (psf)</td>
<td>0</td>
<td>0</td>
<td>60</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>Total (psf)</td>
<td>95</td>
<td>120</td>
<td>100</td>
<td>185</td>
<td>175</td>
</tr>
</tbody>
</table>

See Figure 4.1-4 for mass location.

1.0 psf = 47.9 N/m².

### Table 4.1-2  Line Weights Contributing to Masses on Floor Diaphragms

<table>
<thead>
<tr>
<th>Mass Type</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
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<tr>
<td>From Story Above (plf)</td>
<td>60.0</td>
<td>93.8</td>
<td>93.8</td>
<td>93.8</td>
<td>135.0</td>
</tr>
<tr>
<td>From Story Below (plf)</td>
<td>93.8</td>
<td>93.8</td>
<td>0.0</td>
<td>135.0</td>
<td>1,350.0</td>
</tr>
<tr>
<td>Total (plf)</td>
<td>153.8</td>
<td>187.6</td>
<td>93.8</td>
<td>228.8</td>
<td>1,485.0</td>
</tr>
</tbody>
</table>

See Figure 4.1-4 for mass location.

1.0 plf = 14.6 N/m.
**Figure 4.1-4** Key diagram for computation of floor weights

<table>
<thead>
<tr>
<th>Level</th>
<th>Weight (kips)</th>
<th>Mass (kip·sec²/in.)</th>
<th>Mass Moment of Inertia (in.-kip·sec²/radian)</th>
<th>X Distance to C.M. (in.)</th>
<th>Y Distance to C.M. (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>1,657</td>
<td>4.287</td>
<td>2.072x10⁶</td>
<td>1,260</td>
<td>1,050</td>
</tr>
<tr>
<td>12</td>
<td>1,596</td>
<td>4.130</td>
<td>2.017x10⁶</td>
<td>1,260</td>
<td>1,050</td>
</tr>
<tr>
<td>11</td>
<td>1,596</td>
<td>4.130</td>
<td>2.017x10⁶</td>
<td>1,260</td>
<td>1,050</td>
</tr>
<tr>
<td>10</td>
<td>1,596</td>
<td>4.130</td>
<td>2.017x10⁶</td>
<td>1,260</td>
<td>1,050</td>
</tr>
<tr>
<td>9</td>
<td>3,403</td>
<td>8.807</td>
<td>5.309x10⁶</td>
<td>1,638</td>
<td>1,175</td>
</tr>
<tr>
<td>8</td>
<td>2,331</td>
<td>6.032</td>
<td>3.703x10⁶</td>
<td>1,553</td>
<td>1,145</td>
</tr>
<tr>
<td>7</td>
<td>2,331</td>
<td>6.032</td>
<td>3.703x10⁶</td>
<td>1,553</td>
<td>1,145</td>
</tr>
<tr>
<td>6</td>
<td>2,331</td>
<td>6.032</td>
<td>3.703x10⁶</td>
<td>1,553</td>
<td>1,145</td>
</tr>
<tr>
<td>5</td>
<td>4,320</td>
<td>11.19</td>
<td>9.091x10⁶</td>
<td>1,160</td>
<td>1,206</td>
</tr>
<tr>
<td>4</td>
<td>3,066</td>
<td>7.935</td>
<td>6.356x10⁶</td>
<td>1,261</td>
<td>1,184</td>
</tr>
<tr>
<td>3</td>
<td>3,066</td>
<td>7.935</td>
<td>6.356x10⁶</td>
<td>1,261</td>
<td>1,184</td>
</tr>
<tr>
<td>2</td>
<td>3,097</td>
<td>8.015</td>
<td>6.437x10⁶</td>
<td>1,262</td>
<td>1,181</td>
</tr>
<tr>
<td>G</td>
<td>6,525</td>
<td>16.89</td>
<td>1.503x10⁷</td>
<td>1,265</td>
<td>1,149</td>
</tr>
<tr>
<td>Σ</td>
<td>36,912</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1.0 in. = 25.4 mm, 1.0 kip = 4.45 kN.
4.1.4.2 Damping. Where an equivalent lateral force analysis or a modal response spectrum analysis is performed, the structure’s damping, assumed to be 5 percent of critical, is included in the development of the spectral accelerations $S_S$ and $S_I$. An equivalent viscous damping ratio of 0.05 is appropriate for linear analysis of lightly damaged steel structures.

Where recombinining the individual modal responses in modal response spectrum analysis, the square root of the sum of the squares (SRSS) technique has generally been replaced in practice by the complete quadratic combination (CQC) approach. Indeed, *Standard* Section 12.9.3 requires that the CQC approach be used where the modes are closely spaced. Where using the CQC approach, the analyst must correctly specify a damping factor. This factor, which is entered into the SAP2000 program, must match that used in developing the response spectrum. It should be noted that if zero damping is used in CQC, the results are the same as those for SRSS.

For modal response history analysis, SAP2000 allows an explicit damping ratio to be used in each mode. For this structure, a damping of 5 percent of critical was specified in each mode.

4.1.5 Equivalent Lateral Force Analysis

Prior to performing modal response spectrum or response history analysis, it is necessary to perform an ELF analysis of the structure. This analysis is used for preliminary design, for evaluating torsional regularity, for computing torsional amplification factors (where needed), for application of accidental torsion, for evaluation of P-delta effects and for development of redundancy factors.

The first step in the ELF analysis is to determine the period of vibration of the building. This period can be “accurately” computed from a three-dimensional computer model of the structure. However, it is first necessary to estimate the period using empirical relationships provided by the *Standard*.

*Standard* Equation 12.8-7 is used to estimate the building period:

$$T_a = C_r h_n^x$$

where, from *Standard* Table 12.8-2, $C_r = 0.028$ and $x = 0.8$ for a steel moment frame. Using $h_n = $ the total building height (above grade) = 155.5 ft, $T_a = 0.028(155.5)^{0.8} = 1.59$ seconds\(^1\).

Even where the period is accurately computed from a properly substantiated structural analysis (such as an eigenvalue or Rayleigh analysis), the *Standard* requires that the period used for ELF base shear calculations not exceed $C_u T_a$ where $C_u = 1.4$ (from *Standard* Table 12.8-1 using $S_D = 0.373$). For the structure under consideration, $C_u T_a = 1.4(1.59) = 2.23$ seconds.

Note that where the accurately computed period is less than $C_u T_a$, the computed period should be used. In no case, however, is it necessary to use a period less than $T = T_a = 1.59$ seconds. The use of the Rayleigh method and the eigenvalue method of determining accurate periods of vibration are illustrated in a later part of this example.

In anticipation of the accurately computed period of the building being greater than 2.23 seconds, the ELF analysis is based on a period of vibration equal to $C_u T_a = 2.23$ seconds\(^2\). For the ELF analysis, it is

---

1 The proper computational units for period of vibration are theoretically “seconds/cycle”. However, it is traditional to use units of “seconds,” and this is done in the remainder of this example.
2 As shown later in this example, the computed period is indeed greater than $C_u T_a$.  

---
assumed that the structure is “fixed” at grade level. Hence, the total effective weight of the structure (see Table 4.1-3) is the total weight minus the grade level weight, or 36,920 – 6,526 = 30,394 kips.

4.1.5.1 Base shear and vertical distribution of force. Using Standard Equation 12.8-1, the total seismic base shear is:

\[ V = C_s W \]

where \( W \) is the total weight of the structure. From Standard Equation 12.8-2, the maximum (constant acceleration region) spectral acceleration is:

\[ C_{S_{\max}} = \frac{S_{DS}}{(R/I)} = \frac{0.833}{8/1} = 0.104 \]

Standard Equation 12.8-3 controls in the constant velocity region:

\[ C_s = \frac{S_{DI}}{T(R/I)} = \frac{0.373}{2.23(8/1)} = 0.021 \]

However, the acceleration must not be less than that given by Standard Equation 12.8-5:

\[ C_{S_{\max}} = 0.044 IS_{DS} = 0.01 = 0.044(1)(0.833) = 0.037 \]

The \( C_s \) value determined from Equation 12.8-5 controls the seismic base shear for this building. Using \( W = 30,394 \) kips, \( V = 0.037(30,394) = 1,124 \) kips. The acceleration response spectrum given by the above equations is plotted in Figure 4.1-5.

**Figure 4.1-5** Computed ELF total acceleration response spectrum
While it is reasonable to use Equation 12.8-5 to establish a minimum base shear, the equation should not be used as a basis determining lateral forces used in displacement computations. The effect of using Equation 12.8-5 for displacements is shown in Figure 4.1-6, which represents Equations 12.8-2, 12.8-3 and 12.8-5 in the form of a displacement spectrum. It can be seen from this figure that the dotted line, representing Equation 12.8-5, will predict significantly larger displacements than Equation 12.8-3. The problem with the line represented by Equation 12.8-5 is that it gives an exponentially increasing displacement up to unlimited periods, whereas it is expected that the true spectral displacements will converge towards a constant displacement (the maximum ground displacement) at large periods. In other words, Equation 12.8-5 should not be considered as a branch of the response spectrum—it is simply used to represent the lower bound on design base shear. The Standard does not directly recognize this problem. However, Section 12.8.6.2 allows the deflection analysis of the seismic force-resisting system to be based on the accurately computed fundamental period of vibration, without the $C_u T_a$ upper limit on period. It is the ‘authors’ opinion that this clause may be used to justify drift calculations with forces based on Equation 12.8-3 even when Equation 12.8-5 controls the design base shear. ASCE 7-10 has clarified this issue, by providing an exception that specifically states that Equation 12.8-5 need not be considered for computing drift.

It is important to note, however, that where Equation 12.8-6 controls the design base shear, drifts must be based on lateral forces consistent with Equation 12.8-6. This is due to the fact that Equation 12.8-6 is an approximation of the long period acceleration spectrum for “near field” ground motions (where $S_l$ is likely to be greater than 0.6 g.)

In this example, all ELF analysis is performed using the forces obtained from Equation 12.8-5, but for the purposes of computing drift, the story deflections are computed using the forces from Equation 12.8-3. When using Equation 12.8-3, the upper bound period $C_u T_a$ was used in lieu of the computed period. This allows for a simple “conversion” of displacements where displacements computed from forces based on Equation 12.8-5 are multiplied by the factor $(0.021/0.037 = 0.568)$ to obtain displacements that would be

---

**Figure 4.1-6** Computed ELF relative displacement response spectrum
generated from forces based on Equation 12.8-3 and the $C_s T_s$ limit. If it is found that the factored computed drifts violate the drift limits (which is not the case in this example), it might be advantageous to re-compute the drifts on the basis of Equation 12.8-3 and the computed period $T$.

The seismic base shear computed according to Standard Equation 12.8-1 is distributed along the height of the building using Standard Equations 12.8-11 and 12.8-12:

$$F_x = C_{vx} V$$

and

$$C_{vx} = \frac{w_i h^k}{\sum w_i h_i^k}$$

where $k = 0.75 + 0.5T = 0.75 + 0.5(2.23) = 1.865$. The story forces, story shears and story overturning moments are summarized in Table 4.1-4.

<table>
<thead>
<tr>
<th>Level x</th>
<th>$w_x$ (kips)</th>
<th>$h_x$ (ft)</th>
<th>$w_xh_x^k$</th>
<th>$C_{vx}$</th>
<th>$F_x$ (kips)</th>
<th>$V_x$ (kips)</th>
<th>$M_x$ (ft-kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>1,657</td>
<td>155.5</td>
<td>20,272,144</td>
<td>0.1662</td>
<td>186.9</td>
<td>186.9</td>
<td>2,336</td>
</tr>
<tr>
<td>12</td>
<td>1,596</td>
<td>143.0</td>
<td>16,700,697</td>
<td>0.1370</td>
<td>154.0</td>
<td>340.9</td>
<td>6,597</td>
</tr>
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<td>11</td>
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<td>130.5</td>
<td>14,081,412</td>
<td>0.1155</td>
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<td>118.0</td>
<td>11,670,590</td>
<td>0.0957</td>
<td>107.6</td>
<td>578.4</td>
<td>19,712</td>
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<td>3,403</td>
<td>105.5</td>
<td>20,194,253</td>
<td>0.1656</td>
<td>186.3</td>
<td>764.7</td>
<td>29,271</td>
</tr>
<tr>
<td>8</td>
<td>2,331</td>
<td>93.0</td>
<td>10,933,595</td>
<td>0.0897</td>
<td>100.8</td>
<td>865.5</td>
<td>40,909</td>
</tr>
<tr>
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<td>2,331</td>
<td>80.5</td>
<td>8,353,175</td>
<td>0.0685</td>
<td>77.0</td>
<td>942.5</td>
<td>51,871</td>
</tr>
<tr>
<td>6</td>
<td>2,331</td>
<td>68.0</td>
<td>6,097,775</td>
<td>0.0500</td>
<td>56.2</td>
<td>998.8</td>
<td>64,356</td>
</tr>
<tr>
<td>5</td>
<td>4,324</td>
<td>55.5</td>
<td>7,744,477</td>
<td>0.0635</td>
<td>71.4</td>
<td>1,070.2</td>
<td>77,733</td>
</tr>
<tr>
<td>4</td>
<td>3,066</td>
<td>43.0</td>
<td>3,411,857</td>
<td>0.0280</td>
<td>31.5</td>
<td>1,101.7</td>
<td>91,505</td>
</tr>
<tr>
<td>3</td>
<td>3,066</td>
<td>30.5</td>
<td>1,798,007</td>
<td>0.0147</td>
<td>16.6</td>
<td>1,118.2</td>
<td>103,372</td>
</tr>
<tr>
<td>2</td>
<td>3,097</td>
<td>18.0</td>
<td>679,242</td>
<td>0.0056</td>
<td>6.3</td>
<td>1,124.5</td>
<td>120,964</td>
</tr>
<tr>
<td>Σ</td>
<td>30,394</td>
<td>-</td>
<td>121,937,234</td>
<td>1.00</td>
<td>1124.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Values in column 4 based on exponent $k=1.865$.
1.0 ft = 0.3048 m, 1.0 kip = 4.45 kN.

4.1.5.2 **Accidental torsion and orthogonal loading effects.** Where using the ELF method as the basis for structural design, two effects must be added to the direct lateral forces shown in Table 4.1-4. The first of these effects accounts for the fact that an earthquake can produce inertial forces that act in any direction. For SDC D, E and F buildings, Standard Section 12.5 requires that the structure be investigated for forces that act in the direction that causes the “critical load effects.” Since this direction is not easily defined, the Standard allows the analyst to load the structure with 100 percent of the seismic
force in one direction (along the X axis, for example) simultaneous with the application of 30 percent of the force acting in the orthogonal direction (along the Y axis).

The other requirement is that the structure be modeled with additional forces to account for uncertainties in the location of center of mass and center of rigidity, uneven yielding of vertical systems and the possibility of torsional components of ground motion. For torsionally regular buildings, this requirement, given in Standard Section 12.8.4.2, can be satisfied by applying the equivalent lateral force at an “accidental” eccentricity, where the eccentricity is equal to 5 percent of the overall dimension of the structure in the direction perpendicular to the line of the application of force. For torsionally irregular structures in SDC C, D, E, or F, Standard Section 12.8.4.3 requires that the accidental eccentricity be amplified (although the amplification factor may be 1.0).

According to Standard Table 12.3-1, a torsional irregularity exists if:

$$\frac{\Delta_{\text{max}}}{\Delta_{\text{avg}}} \geq 1.2$$

where $\delta_{\text{max}}$ is the maximum story drift at the edge of the floor diaphragm and $\Delta_{\text{avg}}$ is the average drift at the center of the diaphragm (see Standard Figure 12.8-1). If the ratio of drifts is greater than 1.4, the torsional irregularity is referred to as “extreme.” In computing the drifts, the structure must be loaded with the basic equivalent lateral forces applied at a 5 percent eccentricity.

For main loads acting in the X direction, displacements and drifts were determined on Grid Line D. For the main loads acting in the Y direction, the story displacements on Grid Line 1 were used. Because of the architectural setbacks, the locations for determining displacements associated with $\Delta_{\text{min}}$ and $\Delta_{\text{max}}$ are not always vertically aligned. This situation is shown in Figure 4.1-7, where it is seen that three displacement monitoring stations are required at Levels 5 and 9. The numerical values shown in Figure 4.1-7 are discussed later in relation to Table 4.1-5b.
Figure 4.1-7 Drift monitoring stations for determination of torsional irregularity and torsional amplification (deflections in inches, 1.0 in. = 25.4 mm)

The analysis of the structure for accidental torsion was performed using SAP2000. The same model was used for ELF, modal response spectrum and modal response history analysis. The following approach was used for the mathematical model of the structure:

1. The floor diaphragm was modeled with shell elements, providing nearly rigid behavior in-plane.

2. Flexural, shear, axial and torsional deformations were included in all columns and beams.

3. Beam-column joints were modeled using centerline dimensions. This approximately accounts for deformations in the panel zone.

4. Section properties for the girders were based on bare steel, ignoring composite action. This is a reasonable assumption since most of the girders are on the perimeter of the building and are under reverse curvature.
5. Except for those lateral load-resisting columns that terminate at Levels 5 and 9, all columns were assumed to be fixed at their base.

6. The basement walls and grade level slab were explicitly modeled using 4-node shell elements. This was necessary to allow the interior columns to continue through the basement level. No additional lateral restraint was applied at the grade level; thus, the basement level acts as a very stiff first floor of the structure. This basement level was not relevant for the ELF analysis, but it did influence the modal response spectrum and modal response history analyses as described in later sections of this example.

7. P-delta effects were not included in the mathematical model. These effects are evaluated separately using the procedures provided in Standard Section 12.8.7.

The results of the accidental torsion analysis are shown in Tables 4.1-5a and 4.1-5b. For loading in the X direction, there is no torsional irregularity because all drift ratios ($\Delta_{\text{max}}/\Delta_{\text{avg}}$) are less than 1.2. For loading in the Y direction, the largest ratio of maximum to average story drift is 1.24 at Level 9 of the building. Hence, this structure has a Type 1 torsional irregularity, but only marginally so. See Figure 4.1-7 for the source of the dual displacement values shown for Levels 9 and 5 in Table 4.1-5b.

Even though the torsional irregularity is marginal, Section 12.8.4.3 of the Standard requires that torsional amplification factors be determined for this SDC D building. The results for these calculations, which are based on story displacement, not drift, are presented in Tables 4.1-6a and 4.1-6b for the main load applied in the X and Y directions, respectively. As may be observed, the calculated amplification factors are significantly less than 1.0 at all levels for both directions of loading.

**Table 4.1-5a** Computation for Torsional Irregularity with ELF Loads Acting in X Direction and Torsional Moment Applied Counterclockwise

<table>
<thead>
<tr>
<th>Level</th>
<th>$\delta_1$ (in.)</th>
<th>$\delta_2$ (in.)</th>
<th>$\Delta_1$ (in.)</th>
<th>$\Delta_2$ (in.)</th>
<th>$\Delta_{\text{avg}}$ (in.)</th>
<th>$\Delta_{\text{max}}$ (in.)</th>
<th>$\Delta_{\text{max}}/\Delta_{\text{avg}}$</th>
<th>Irregularity</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>7.27</td>
<td>6.15</td>
<td>0.34</td>
<td>0.29</td>
<td>0.31</td>
<td>0.34</td>
<td>1.08</td>
<td>None</td>
</tr>
<tr>
<td>12</td>
<td>6.93</td>
<td>5.87</td>
<td>0.48</td>
<td>0.42</td>
<td>0.45</td>
<td>0.48</td>
<td>1.07</td>
<td>None</td>
</tr>
<tr>
<td>11</td>
<td>6.44</td>
<td>5.45</td>
<td>0.60</td>
<td>0.51</td>
<td>0.55</td>
<td>0.60</td>
<td>1.07</td>
<td>None</td>
</tr>
<tr>
<td>10</td>
<td>5.85</td>
<td>4.93</td>
<td>0.66</td>
<td>0.56</td>
<td>0.61</td>
<td>0.66</td>
<td>1.08</td>
<td>None</td>
</tr>
<tr>
<td>9</td>
<td>5.19</td>
<td>4.37</td>
<td>0.65</td>
<td>0.54</td>
<td>0.59</td>
<td>0.65</td>
<td>1.10</td>
<td>None</td>
</tr>
<tr>
<td>8</td>
<td>4.54</td>
<td>3.84</td>
<td>0.69</td>
<td>0.58</td>
<td>0.64</td>
<td>0.69</td>
<td>1.09</td>
<td>None</td>
</tr>
<tr>
<td>7</td>
<td>3.84</td>
<td>3.26</td>
<td>0.70</td>
<td>0.59</td>
<td>0.65</td>
<td>0.70</td>
<td>1.09</td>
<td>None</td>
</tr>
<tr>
<td>6</td>
<td>3.14</td>
<td>2.67</td>
<td>0.69</td>
<td>0.58</td>
<td>0.63</td>
<td>0.69</td>
<td>1.09</td>
<td>None</td>
</tr>
<tr>
<td>5</td>
<td>2.46</td>
<td>2.09</td>
<td>0.60</td>
<td>0.50</td>
<td>0.55</td>
<td>0.60</td>
<td>1.09</td>
<td>None</td>
</tr>
<tr>
<td>4</td>
<td>1.86</td>
<td>1.60</td>
<td>0.59</td>
<td>0.50</td>
<td>0.55</td>
<td>0.59</td>
<td>1.08</td>
<td>None</td>
</tr>
<tr>
<td>3</td>
<td>1.27</td>
<td>1.10</td>
<td>0.58</td>
<td>0.49</td>
<td>0.53</td>
<td>0.58</td>
<td>1.08</td>
<td>None</td>
</tr>
<tr>
<td>2</td>
<td>0.69</td>
<td>0.61</td>
<td>0.69</td>
<td>0.61</td>
<td>0.65</td>
<td>0.69</td>
<td>1.06</td>
<td>None</td>
</tr>
</tbody>
</table>

1.0 in. = 25.4 mm.
Table 4.1-5b  Computation for Torsional Irregularity with ELF Loads Acting in Y Direction and Torsional Moment Applied Clockwise

<table>
<thead>
<tr>
<th>Level</th>
<th>$\delta_1$ (in.)</th>
<th>$\delta_2$ (in.)</th>
<th>$\Delta_1$ (in.)</th>
<th>$\Delta_2$ (in.)</th>
<th>$\Delta_{avg}$ (in.)</th>
<th>$\Delta_{max}$ (in.)</th>
<th>$\Delta_{max}/\Delta_{avg}$</th>
<th>Irregularity</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>5.19</td>
<td>4.77</td>
<td>0.15</td>
<td>0.14</td>
<td>0.15</td>
<td>0.15</td>
<td>1.03</td>
<td>None</td>
</tr>
<tr>
<td>12</td>
<td>5.03</td>
<td>4.63</td>
<td>0.25</td>
<td>0.23</td>
<td>0.24</td>
<td>0.25</td>
<td>1.03</td>
<td>None</td>
</tr>
<tr>
<td>11</td>
<td>4.79</td>
<td>4.40</td>
<td>0.31</td>
<td>0.29</td>
<td>0.30</td>
<td>0.31</td>
<td>1.04</td>
<td>None</td>
</tr>
<tr>
<td>10</td>
<td>4.48</td>
<td>4.11</td>
<td>0.38</td>
<td>0.34</td>
<td>0.36</td>
<td>0.38</td>
<td>1.06</td>
<td>None</td>
</tr>
<tr>
<td>9</td>
<td>4.10</td>
<td>3.77, 3.55</td>
<td>0.46</td>
<td>0.28</td>
<td>0.37</td>
<td>0.46</td>
<td>1.24</td>
<td>Irregularity</td>
</tr>
<tr>
<td>8</td>
<td>3.64</td>
<td>3.26</td>
<td>0.54</td>
<td>0.36</td>
<td>0.45</td>
<td>0.54</td>
<td>1.20</td>
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<tr>
<td>7</td>
<td>3.09</td>
<td>2.90</td>
<td>0.56</td>
<td>0.39</td>
<td>0.47</td>
<td>0.56</td>
<td>1.18</td>
<td>None</td>
</tr>
<tr>
<td>6</td>
<td>2.53</td>
<td>2.51</td>
<td>0.60</td>
<td>0.42</td>
<td>0.51</td>
<td>0.60</td>
<td>1.18</td>
<td>None</td>
</tr>
<tr>
<td>5</td>
<td>1.93, 1.95</td>
<td>2.09</td>
<td>0.41</td>
<td>0.47</td>
<td>0.44</td>
<td>0.47</td>
<td>1.06</td>
<td>None</td>
</tr>
<tr>
<td>4</td>
<td>1.53</td>
<td>1.62</td>
<td>0.47</td>
<td>0.50</td>
<td>0.48</td>
<td>0.50</td>
<td>1.03</td>
<td>None</td>
</tr>
<tr>
<td>3</td>
<td>1.07</td>
<td>1.12</td>
<td>0.47</td>
<td>0.50</td>
<td>0.48</td>
<td>0.50</td>
<td>1.03</td>
<td>None</td>
</tr>
<tr>
<td>2</td>
<td>0.60</td>
<td>0.63</td>
<td>0.60</td>
<td>0.63</td>
<td>0.61</td>
<td>0.63</td>
<td>1.03</td>
<td>None</td>
</tr>
</tbody>
</table>

1.0 in. = 25.4 mm.

Table 4.1-6a  Amplification Factor $A_x$ for Accidental Torsional Moment Loads Acting in the X Direction and Torsional Moment Applied Counterclockwise

<table>
<thead>
<tr>
<th>Level</th>
<th>$\delta_1$ (in.)</th>
<th>$\delta_2$ (in.)</th>
<th>$\delta_{avg}$ (in.)</th>
<th>$\delta_{max}$ (in.)</th>
<th>$A_x$ calculated</th>
<th>$A_x$ used</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>7.27</td>
<td>6.15</td>
<td>6.71</td>
<td>7.27</td>
<td>0.81</td>
<td>1.00</td>
</tr>
<tr>
<td>12</td>
<td>6.93</td>
<td>5.87</td>
<td>6.40</td>
<td>6.93</td>
<td>0.81</td>
<td>1.00</td>
</tr>
<tr>
<td>11</td>
<td>6.44</td>
<td>5.45</td>
<td>5.95</td>
<td>6.44</td>
<td>0.82</td>
<td>1.00</td>
</tr>
<tr>
<td>10</td>
<td>5.85</td>
<td>4.93</td>
<td>5.39</td>
<td>5.85</td>
<td>0.82</td>
<td>1.00</td>
</tr>
<tr>
<td>9</td>
<td>5.19</td>
<td>4.37</td>
<td>4.78</td>
<td>5.19</td>
<td>0.82</td>
<td>1.00</td>
</tr>
<tr>
<td>8</td>
<td>4.54</td>
<td>3.84</td>
<td>4.19</td>
<td>4.54</td>
<td>0.82</td>
<td>1.00</td>
</tr>
<tr>
<td>7</td>
<td>3.84</td>
<td>3.26</td>
<td>3.55</td>
<td>3.84</td>
<td>0.81</td>
<td>1.00</td>
</tr>
<tr>
<td>6</td>
<td>3.14</td>
<td>2.67</td>
<td>2.90</td>
<td>3.14</td>
<td>0.81</td>
<td>1.00</td>
</tr>
<tr>
<td>5</td>
<td>2.46</td>
<td>2.09</td>
<td>2.27</td>
<td>2.46</td>
<td>0.81</td>
<td>1.00</td>
</tr>
<tr>
<td>4</td>
<td>1.86</td>
<td>1.60</td>
<td>1.73</td>
<td>1.86</td>
<td>0.80</td>
<td>1.00</td>
</tr>
<tr>
<td>3</td>
<td>1.27</td>
<td>1.10</td>
<td>1.18</td>
<td>1.27</td>
<td>0.80</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>0.69</td>
<td>0.61</td>
<td>0.65</td>
<td>0.69</td>
<td>0.79</td>
<td>1.00</td>
</tr>
</tbody>
</table>

1.0 in. = 25.4 mm.
4.1.5.3 Drift and P-delta effects. Using the basic structural configuration shown in Figure 4.1-1 and the equivalent lateral forces shown in Table 4.1-4, the total story deflections were computed as described in the previous section. In this section, story drifts are computed and compared to the allowable drifts specified by the Standard.

For structures with “significant torsional effects”, Standard Section 12.12.1 requires that the maximum drifts include torsional effects, meaning that the accidental torsion, amplified as appropriate, must be included in the drift analysis. The same section of the Standard requires that deflections used to compute drift should be taken at the edges of the structure if the structure is torsionally irregular. For torsionally regular buildings, the drifts may be based on deflections at the center of mass of adjacent levels.

As the structure under consideration is only marginally irregular in torsion, the lateral loads were placed at the center of mass and total drifts are based on center of mass deflections and not deflections at the edges of the floor plate. Using the centers of mass of the floor plates to compute story drift is awkward where the centers of mass of the upper and lower floor plates are not aligned vertically. For this reason, the story drift is computed as the difference between displacements of the center of mass of the upper level diaphragm and the displacement at a point on the lower diaphragm which is located directly below the center of mass of the upper level diaphragm. Note that computation of drift in this manner has been adopted in ASCE 7-10 Section 12.8-6.

The values in Column 1 of Tables 4.1-7 and 4.1-8 are the total story displacements ($\delta$) at the center of mass of the story as reported by SAP2000 and the values in Column 2 are the story drifts ($\Delta$) computed from these numbers in the manner described earlier. The true elastic “amplified” story drift, which by assumption is equal to $C_d (= 5.5)$ times the SAP2000 drift, is shown in Column 3. As discussed above in Section 4.1.5.1, the values in Column 4 are multiplied by 0.568 to scale the results to the base shear computed using Standard Equation 12.8-3.

The allowable story drift of 2.0 percent of the story height per Standard Table 12.12-1 is shown in Column 5. (Recall that this building is assigned to Occupancy Category II.) It is clear from Tables 4.1-7 and 4.1-8 that the allowable drift is not exceeded at any level. It is also evident that the allowable drifts

<table>
<thead>
<tr>
<th>Level</th>
<th>$\delta_1$ (in.)</th>
<th>$\delta_2$ (in.)</th>
<th>$\delta_{avg}$ (in.)</th>
<th>$\delta_{max}$ (in.)</th>
<th>$A_x$, calculated</th>
<th>$A_x$, used</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
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<td>4.77</td>
<td>4.98</td>
<td>5.19</td>
<td>0.75</td>
<td>1.00</td>
</tr>
<tr>
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<td>5.03</td>
<td>4.63</td>
<td>4.83</td>
<td>5.03</td>
<td>0.75</td>
<td>1.00</td>
</tr>
<tr>
<td>11</td>
<td>4.79</td>
<td>4.40</td>
<td>4.59</td>
<td>4.79</td>
<td>0.76</td>
<td>1.00</td>
</tr>
<tr>
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<td>4.11</td>
<td>4.29</td>
<td>4.48</td>
<td>0.76</td>
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<tr>
<td>9</td>
<td>4.10</td>
<td>3.55</td>
<td>3.82</td>
<td>4.10</td>
<td>0.80</td>
<td>1.00</td>
</tr>
<tr>
<td>8</td>
<td>3.64</td>
<td>3.26</td>
<td>3.45</td>
<td>3.64</td>
<td>0.77</td>
<td>1.00</td>
</tr>
<tr>
<td>7</td>
<td>3.09</td>
<td>2.90</td>
<td>3.00</td>
<td>3.09</td>
<td>0.74</td>
<td>1.00</td>
</tr>
<tr>
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<td>2.51</td>
<td>2.52</td>
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<tr>
<td>5</td>
<td>1.95</td>
<td>2.09</td>
<td>2.02</td>
<td>2.09</td>
<td>0.74</td>
<td>1.00</td>
</tr>
<tr>
<td>4</td>
<td>1.53</td>
<td>1.62</td>
<td>1.58</td>
<td>1.62</td>
<td>0.73</td>
<td>1.00</td>
</tr>
<tr>
<td>3</td>
<td>1.07</td>
<td>1.12</td>
<td>1.10</td>
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<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>0.60</td>
<td>0.63</td>
<td>0.61</td>
<td>0.63</td>
<td>0.73</td>
<td>1.00</td>
</tr>
</tbody>
</table>

1.0 in. = 25.4 mm.
would not have been exceeded even if accidental torsion effects were included in the drift calculations, with the drift determined at the edge of the building.

Table 4.1-7 ELF Drift for Building Responding in X Direction

<table>
<thead>
<tr>
<th>Level</th>
<th>1 Total drift from SAP2000 (in.)</th>
<th>2 Story drift from SAP2000 (in.)</th>
<th>3 Amplified story drift (in.)</th>
<th>4 Amplified drift times 0.568 (in.)</th>
<th>5 Allowable drift (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>6.67</td>
<td>0.32</td>
<td>1.74</td>
<td>0.99</td>
<td>3.00</td>
</tr>
<tr>
<td>12</td>
<td>6.35</td>
<td>0.45</td>
<td>2.48</td>
<td>1.41</td>
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</tr>
<tr>
<td>11</td>
<td>5.90</td>
<td>0.56</td>
<td>3.07</td>
<td>1.75</td>
<td>3.00</td>
</tr>
<tr>
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<td>5.34</td>
<td>0.62</td>
<td>3.39</td>
<td>1.92</td>
<td>3.00</td>
</tr>
<tr>
<td>9</td>
<td>4.73</td>
<td>0.58</td>
<td>3.20</td>
<td>1.82</td>
<td>3.00</td>
</tr>
<tr>
<td>8</td>
<td>4.15</td>
<td>0.63</td>
<td>3.47</td>
<td>1.97</td>
<td>3.00</td>
</tr>
<tr>
<td>7</td>
<td>3.52</td>
<td>0.64</td>
<td>3.54</td>
<td>2.01</td>
<td>3.00</td>
</tr>
<tr>
<td>6</td>
<td>2.87</td>
<td>0.63</td>
<td>3.47</td>
<td>1.97</td>
<td>3.00</td>
</tr>
<tr>
<td>5</td>
<td>2.24</td>
<td>0.54</td>
<td>2.95</td>
<td>1.67</td>
<td>3.00</td>
</tr>
<tr>
<td>4</td>
<td>1.71</td>
<td>0.54</td>
<td>2.97</td>
<td>1.69</td>
<td>3.00</td>
</tr>
<tr>
<td>3</td>
<td>1.17</td>
<td>0.53</td>
<td>2.90</td>
<td>1.65</td>
<td>3.00</td>
</tr>
<tr>
<td>2</td>
<td>0.64</td>
<td>0.64</td>
<td>3.51</td>
<td>2.00</td>
<td>4.32</td>
</tr>
</tbody>
</table>

Column 4 adjusts for Standard Eq. 12.8-3 (for drift) vs 12.8-5 (for strength). 1.0 in. = 25.4 mm.

Table 4.1-8 ELF Drift for Building Responding in Y Direction

<table>
<thead>
<tr>
<th>Level</th>
<th>1 Total drift from SAP2000 (in.)</th>
<th>2 Story drift from SAP2000 (in.)</th>
<th>3 Amplified story drift (in.)</th>
<th>4 Amplified drift times 0.568 (in.)</th>
<th>5 Allowable drift (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>4.86</td>
<td>0.15</td>
<td>0.81</td>
<td>0.46</td>
<td>3.00</td>
</tr>
<tr>
<td>12</td>
<td>4.71</td>
<td>0.24</td>
<td>1.30</td>
<td>0.74</td>
<td>3.00</td>
</tr>
<tr>
<td>11</td>
<td>4.47</td>
<td>0.30</td>
<td>1.64</td>
<td>0.93</td>
<td>3.00</td>
</tr>
<tr>
<td>10</td>
<td>4.17</td>
<td>0.36</td>
<td>1.96</td>
<td>1.11</td>
<td>3.00</td>
</tr>
<tr>
<td>9</td>
<td>3.82</td>
<td>0.37</td>
<td>2.05</td>
<td>1.16</td>
<td>3.00</td>
</tr>
<tr>
<td>8</td>
<td>3.44</td>
<td>0.46</td>
<td>2.54</td>
<td>1.44</td>
<td>3.00</td>
</tr>
<tr>
<td>7</td>
<td>2.98</td>
<td>0.48</td>
<td>2.64</td>
<td>1.50</td>
<td>3.00</td>
</tr>
<tr>
<td>6</td>
<td>2.50</td>
<td>0.48</td>
<td>2.62</td>
<td>1.49</td>
<td>3.00</td>
</tr>
<tr>
<td>5</td>
<td>2.03</td>
<td>0.45</td>
<td>2.49</td>
<td>1.42</td>
<td>3.00</td>
</tr>
<tr>
<td>4</td>
<td>1.57</td>
<td>0.48</td>
<td>2.66</td>
<td>1.51</td>
<td>3.00</td>
</tr>
<tr>
<td>3</td>
<td>1.09</td>
<td>0.48</td>
<td>2.64</td>
<td>1.50</td>
<td>3.00</td>
</tr>
<tr>
<td>2</td>
<td>0.61</td>
<td>0.61</td>
<td>3.35</td>
<td>1.90</td>
<td>4.32</td>
</tr>
</tbody>
</table>

Column 4 adjusts for Standard Eq. 12.8-3 (for drift) versus Eq. 12.8-5 (for strength). 1.0 in. = 25.4 mm.
4.1.5.3.1 Using ELF forces and drift to compute accurate period. Before continuing with the example, it is advisable to use the computed drifts to more accurately estimate the fundamental periods of vibration of the building. This will serve as a check on the “exact” periods computed by eigenvalue extraction in SAP2000. A Rayleigh analysis will be used to estimate the periods. This procedure, which usually is very accurate, is derived as follows:

The exact frequency of vibration $\omega$ (a scalar), in units of radians/second, is found from the following eigenvalue equation:

$$K\phi = \omega^2 M \phi$$

where $K$ is the structure stiffness matrix, $M$ is the (diagonal) mass matrix and $\phi$ is a vector containing the components of the mode shape associated with $\omega$.

If an approximate mode shape $\delta$ is used instead of $\phi$, where $\delta$ is the deflected shape under the equivalent lateral forces $F$, the frequency $\omega$ can be closely approximated. Making the substitution of $\delta$ for $\phi$, premultiplying both sides of the above equation by $\delta^T$ (the transpose of the displacement vector), noting that $F = K\delta$ and $M = W/g$, the following is obtained:

$$\delta^T F = \omega^2 \delta^T M \delta = \frac{\omega^2}{g} \delta^T W \delta$$

where $W$ is a vector containing the story weights and $g$ is the acceleration due to gravity (a scalar). After rearranging terms, this gives:

$$\omega = \sqrt{\frac{g}{\delta^T W \delta}}$$

Using the relationship between period and frequency, $T = \frac{2\pi}{\omega}$

Using $F$ from Table 4.1-4 and $\delta$ from Column 1 of Tables 4.1-7 and 4.1-8, the periods of vibration are computed as shown in Tables 4.1-9 and 4.1-10 for the structure loaded in the X and Y directions, respectively. As may be seen from the tables, the X direction period of 2.85 seconds and the Y direction period of 2.56 seconds are significantly greater than the approximate period of $T_a = 1.59$ seconds and also exceed the upper limit on period of $C_u T_a = 2.23$ seconds.

Table 4.1-9 Rayleigh Analysis for X Direction Period of Vibration

<table>
<thead>
<tr>
<th>Level</th>
<th>Drift, $\delta$ (in.)</th>
<th>Force, $F$ (kips)</th>
<th>Weight, $W$ (kips)</th>
<th>$\delta F$ (in.-kips)</th>
<th>$\delta W/g$ (in.-kips-sec$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>6.67</td>
<td>186.9</td>
<td>1,657</td>
<td>1,247</td>
<td>191</td>
</tr>
<tr>
<td>12</td>
<td>6.35</td>
<td>154.0</td>
<td>1,596</td>
<td>979</td>
<td>167</td>
</tr>
<tr>
<td>11</td>
<td>5.90</td>
<td>129.9</td>
<td>1,596</td>
<td>767</td>
<td>144</td>
</tr>
<tr>
<td>10</td>
<td>5.34</td>
<td>107.6</td>
<td>1,596</td>
<td>575</td>
<td>118</td>
</tr>
<tr>
<td>9</td>
<td>4.73</td>
<td>186.3</td>
<td>3,403</td>
<td>881</td>
<td>197</td>
</tr>
<tr>
<td>8</td>
<td>4.15</td>
<td>100.8</td>
<td>2,331</td>
<td>418</td>
<td>104</td>
</tr>
</tbody>
</table>
### Table 4.1-9 Rayleigh Analysis for X Direction Period of Vibration

<table>
<thead>
<tr>
<th>Level</th>
<th>Drift, δ (in.)</th>
<th>Force, F (kips)</th>
<th>Weight, W (kips)</th>
<th>δF (in.-kips)</th>
<th>δW/g (in.-kips-sec²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>3.52</td>
<td>77.0</td>
<td>2,331</td>
<td>271</td>
<td>75</td>
</tr>
<tr>
<td>6</td>
<td>2.87</td>
<td>56.2</td>
<td>2,331</td>
<td>162</td>
<td>50</td>
</tr>
<tr>
<td>5</td>
<td>2.24</td>
<td>71.4</td>
<td>4,324</td>
<td>160</td>
<td>56</td>
</tr>
<tr>
<td>4</td>
<td>1.71</td>
<td>31.5</td>
<td>3,066</td>
<td>54</td>
<td>23</td>
</tr>
<tr>
<td>3</td>
<td>1.17</td>
<td>16.6</td>
<td>3,066</td>
<td>19</td>
<td>11</td>
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<tr>
<td>2</td>
<td>0.64</td>
<td>6.3</td>
<td>3,097</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Σ</td>
<td></td>
<td></td>
<td></td>
<td>5,536</td>
<td>1,138</td>
</tr>
</tbody>
</table>

\( \omega = (5,536/1,138)^{0.5} = 2.21 \text{ rad/sec.} \quad T = 2\pi/\omega = 2.85 \text{ sec.} \)

1.0 in. = 25.4 mm, 1.0 kip = 4.45 kN.

### Table 4.1-10 Rayleigh Analysis for Y Direction Period of Vibration

<table>
<thead>
<tr>
<th>Level</th>
<th>Drift, δ (in.)</th>
<th>Force, F (kips)</th>
<th>Weight, W (kips)</th>
<th>δF (in.-kips)</th>
<th>δW/g (in.-kips-sec²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>4.86</td>
<td>186.9</td>
<td>1,657</td>
<td>908</td>
<td>101</td>
</tr>
<tr>
<td>12</td>
<td>4.71</td>
<td>154.0</td>
<td>1,596</td>
<td>725</td>
<td>92</td>
</tr>
<tr>
<td>11</td>
<td>4.47</td>
<td>129.9</td>
<td>1,596</td>
<td>581</td>
<td>83</td>
</tr>
<tr>
<td>10</td>
<td>4.17</td>
<td>107.6</td>
<td>1,596</td>
<td>449</td>
<td>72</td>
</tr>
<tr>
<td>9</td>
<td>3.82</td>
<td>186.3</td>
<td>3,403</td>
<td>711</td>
<td>128</td>
</tr>
<tr>
<td>8</td>
<td>3.44</td>
<td>100.8</td>
<td>2,331</td>
<td>347</td>
<td>72</td>
</tr>
<tr>
<td>7</td>
<td>2.98</td>
<td>77.0</td>
<td>2,331</td>
<td>230</td>
<td>54</td>
</tr>
<tr>
<td>6</td>
<td>2.50</td>
<td>56.2</td>
<td>2,331</td>
<td>141</td>
<td>38</td>
</tr>
<tr>
<td>5</td>
<td>2.03</td>
<td>71.4</td>
<td>4,324</td>
<td>145</td>
<td>46</td>
</tr>
<tr>
<td>4</td>
<td>1.57</td>
<td>31.5</td>
<td>3,066</td>
<td>49</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>1.09</td>
<td>16.6</td>
<td>3,066</td>
<td>18</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>0.61</td>
<td>6.3</td>
<td>3,097</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Σ</td>
<td></td>
<td></td>
<td></td>
<td>4,307</td>
<td>716</td>
</tr>
</tbody>
</table>

\( \omega = (4,307/716)^{0.5} = 2.45 \text{ rad/sec.} \quad T = 2\pi/\omega = 2.56 \text{ sec.} \)

1.0 in. = 25.4 mm, 1.0 kip = 4.45 kN.

#### 4.1.5.3.2 P-delta effects

P-delta effects are computed for the X direction response in Table 4.1-11. The last column of the table shows the story stability ratio computed according to Standard Equation 12.8-16:

\[
\theta = \frac{P_{d} \Delta I}{V_{x} h_{xx} C_{d}}
\]

Note that the \( I \) in the numerator of Equation 12.8-16 was inadvertently omitted in early printings of the Standard.
Standard Equation 12.8-17 places an upper limit on \( \theta \):

\[
\theta_{\text{max}} = \frac{0.5}{\beta C_d}
\]

where \( \beta \) is the ratio of shear demand to shear capacity for the story. Conservatively taking \( \beta = 1.0 \) and using \( C_d = 5.5 \), \( \theta_{\text{max}} = 0.091 \).

The \( \Delta \) terms in Table 4.1-11 are taken from the fourth column of Table 4.1-7 because these are consistent with the ELF story shears of Table 4.1-4 and thereby represent the true lateral stiffness of the system. (If 0.568 times the story drifts were used, then 0.568 times the story shears also would need to be used. Hence, the 0.568 factor would cancel out since it would appear in both the numerator and denominator.)

The deflections used in P-delta stability ratio calculations must include the deflection amplifier \( C_d \).

The live load \( P_L \) in Table 4.1-11 is based on a 20 psf uniform live load over 100 percent of the floor and roof area. This live load is somewhat conservative because Section 12.8.7 of the Standard states that the gravity load should be the “total vertical design load”. For a 50 psf live load for office buildings, a live load reduction factor of 0.4 would be applicable for each level (see Standard Sec. 4.8), producing a reduced live load of 20 psf at the floor levels. This could be further reduced by a factor of 0.5 as allowed by Section 2.3.2, bringing the effective live load to 10 psf. This value is close to the mean survey live load of 10.9 psf for office buildings, as listed in Table C4.2 of the Standard. Several publications, including ASCE 41 include 25 percent of the unreduced live load in P-delta calculations. This would result in a 12.5 psf live load for the current example.

<table>
<thead>
<tr>
<th>Level</th>
<th>( h_{sx} ) (in.)</th>
<th>( \Delta ) (in.)</th>
<th>( P_D ) (kips)</th>
<th>( P_L ) (kips)</th>
<th>( P_T ) (kips)</th>
<th>( P_X ) (kips)</th>
<th>( V_X ) (kips)</th>
<th>( \theta_X )</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>150</td>
<td>1.74</td>
<td>1,656.5</td>
<td>315.0</td>
<td>1,971.5</td>
<td>1,971.5</td>
<td>186.9</td>
<td>0.022</td>
</tr>
<tr>
<td>12</td>
<td>150</td>
<td>2.48</td>
<td>1,595.8</td>
<td>315.0</td>
<td>1,910.8</td>
<td>3,882.3</td>
<td>340.9</td>
<td>0.034</td>
</tr>
<tr>
<td>11</td>
<td>150</td>
<td>3.07</td>
<td>1,595.8</td>
<td>315.0</td>
<td>1,910.8</td>
<td>5,793.1</td>
<td>470.8</td>
<td>0.046</td>
</tr>
<tr>
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<td>150</td>
<td>3.39</td>
<td>1,595.8</td>
<td>315.0</td>
<td>1,910.8</td>
<td>7,703.9</td>
<td>578.4</td>
<td>0.055</td>
</tr>
<tr>
<td>9</td>
<td>150</td>
<td>3.20</td>
<td>3,403.0</td>
<td>465.0</td>
<td>3,868.0</td>
<td>11,571.9</td>
<td>764.7</td>
<td>0.059</td>
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<td>3.47</td>
<td>2,330.8</td>
<td>465.0</td>
<td>2,795.8</td>
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<td>0.070</td>
</tr>
<tr>
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<td>150</td>
<td>3.54</td>
<td>2,330.8</td>
<td>465.0</td>
<td>2,795.8</td>
<td>17,163.5</td>
<td>942.5</td>
<td>0.078</td>
</tr>
<tr>
<td>6</td>
<td>150</td>
<td>3.47</td>
<td>2,330.8</td>
<td>465.0</td>
<td>2,795.8</td>
<td>19,959.3</td>
<td>998.8</td>
<td>0.084</td>
</tr>
<tr>
<td>5</td>
<td>150</td>
<td>2.95</td>
<td>4,323.8</td>
<td>615.0</td>
<td>4,938.8</td>
<td>24,898.1</td>
<td>1,070.2</td>
<td>0.083</td>
</tr>
<tr>
<td>4</td>
<td>150</td>
<td>2.97</td>
<td>3,066.1</td>
<td>615.0</td>
<td>3,681.1</td>
<td>28,579.2</td>
<td>1,101.7</td>
<td>0.093</td>
</tr>
<tr>
<td>3</td>
<td>150</td>
<td>2.90</td>
<td>3,066.1</td>
<td>615.0</td>
<td>3,681.1</td>
<td>32,260.3</td>
<td>1,118.2</td>
<td>0.101</td>
</tr>
<tr>
<td>2</td>
<td>216</td>
<td>3.51</td>
<td>3,097.0</td>
<td>615.0</td>
<td>3,712.0</td>
<td>35,972.3</td>
<td>1,124.5</td>
<td>0.095</td>
</tr>
</tbody>
</table>

1.0 in. = 25.4 mm, 1.0 kip = 4.45 kN.

The stability ratio just exceeds 0.091 at Levels 2 through 4. However, the live loads were somewhat conservative and \( \beta \) was very conservatively taken as 1.0. Because a more refined analysis would provide somewhat lower live loads and a lower value of \( \beta \), we will proceed assuming that P-delta effects are not a
problem for this structure. Calculations for P-delta effects under Y direction loading gave no story stability ratios greater than 0.091 and for brevity, those results are not included herein.

It is important to note that for this structure, P-delta effects are a potential issue even though drift limits were easily satisfied. This is often the case when drift limits are based on lateral loads that have been computed using the computed period of vibration (without the $C_uT_a$ limit, or without the use of Equation 12.8.5). It is the authors’ experience that this is a typical situation in the analysis of steel special steel moment frame systems.

Part 1 of the Provisions recommends that a significant change be made to the current P-delta approach. In the recommended approach, Equation 12.8-16 is still used to determine the magnitude of P-delta effects. However, if the stability ratio at any level is greater than 0.1, the designer must either redesign the building such that the stability ratio is less than 0.1, or must perform a static pushover analysis and demonstrate that the slope of the pushover curve of the structure is continuously positive up to the “target displacement” computed in accordance with the requirements of ASCE 41.

4.1.5.4 Computation of member forces. Before member forces may be computed, the proper load cases and combinations of load must be identified such that all critical seismic effects are captured in the analysis.

4.1.5.4.1 Orthogonal loading effects and accidental torsion. For SDC D structures with a Type 5 horizontal structural irregularity, Section 12.5.3 of the Standard requires that orthogonal load effects be considered. For the purposes of this example, it is assumed that such an irregularity does exist because the layout of the frames is not symmetric. (ASCE 7-10 has eliminated non-symmetry as a trigger for invoking the nonparallel system horizontal irregularity. However, as the structural system under consideration has several intersecting frames, it would be advisable to perform the orthogonal load analysis as required under Section 12.5.4 of both the 2005 and 2010 versions of the Standard.)

When orthogonal load effects are included in the analysis, four directions of seismic force (+X, -X, +Y, -Y) must be considered and for each direction of force, there are two possible directions in which the accidental eccentricity can apply (causing positive or negative torsion). This requires a total of eight possible combinations of direct force plus accidental torsion. Where the 30 percent orthogonal loading rule is applied (see Standard Sec. 12.5.3 Item “a”), the number of load combinations increases to 16 because, for each direct application of load, a positive or negative orthogonal loading can exist. Orthogonal loads are applied without accidental eccentricity.

Figure 4.1-8 illustrates the basic possibilities of application of load. Although this figure shows 16 different load conditions, it may be observed that eight of these conditions—7, 8, 5, 6, 15, 16, 13 and 14—are negatives (opposite signs throughout) of conditions 1, 2, 3, 4, 9, 10, 11 and 12, respectively.
4.1.5.4.2 Load combinations. The basic load combinations for this structure are designated in Chapter 2 of the Standard. Two sets of combinations are provided: one for strength design and the other for allowable stress design. The strength-based combinations that are related to seismic effects are the following:

\[ 1.2D + 1.0E + 1.0L + 0.2S \]

\[ 0.9D + 1.0E + 1.6H \]

The factor on live load, \( L \), may be reduced to 0.5 if the nominal live load is less than 100 psf. The load due to lateral earth pressure, \( H \), may need to be considered when designing the basement walls.
Section 12.4 of the *Standard* divides the earthquake load, $E$, into two components, $E_h$ and $E_v$, where the subscripts $h$ and $v$ represent horizontal and vertical seismic effects, respectively. These components are defined as follows:

$$E_h = \rho Q_E$$

$$E_v = 0.2S_{DS}D$$

where $Q_E$ is the earthquake load effect and $\rho$ is a redundancy factor, described later.

When the above components are substituted into the basic load combinations, the load combinations for strength design with a factor of 0.5 used for live load and with the $H$ load removed are as follows:

$$(1.2 + 0.2S_{DS})D + \rho Q_E + 0.5L + 0.2S$$

$$(0.9 - 0.2S_{DS})D + \rho Q_E$$

Using $S_{DS} = 0.833$ and assuming the snow load is negligible in Stockton, California, the basic load combinations for strength design become:

$$1.37D + 0.5L + \rho Q_E$$

$$0.73D + \rho Q_E$$

The redundancy factor, $\rho$, is determined in accordance with *Standard* Section 12.3.4. This factor will take a value of 1.0 or 1.3, with the value depending on a variety of conditional tests. None of the conditions specified in Section 12.3.4.1 are applicable, so $\rho$ may not be automatically taken as 1.0, and the more detailed evaluation specified in Section 12.3.4.2 is required. Subparagraph “b” of Section 12.3.4.2 applies to this building (because of the plan irregularities) and therefore, the evaluation described in the second row of Table 12.3-3 must be performed. It can be seen from inspection that the removal of a single beam from the perimeter moment frames will not cause a reduction in strength of 33 percent, nor will an extreme torsional irregularly result from the removal of the beam. Hence, the redundancy factor may be taken as 1.0 for this structure.

Hence, the final load conditions to be used for design are as follows:

$$1.37D + 0.5L + 1.0Q_E$$

$$0.73D + 1.0Q_E$$

The first load condition will produce the maximum negative moments (tension on the top) at the face of the supports in the girders and maximum compressive forces in columns. The second load condition will produce the maximum positive moments (or minimum negative moment) at the face of the supports of the girders and maximum tension (or minimum compression) in the columns. In addition to the above load condition, the gravity-only load combinations as specified in the *Standard* also must be checked. Due to the relatively short spans in the moment frames, however, it is not expected that the non-seismic load combinations will control.
4.1.5.4.3 Setting up the load combinations in SAP2000. The load combinations required for the analysis are shown in Table 4.1-12.

It should be noted that 32 different load combinations are required only if one wants to maintain the signs in the member force output, thereby providing complete design envelopes for all members. As mentioned later, these signs are lost in response spectrum analysis and as a result, it is possible to capture the effects of dead load plus live load plus-or-minus earthquake load in a single SAP2000 run containing only four load combinations.

<table>
<thead>
<tr>
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<td></td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>[14]</td>
<td></td>
</tr>
</tbody>
</table>

*Numbers in brackets [ ] in represent load cases shown in Figure 4.1-8.
4.1.5.4.4 Member forces. For this portion of the analysis, the earthquake shears in the girders along Gridline 1 are computed. This analysis considers 100 percent of the X direction forces applied in combination with 30 percent of the (positive or negative) Y direction forces. The accidental torsion is not included and will be considered separately. The results of the member force analysis are shown in Figure 4.1-9a. In a later part of this example, the girder shears are compared to those obtained from modal response spectrum and modal response history analyses.

Beam shears in the same frame, due to accidental torsion only, are shown in Figure 4.1-9b. The eccentricity was set to produce clockwise torsions (when viewed from above) on the floor plates. These shears would be added to the shears shown in Figure 4.1-9a to produce the total seismic beam shears in the frame. The same torsional shears (from Table 4.1-9b) will be used in the modal response spectrum and modal response history analyses.

<table>
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<td>18.9</td>
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<td>28.1</td>
<td>29.5</td>
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<td>11-10</td>
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<td>33.1</td>
<td>35.7</td>
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<td>10-9</td>
<td>34.8</td>
<td>34.7</td>
<td>32.2</td>
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<td>9-8</td>
<td>36.4</td>
<td>35.9</td>
<td>33.9</td>
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<td>8-7</td>
<td>41.2</td>
<td>40.1</td>
<td>38.4</td>
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<td>7-6</td>
<td>43.0</td>
<td>40.6</td>
<td>39.3</td>
</tr>
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<td>6-5</td>
<td>14.1</td>
<td>33.1</td>
<td>33.8</td>
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<td>33.3</td>
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<td>22.9</td>
<td>36.9</td>
<td>34.1</td>
</tr>
<tr>
<td>2 - G</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure 4.1-9a** Seismic shears (kips) in girders on Frame Line 1 as computed using ELF analysis (analysis includes orthogonal loading but excludes accidental torsion)
Chapter 4: Structural Analysis

4.1.6 Modal Response Spectrum Analysis

The first step in the modal response spectrum analysis is the computation of the natural mode shapes and associated periods of vibration. Using the structural masses from Table 4.1-4 and the same mathematical model as used for the ELF and the Rayleigh analyses, the mode shapes and frequencies are automatically computed by SAP2000. This mathematical model included the basement as a separate level. (See Section 4.1.5.2 of this example for a description of the mathematical model used in the analysis). The basement walls were fixed at the base but were unrestrained at grade level. Thus, the basement level is treated as a separate story in the analysis. However, the lateral stiffness of the basement level is significantly greater than that of the upper levels and this causes complications when interpreting the requirements of Standard Section 12.9.1. As shown later, the explicit modeling of the basement can also lead to some unexpected results in the modal response history analysis of the structure.

The periods of vibration for the first 12 modes, computed from an eigenvalue analysis, are summarized in the second column of Table 4.1-13. The first eight mode shapes are shown in Figure 4.1-10. The first mode period, 2.87 seconds, corresponds to vibration primarily in the X direction and the second period, 2.60 seconds, corresponds to vibration in the Y direction. The third mode, with a period of 1.57 seconds, is almost purely torsion. The directionality of the modes may be inferred from the effective mass values shown in Columns 3 through 5 of the tables, as well as from the mode shapes. There is very little lateral-torsional coupling in any of the first 12 modes, which is somewhat surprising because of the shifted centers of mass associated with the plan offsets.

Figure 4.1-9b Seismic shears in girders (kips) from clockwise torsion only
The X- and Y-translation periods of 2.87 and 2.60 seconds, respectively, are somewhat longer than the upper limit on the approximate period, $C_u T_{ap}$ of 2.23 seconds.

The first and second mode periods are virtually identical to the periods computed by Rayleigh analysis (2.85 and 2.56 seconds in the X and Y directions, respectively). The closeness of the Rayleigh and eigenvalue periods for this building arises from the fact that the first and second modes of vibration act primarily along the orthogonal axes. Had the first and second modes not acted along the orthogonal axes, the Rayleigh periods (based on loads and displacements in the X and Y directions) would have been somewhat less accurate.

Standard Section 12.9.1 specifies that “the analysis shall include a sufficient number of modes to obtain a combined modal mass participation of at least 90 percent of the total mass in each of the orthogonal horizontal directions of response considered by the model”. Usually, this is a straightforward requirement and the first twelve modes would be sufficient for a 12-story building. For this building, however, twelve modes capture only about 82 percent of the X and Y direction mass. (The effective mass as a fraction of total mass is shown in brackets [ ] in Columns 3 through 5 of Tables 4.1-13 and 4.1-14.) Most of the remaining effective mass is in the grade-level slab and in the basement walls. This mass does not show up until Mode 112 in the Y direction and Mode 118 in the X direction. This is shown in Table 12.1-14, which provides the periods and effective modal masses in Modes 108 through 119. The intermediate modes (13 through 107) represent primarily vertical vibration of various portions of the floor diaphragms.

Analyzing the system with 120 or more modes might provide useful information on the response of the basement level, including shears through the basement and total system base shears at the base of the basement. However, there would be some difficulty in interpreting the results because the model did not include sub-grade soil that would be in contact with the basement walls and which would absorb part of the base shear. Additionally, the computed response of the upper 12 levels of the building, which is the main focus of this analysis, is virtually identical for the 12 and the 120 mode analyses. For this reason, the modal response spectrum analysis discussed in this example was run with only the first 12 modes listed in Table 4.1-13.
Chapter 4: Structural Analysis

Figure 4.1-10 First eight mode shapes

Table 4.1-13 Computed Periods and Effective Mass Factors (Lower Modes)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Period (sec.)</th>
<th>X Translation</th>
<th>Y Translation</th>
<th>Z Rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.87</td>
<td>0.6446 [0.64]</td>
<td>0.0003 [0.00]</td>
<td>0.0028 [0.00]</td>
</tr>
<tr>
<td>2</td>
<td>2.60</td>
<td>0.0003 [0.65]</td>
<td>0.6804 [0.68]</td>
<td>0.0162 [0.02]</td>
</tr>
<tr>
<td>3</td>
<td>1.57</td>
<td>0.0035 [0.65]</td>
<td>0.0005 [0.68]</td>
<td>0.5806 [0.60]</td>
</tr>
<tr>
<td>4</td>
<td>1.15</td>
<td>0.1085 [0.76]</td>
<td>0.0000 [0.68]</td>
<td>0.0000 [0.60]</td>
</tr>
<tr>
<td>5</td>
<td>0.975</td>
<td>0.0000 [0.76]</td>
<td>0.0939 [0.78]</td>
<td>0.0180 [0.62]</td>
</tr>
<tr>
<td>6</td>
<td>0.705</td>
<td>0.0263 [0.78]</td>
<td>0.0000 [0.78]</td>
<td>0.0271 [0.64]</td>
</tr>
<tr>
<td>7</td>
<td>0.682</td>
<td>0.0056 [0.79]</td>
<td>0.0006 [0.79]</td>
<td>0.0687 [0.71]</td>
</tr>
<tr>
<td>8</td>
<td>0.573</td>
<td>0.0000 [0.79]</td>
<td>0.0188 [0.79]</td>
<td>0.0123 [0.73]</td>
</tr>
</tbody>
</table>
4.1.6.1 Response spectrum coordinates and computation of modal forces. The coordinates of the response spectrum are based on Standard Section 11.4.5. This spectrum consists of three parts (for periods less than $T_L = 8.0$ seconds) as follows:

- For periods less than $T_0$:
  \[ S_a = 0.6 \frac{S_{DS}}{T_0} T + 0.4 S_{DS} \]

- For periods between $T_0$ and $T_S$:
  \[ S_a = S_{DS} \]

- For periods greater than $T_S$:
  \[ S_a = \frac{S_{D1}}{T} \]

where $T_0 = 0.2 S_{D1}/S_{DS}$ and $T_S = S_D/S_{DS}$. 

---

**Table 4.1-13** Computed Periods and Effective Mass Factors (Lower Modes)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Period (sec.)</th>
<th>Effective Mass Factor [Accum Mass Factor]</th>
<th>X Translation</th>
<th>Y Translation</th>
<th>Z Rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>0.434</td>
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<td>0.0129 [0.80]</td>
<td>0.0000 [0.79]</td>
<td>0.0084 [0.73]</td>
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<tr>
<td>10</td>
<td>0.387</td>
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<td>0.0048 [0.81]</td>
<td>0.0000 [0.79]</td>
<td>0.0191 [0.75]</td>
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<tr>
<td>11</td>
<td>0.339</td>
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<td>0.0000 [0.81]</td>
<td>0.0193 [0.81]</td>
<td>0.0010 [0.75]</td>
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<tr>
<td>12</td>
<td>0.300</td>
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<td>0.0089 [0.82]</td>
<td>0.0000 [0.81]</td>
<td>0.0003 [0.75]</td>
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</tbody>
</table>

**Table 4.1-14** Computed Periods and Effective Mass Factors (Higher Modes)

<table>
<thead>
<tr>
<th>Mode</th>
<th>Period (sec.)</th>
<th>Effective Mass Factor [Accum Effective Mass]</th>
<th>X Translation</th>
<th>Y Translation</th>
<th>Z Rotation</th>
</tr>
</thead>
<tbody>
<tr>
<td>108</td>
<td>0.0693</td>
<td></td>
<td>0.0000 [0.83]</td>
<td>0.0000 [0.83]</td>
<td>0.0000 [0.79]</td>
</tr>
<tr>
<td>109</td>
<td>0.0673</td>
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<td>0.0000 [0.83]</td>
<td>0.0000 [0.83]</td>
<td>0.0000 [0.79]</td>
</tr>
<tr>
<td>110</td>
<td>0.0671</td>
<td></td>
<td>0.0000 [0.83]</td>
<td>0.0354 [0.86]</td>
<td>0.0000 [0.79]</td>
</tr>
<tr>
<td>111</td>
<td>0.0671</td>
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<td>0.0000 [0.83]</td>
<td>0.0044 [0.87]</td>
<td>0.0000 [0.79]</td>
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<td>112</td>
<td>0.0669</td>
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<td>0.0000 [0.83]</td>
<td>0.1045 [0.97]</td>
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<td>113</td>
<td>0.0663</td>
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<td>0.0008 [0.83]</td>
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<td>0.0035 [0.80]</td>
</tr>
<tr>
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<td>0.0000 [0.98]</td>
<td>0.0000 [0.97]</td>
<td>0.0000 [0.80]</td>
</tr>
</tbody>
</table>
Using \( S_{DS} = 0.833 \) and \( S_{Df} = 0.373 \), \( T_S = 0.448 \) seconds and \( T_0 = 0.089 \) seconds. The computed response spectrum coordinates for several period values are shown in Table 4.1-15 and the response spectrum, shown with and without the \( I/R = 1/8 \) modification, is plotted in Figure 4.1-11. The spectrum does not include the high period limit on \( C_s \left( 0.044 I S_{DS} \right) \), which controlled the ELF base shear for this structure and which ultimately will control the scaling of the results from the response spectrum analysis. (Recall that if the computed base shear falls below 85 percent of the ELF base shear, the computed response must be scaled up such that the computed base shear equals 85 percent of the ELF base shear.)

**Table 4.1-15** Response Spectrum Coordinates

<table>
<thead>
<tr>
<th>( T_m ) (sec.)</th>
<th>( S_a )</th>
<th>( S_a(I/R) )</th>
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</thead>
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<tr>
<td>0.000</td>
<td>0.333</td>
<td>0.0416</td>
</tr>
<tr>
<td>0.089 (( T_0 ))</td>
<td>0.833</td>
<td>0.104</td>
</tr>
<tr>
<td>0.448 (( T_S ))</td>
<td>0.833</td>
<td>0.104</td>
</tr>
<tr>
<td>1.000</td>
<td>0.373</td>
<td>0.0446</td>
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<tr>
<td>1.500</td>
<td>0.249</td>
<td>0.0311</td>
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<tr>
<td>2.000</td>
<td>0.186</td>
<td>0.0235</td>
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<td>2.500</td>
<td>0.149</td>
<td>0.0186</td>
</tr>
<tr>
<td>3.000</td>
<td>0.124</td>
<td>0.0155</td>
</tr>
</tbody>
</table>

\( I = 1, R = 8.0. \)

**Figure 4.1-11** Total acceleration response spectrum used in analysis

Using the response spectrum coordinates listed in Column 3 of Table 4.1-15, the response spectrum analysis was carried out using SAP2000. As mentioned above, the first 12 modes of response were computed and superimposed using the CQC approach. A modal damping ratio of 5 percent of critical was used in the CQC calculations.
Two analyses were carried out. The first directed the seismic motion along the X axis of the structure and the second directed the motion along the Y axis. Combinations of these two loadings plus accidental torsion are discussed later.

4.1.6.1 Dynamic base shear. After specifying member groups, SAP2000 automatically computes the CQC story shears. Groups were defined such that total shears would be obtained for each story of the structure. The shears at the base of the first story above grade are reported as follows:

- X direction base shear = 438.1 kips
- Y direction base shear = 492.8 kips

These values are much lower that the ELF base shear of 1,124 kips. Recall that the ELF base shear was controlled by Standard Equation 12.8-5. The modal response spectrum shears are less than the ELF shears because the fundamental periods of the structure used in the response spectrum analysis (2.87 seconds and 2.6 seconds in the X and Y directions, respectively) are greater than the upper limit empirical period, $C_{\alpha} T_{\alpha}$, of 2.23 seconds and because the response spectrum of Figure 4.1-11 does not include the minimum base shear limit imposed by Standard Equation 12.8-5.

According to Standard Section 12.9.4, the base shears from the modal response spectrum analysis must not be less than 85 percent of that computed from the ELF analysis. If the response spectrum shears are lower than the ELF shear, then the computed shears must be scaled up such that the response spectrum base shear is 85 percent of that computed from the ELF analysis.

Hence, the required scale factors are as follows:

- X direction scale factor = $0.85(1124)/438.1 = 2.18$
- Y direction scale factor = $0.85(1124)/492.8 = 1.94$

The computed and scaled story shears are as shown in Table 4.1-16. Since the base shears for the ELF and the modal analysis are different (due to the 0.85 factor), direct comparisons cannot be made between Table 4.1-16 and Table 4.1-4. However, it is clear that the vertical distribution of forces is somewhat similar where computed by ELF and modal response spectrum.

<table>
<thead>
<tr>
<th>Story</th>
<th>X Direction (SF = 2.18)</th>
<th>Y Direction (SF = 1.94)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unscaled Shear (kips)</td>
<td>Scaled Shear (kips)</td>
</tr>
<tr>
<td>R-12</td>
<td>82.7</td>
<td>180</td>
</tr>
<tr>
<td>12-11</td>
<td>130.9</td>
<td>286</td>
</tr>
<tr>
<td>11-10</td>
<td>163.8</td>
<td>357</td>
</tr>
<tr>
<td>10-9</td>
<td>191.4</td>
<td>418</td>
</tr>
<tr>
<td>9-8</td>
<td>240.1</td>
<td>524</td>
</tr>
<tr>
<td>8-7</td>
<td>268.9</td>
<td>587</td>
</tr>
<tr>
<td>7-6</td>
<td>292.9</td>
<td>639</td>
</tr>
<tr>
<td>6-5</td>
<td>316.1</td>
<td>690</td>
</tr>
<tr>
<td>5-4</td>
<td>359.5</td>
<td>784</td>
</tr>
</tbody>
</table>
Table 4.1-16 Story Shears from Modal Response Spectrum Analysis

<table>
<thead>
<tr>
<th>Story</th>
<th>X Direction (SF = 2.18)</th>
<th>Y Direction (SF = 1.94)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unscaled Shear (kips)</td>
<td>Scaled Shear (kips)</td>
</tr>
<tr>
<td>4-3</td>
<td>384.8</td>
<td>840</td>
</tr>
<tr>
<td>3-2</td>
<td>401.4</td>
<td>895</td>
</tr>
<tr>
<td>2-G</td>
<td>438.1</td>
<td>956</td>
</tr>
</tbody>
</table>

1.0 kip = 4.45 kN.

4.1.6.2 Drift and P-delta effects. According to Standard Section 12.9.4, the computed displacements and drift (as based on the response spectrum of Figure 4.1-11) need not be scaled by the base shear factors (SF) of 2.18 and 1.94 for the structure loaded in the X and Y directions, respectively. This provides consistency with Section 12.8.6.2, which allows drift from an ELF analysis to be based on the computed period without the upper limit $C_u T_a$.

Section 12.9.4.2 of ASCE 7-10 requires that drifts from a response spectrum analysis be scaled only if Equation 12.8-6 controls the value of $C_S$. In this example, Equation 12.8-5 controlled the base shear, so drifts need not be scaled in ASCE 7-10.

In Tables 4.1-17 and 4.1-18, the story displacement from the response spectrum analysis, the story drift, the amplified story drift (as multiplied by $C_d = 5.5$) and the allowable story drift are listed. As before the story drifts represent the differences in the displacement at the center of mass of one level, and the displacement at vertical projection of that point at the level below. These values were determined in each mode and then combined using CQC. As may be observed from the tables, the allowable drift is not exceeded at any level.

Table 4.1-17 Response Spectrum Drift for Building Responding in X Direction

<table>
<thead>
<tr>
<th>Level</th>
<th>Total Drift from R.S. Analysis (in.)</th>
<th>Story Drift (in.)</th>
<th>Story Drift $\times C_d$ (in.)</th>
<th>Allowable Story Drift (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>2.23</td>
<td>0.12</td>
<td>0.66</td>
<td>3.00</td>
</tr>
<tr>
<td>12</td>
<td>2.10</td>
<td>0.16</td>
<td>0.89</td>
<td>3.00</td>
</tr>
<tr>
<td>11</td>
<td>1.94</td>
<td>0.19</td>
<td>1.03</td>
<td>3.00</td>
</tr>
<tr>
<td>10</td>
<td>1.76</td>
<td>0.20</td>
<td>1.08</td>
<td>3.00</td>
</tr>
<tr>
<td>9</td>
<td>1.56</td>
<td>0.18</td>
<td>0.98</td>
<td>3.00</td>
</tr>
<tr>
<td>8</td>
<td>1.38</td>
<td>0.19</td>
<td>1.06</td>
<td>3.00</td>
</tr>
<tr>
<td>7</td>
<td>1.19</td>
<td>0.20</td>
<td>1.08</td>
<td>3.00</td>
</tr>
<tr>
<td>6</td>
<td>0.99</td>
<td>0.20</td>
<td>1.08</td>
<td>3.00</td>
</tr>
<tr>
<td>5</td>
<td>0.80</td>
<td>0.18</td>
<td>0.97</td>
<td>3.00</td>
</tr>
<tr>
<td>4</td>
<td>0.62</td>
<td>0.19</td>
<td>1.02</td>
<td>3.00</td>
</tr>
<tr>
<td>3</td>
<td>0.43</td>
<td>0.19</td>
<td>1.05</td>
<td>3.00</td>
</tr>
<tr>
<td>2</td>
<td>0.24</td>
<td>0.24</td>
<td>1.34</td>
<td>4.32</td>
</tr>
</tbody>
</table>

1.0 in. = 25.4 mm.
Table 4.1-18 Response Spectrum Drift for Building Responding in Y Direction

<table>
<thead>
<tr>
<th>Level</th>
<th>Total Drift from R.S. Analysis (in.)</th>
<th>Story Drift (in.)</th>
<th>Story Drift × Cd (in.)</th>
<th>Allowable Story Drift (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>1.81</td>
<td>0.06</td>
<td>0.32</td>
<td>3.00</td>
</tr>
<tr>
<td>12</td>
<td>1.76</td>
<td>0.09</td>
<td>0.49</td>
<td>3.00</td>
</tr>
<tr>
<td>11</td>
<td>1.67</td>
<td>0.11</td>
<td>0.58</td>
<td>3.00</td>
</tr>
<tr>
<td>10</td>
<td>1.56</td>
<td>0.12</td>
<td>0.67</td>
<td>3.00</td>
</tr>
<tr>
<td>9</td>
<td>1.44</td>
<td>0.13</td>
<td>0.70</td>
<td>3.00</td>
</tr>
<tr>
<td>8</td>
<td>1.31</td>
<td>0.16</td>
<td>0.87</td>
<td>3.00</td>
</tr>
<tr>
<td>7</td>
<td>1.15</td>
<td>0.17</td>
<td>0.91</td>
<td>3.00</td>
</tr>
<tr>
<td>6</td>
<td>0.99</td>
<td>0.17</td>
<td>0.92</td>
<td>3.00</td>
</tr>
<tr>
<td>5</td>
<td>0.92</td>
<td>0.17</td>
<td>0.93</td>
<td>3.00</td>
</tr>
<tr>
<td>4</td>
<td>0.65</td>
<td>0.19</td>
<td>1.04</td>
<td>3.00</td>
</tr>
<tr>
<td>3</td>
<td>0.46</td>
<td>0.20</td>
<td>1.08</td>
<td>3.00</td>
</tr>
<tr>
<td>2</td>
<td>0.26</td>
<td>0.26</td>
<td>1.44</td>
<td>4.32</td>
</tr>
</tbody>
</table>

1.0 in. = 25.4 mm.

According to Standard Section 12.9.6, P-delta effects should be checked using the ELF method. This implies that such effects should not be determined using the results from the modal response spectrum analysis. Thus, the results already shown and discussed in Table 4.1-11 of this example are applicable.

Nevertheless, P-delta effects can be assessed using the results of the modal response spectrum analysis if the displacements, drifts and story shears are used as computed from the response spectrum analysis, without the base shear scale factors. However, when computing the stability ratio, the drifts must include the amplifier $C_d$ (because of the presence of $C_d$ in the denominator of Standard Equation 12.8-16). Using this approach, P-delta effects were computed for the X direction response as shown in Table 4.1-19. Note that the stability factors are very similar to those given in Table 4.1-11. As with Table 4.1-11, the stability factors from Table 4.1-19 exceed the limit ($\theta_{max} = 0.091$) only at the bottom three levels of the structure and are only marginally above the limit. Since the $\beta$ factor was conservatively set at 1.0 inch for computing the limit, it is likely that a refined analysis for $\beta$ would indicate that P-delta effects are not of particular concern for this structure.

Table 4.1-19 Computation of P-delta Effects for X Direction Response

<table>
<thead>
<tr>
<th>Level</th>
<th>$h_s$ (in.)</th>
<th>$\Delta$ (in.)</th>
<th>$P_D$ (kips)</th>
<th>$P_L$ (kips)</th>
<th>$P_T$ (kips)</th>
<th>$P_X$ (kips)</th>
<th>$V_X$ (kips)</th>
<th>$\theta_X$</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>150</td>
<td>0.66</td>
<td>1,656.5</td>
<td>315.0</td>
<td>1,971.5</td>
<td>1,971.5</td>
<td>82.7</td>
<td>0.019</td>
</tr>
<tr>
<td>12</td>
<td>150</td>
<td>0.89</td>
<td>1,595.8</td>
<td>315.0</td>
<td>1,910.8</td>
<td>3,882.3</td>
<td>130.9</td>
<td>0.032</td>
</tr>
<tr>
<td>11</td>
<td>150</td>
<td>1.03</td>
<td>1,595.8</td>
<td>315.0</td>
<td>1,910.8</td>
<td>5,793.1</td>
<td>163.8</td>
<td>0.044</td>
</tr>
<tr>
<td>10</td>
<td>150</td>
<td>1.08</td>
<td>1,595.8</td>
<td>315.0</td>
<td>1,910.8</td>
<td>7,703.9</td>
<td>191.4</td>
<td>0.053</td>
</tr>
<tr>
<td>9</td>
<td>150</td>
<td>0.98</td>
<td>3,403.0</td>
<td>465.0</td>
<td>3,868.0</td>
<td>11,571.9</td>
<td>240.1</td>
<td>0.057</td>
</tr>
<tr>
<td>8</td>
<td>150</td>
<td>1.06</td>
<td>2,330.8</td>
<td>465.0</td>
<td>2,795.8</td>
<td>14,367.7</td>
<td>268.9</td>
<td>0.069</td>
</tr>
<tr>
<td>7</td>
<td>150</td>
<td>1.08</td>
<td>2,330.8</td>
<td>465.0</td>
<td>2,795.8</td>
<td>17,163.5</td>
<td>292.9</td>
<td>0.077</td>
</tr>
<tr>
<td>6</td>
<td>150</td>
<td>1.08</td>
<td>2,330.8</td>
<td>465.0</td>
<td>2,795.8</td>
<td>19,959.3</td>
<td>316.1</td>
<td>0.083</td>
</tr>
<tr>
<td>5</td>
<td>150</td>
<td>0.97</td>
<td>4,323.8</td>
<td>615.0</td>
<td>4,938.8</td>
<td>24,898.1</td>
<td>359.5</td>
<td>0.081</td>
</tr>
</tbody>
</table>
### Table 4.1-19 Computation of P-delta Effects for X Direction Response

<table>
<thead>
<tr>
<th>Level</th>
<th>$h_{st}$ (in.)</th>
<th>$\Delta$ (in.)</th>
<th>$P_D$ (kips)</th>
<th>$P_L$ (kips)</th>
<th>$P_T$ (kips)</th>
<th>$P_X$ (kips)</th>
<th>$V_X$ (kips)</th>
<th>$\theta_X$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>150</td>
<td>1.02</td>
<td>3,066.1</td>
<td>615.0</td>
<td>3,681.1</td>
<td>28,579.2</td>
<td>384.8</td>
<td>0.092</td>
</tr>
<tr>
<td>3</td>
<td>150</td>
<td>1.05</td>
<td>3,066.1</td>
<td>615.0</td>
<td>3,681.1</td>
<td>32,260.3</td>
<td>401.9</td>
<td>0.102</td>
</tr>
<tr>
<td>2</td>
<td>216</td>
<td>1.34</td>
<td>3,097.0</td>
<td>615.0</td>
<td>3,712.0</td>
<td>35,972.3</td>
<td>438.1</td>
<td>0.093</td>
</tr>
</tbody>
</table>

1.0 in. = 25.4 mm, 1.0 kip = 4.45 kN.

### 4.1.6.3 Torsion, orthogonal loading and load combinations.

To determine member design forces, it is necessary to add the effects of accidental torsion and orthogonal loading into the analysis. When including accidental torsion in modal response spectrum analysis, there are generally two approaches that can be taken:

- Displace the center of mass of the floor plate plus or minus 5 percent of the plate dimension perpendicular to the direction of the applied response spectrum. As there are four possible mass locations, this will require four separate modal analyses for torsion with each analysis using a different set of mode shapes and frequencies.

- Compute the effects of accidental torsion by creating a load condition with the accidental story torques applied as static forces. Member forces created by the accidental torsion are then added directly to the results of the response spectrum analysis. As with the displaced mass method, there are four possible ways to apply the accidental torsion: plus and minus torsion for primary loads in the X and Y directions. Where scaling of the modal response spectrum design forces is required, the torsional loading used for accidental torsion analysis should be multiplied by 0.85.

Each of the above approaches has advantages and disadvantages. The primary disadvantage of the first approach is a practical one: most computer programs do not allow for the extraction of member force maxima from more than one run where the different runs incorporate a different set of mode shapes and frequencies. An advantage of the approach stipulated in *Standard* Section 12.9.5 is that accidental torsion need not be amplified (when otherwise required by *Standard* Section 12.8.4.3) because the accidental torsion effect is amplified within the dynamic analysis.

For structures that are torsionally regular and which will not require amplification of torsion, the second approach may be preferred. A disadvantage of the approach is the difficulty of combining member forces from a CQC analysis (all results positive), and a separate static torsion analysis (member forces have positive and negative signs as appropriate).

In the analysis that follows, the second approach has been used because the structure has excellent torsional rigidity, and amplification of accidental torsion is not required (all amplification factors = 1.0).

There are two possible methods for applying the orthogonal loading rule:

- Run two separate response spectrum analyses, one in the X direction and one in the Y direction, with CQC being used for modal combinations in each analysis. Using a direct sum, combine 100 percent of the scaled X direction results with 30 percent of the scaled Y direction results. Perform a similar analysis using 100 percent of the scaled Y direction forces and 30 percent of the scaled X direction forces. All seismic effects can be considered in only two dynamic load cases (one response spectrum analysis in each direction) and two torsion cases (resulting from loads applied at a 5 percent eccentricity in each direction). These are shown in Figure 4.1-12.
Run two separate response spectrum analyses, one in the X direction and one in the Y direction, with CQC being used for modal combinations in each analysis. Using SRSS, combine 100 percent of the scaled X direction results with 100 percent of the scaled Y direction results (Wilson, 2004).

![Figure 4.1-12 Load combinations for response spectrum analysis](image)

### 4.1.6.4 Member design forces

Earthquake shear forces in the beams of Frame 1 are given in Figure 4.1-13. These member forces are based on 2.18 times the spectrum applied in the X direction and 1.94 times of the spectrum applied independently in the Y direction. Individual member forces from the X and Y directions are obtained by CQC for that analysis and these forces are combined by SRSS. To account of accidental torsion, the forces in Figure 4.1-13 should be added to 0.85 times the forces shown in Figure 4.1-9b.
### Modal Response History Analysis

**Before beginning this section, it is important to note that the analysis performed here is based on the requirements of Chapter 16 of ASCE 7-10. This version contains several important updates that removed inconstancies and omissions that were present in Chapter 16 of ASCE 7-05.**

In modal response history analysis, the original set of coupled equations of motion is transformed into a set of uncoupled “modal” equations, an explicit displacement history is computed for each mode, the modal histories are transformed back into the original coordinate system, and these responses are added together to produce the response history of the displacements at each of the original degrees of freedom. These displacement histories may then be used to determine histories of story drift, member forces, or story shears.

Requirements for response history analysis are provided in Chapter 16 of ASCE 7-10. The same mathematical model of the structure used for the ELF and response spectrum analysis is used for the response history analysis. Five percent damping was used in each mode and as with the response spectrum method, 12 modes were used in the analysis. These 12 modes captured more than 90 percent of the mass of the structure above grade. Several issues related to a reanalysis of the structure with 120 modes are described later.

As allowed by ASCE 7-10 Section 16.1, the structure is analyzed using three different pairs of ground acceleration histories. The development of a proper suite of ground motions is one of the most critical.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8.41</td>
<td>8.72</td>
</tr>
<tr>
<td>R-12</td>
<td>14.9</td>
<td>15.6</td>
</tr>
<tr>
<td>12-11</td>
<td>21.5</td>
<td>21.6</td>
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<td>11-10</td>
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<td>9-8</td>
<td>23.7</td>
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</tr>
<tr>
<td>8-7</td>
<td>26.9</td>
<td>26.1</td>
</tr>
<tr>
<td>7-6</td>
<td>28.4</td>
<td>26.8</td>
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<td>6-5</td>
<td>10.1</td>
<td>22.4</td>
</tr>
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<td>5-4</td>
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<td>4-3</td>
<td>18.5</td>
<td>27.5</td>
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<td>3-2</td>
<td>18.5</td>
<td>29.1</td>
</tr>
<tr>
<td>2 - G</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Figure 4.1-13* Seismic shears in girders (kips) as computed using response spectrum analysis (analysis includes orthogonal loading but excludes accidental torsion)
and difficult aspects of response history approaches. The motions should be characteristic of the site and should be from real (or simulated) ground motions that have a magnitude, distance and source mechanism consistent with those that control the maximum considered earthquake (MCE).

For the purposes of this example, however, the emphasis is on the implementation of the response history approach rather than on selection of realistic ground motions. For this reason, the motion suite developed for Example 4.2 is also used for the present example. The structure for Example 4.2 is situated in Seattle, Washington and uses three pairs of motions developed specifically for the site. The use of the Seattle motions for a Stockton building analysis is, of course, not strictly consistent with the requirements of the Standard. However, a realistic comparison may still be made between the ELF, response spectrum and response history approaches.

4.1.7.1 The Seattle ground motion suite. It is beneficial to provide some basic information on the Seattle motion suites in Table 4.1-20a below. Refer to Figures 4.2-40 through 4.2-42 for additional information, including plots of the ground acceleration histories and 5-percent damped response spectra for each component of each motion.

The acceleration histories for each source motion were downloaded from the PEER NGA Strong Ground Motion Database:

http://peer.berkeley.edu/products/strong_ground_motion_db.html

The PEER NGA record number is provided in the first column of the table. Note that the magnitude, epicenter distance and site class were obtained from the NGA Flatfile (a large Excel file that contains information about each NGA record).

<table>
<thead>
<tr>
<th>NGA Record Number</th>
<th>Magnitude [Epicenter Distance, km]</th>
<th>Site Class</th>
<th>Number of Points and Digitization Increment</th>
<th>Component Source Motion</th>
<th>PGA (g)</th>
<th>Record Name (This Example)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0879</td>
<td>7.28 [44]</td>
<td>C</td>
<td>9625 @ 0.005 sec</td>
<td>Landers/LCN260*</td>
<td>0.727</td>
<td>A00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Landers/LCN345*</td>
<td>0.789</td>
<td>A90</td>
</tr>
<tr>
<td>0725</td>
<td>6.54 [11.2]</td>
<td>D</td>
<td>2230 @ 0.01 sec</td>
<td>SUPERST/B-POE270</td>
<td>0.446</td>
<td>B00</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>SUPERST/B-POE360</td>
<td>0.300</td>
<td>B90</td>
</tr>
<tr>
<td>0139</td>
<td>7.35 [21]</td>
<td>C</td>
<td>1192 @ 0.02 sec</td>
<td>TABAS/DAY-LN</td>
<td>0.328</td>
<td>C00</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>TABAS/DAY-TR</td>
<td>0.406</td>
<td>C90</td>
</tr>
</tbody>
</table>

*Note that the two components of motion for the Landers earthquake are apparently separated by an 85 degree angle, not 90 degrees as is traditional. It is not known whether these are true orientations or whether there is an error in the descriptions provided in the NGA database.

Before the ground motions may be used in the response history analysis, they must be scaled for compatibility with the design spectrum. The scaling procedures for three-dimensional dynamic analysis are provided in Section 16.1.3.2 of ASCE 7-10. These requirements are provided verbatim as follows:

5See Sec. 3.2.6.2 of this volume of design examples for a detailed discussion of the selection and scaling of ground motions.
“For each pair of horizontal ground motion components a square root of the sum of the squares (SRSS) spectrum shall be constructed by taking the SRSS of the 5-percent damped response spectra for the scaled components (where an identical scale factor is applied to both components of a pair). Each pair of motions shall be scaled such that for each period in the range from 0.27 to 1.5T, the average of the SRSS spectra from all horizontal component pairs does not fall below the corresponding ordinate of the design response spectrum, determined in accordance with Section 11.4.5 or 11.4.7.”

ASCE 7-10 does not provide clear guidance as to which fundamental period, $T$, should be used for determining $0.2T$ and $1.5T$ when the periods of vibration are different in the two orthogonal directions of analysis. This issue is resolved herein by taking $T$ as the average of the computed periods in the two principal directions. For this example, the average period, referred to as $T_{avg}$, is $0.5(2.87 + 2.60) = 2.74$ seconds. (Another possibility would be to use the shorter of the two fundamental periods for computing $0.2T$ and the longer of the two fundamental periods for computing $1.5T$.)

It is also noted that the scaling procedure provided by ASCE 7-10 does not provide a unique set of scale factors for each set of ground motions. This “degree of freedom” in the scaling process may be eliminated by providing a six-step procedure, as described below:

1. Compute the 5 percent damped pseudo-acceleration spectrum for each unscaled component of each pair of ground motions in the set and produce the SRSS spectrum for each pair of motions within the set.

2. Using the same period values used to compute the ground motion spectra, compute the design spectrum following the procedures in Standard Section 11.4.5. This spectrum is designated as the “target spectrum”.

3. Scale each SRSS spectrum such that the spectral ordinate of the scaled spectrum at $T_{avg}$ is equal to the spectral ordinate of the design spectrum at the same period. Each SRSS spectrum will have a unique scale factor, $S_{1i}$, where $i$ is the number of the pair ($i$ ranges from 1 to 3 for the current example).

4. Create a new spectrum that is the average of the $S_1$ scaled SRSS spectra. This spectrum is designated as the “average $S_1$ scaled SRSS spectrum” and should have the same spectral ordinate as the target spectrum at the period $T_{avg}$.

5. For each spectral ordinate in the period range $0.2T_{avg}$ to $1.5T_{avg}$, divide the ordinate of the target spectrum by the corresponding ordinate of the average $S_1$ scaled SRSS spectrum, producing a set of spectral ratios over the range $0.2T_{avg}$ to $1.5T_{avg}$. The largest value among these ratios is designated as $S_2$.

6. Multiply the factor $S_{1i}$ determined in Step 3 for each pair in the set by the factor $S_2$ determined in Step 5. This product, $SS_i = S_{1i} \times S_2$ is the scale factor that should be applied to each component of ground motion in pair $i$ of the set.

The results of the scaling process are summarized in Table 4.1-20b and in Figures 4.1-14 through 4.1-18.

---

4 Elimination of the degree of freedom results in consistent scale factors for all persons using the process. This consistency is not required by ASCE 7 and experienced analysts may wish to use the “degree of freedom” to reduce or increase the influence of a given ground motion.
### Table 4.1-20b Result of 3D Scaling Process

<table>
<thead>
<tr>
<th>Set Number</th>
<th>Designation</th>
<th>SRSS Ordinate at $T = T_{Avg}$ (g)</th>
<th>Target Ordinate at $T = T_{Avg}$ (g)</th>
<th>S1</th>
<th>S2</th>
<th>SS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A00 &amp; A90</td>
<td>0.335</td>
<td>0.136</td>
<td>0.407</td>
<td>1.184</td>
<td>0.482</td>
</tr>
<tr>
<td>2</td>
<td>B00 &amp; B90</td>
<td>0.191</td>
<td>0.136</td>
<td>0.712</td>
<td>1.184</td>
<td>0.843</td>
</tr>
<tr>
<td>3</td>
<td>C00 &amp; C90</td>
<td>0.104</td>
<td>0.136</td>
<td>1.310</td>
<td>1.184</td>
<td>1.551</td>
</tr>
</tbody>
</table>

Figure 4.1-14 shows the unscaled SRSS spectra for each component pair, together with the target spectrum. Figure 4.1-15 shows the average S1 scaled SRSS spectrum and the target spectrum, where it may be seen that both spectra have a common ordinate at the average period of 2.74 seconds. Figure 4.1-16 is a plot of the spectral ratios computed in Step 5. Figure 4.1-17 is a plot of the SS scaled average SRSS spectrum, together with the target spectrum. From this plot it may be seen that all ordinates of the SS scaled average SRSS spectrum are greater than or equal to the ordinate of the target spectrum over the period range $0.2T_{Avg}$ to $1.5T_{Avg}$. The “controlling” period at which the two spectra in Figure 4.1-17 have exactly the same ordinate is approximately 1.6 seconds.

Figure 4.1-18a shows the SS scaled spectra for the “00” components of each earthquake, together with the target spectrum. Figure 4.1-18b is similar, but shows the “90” components of the ground motions. Also shown in these plots are vertical lines that represent the first 12 periods of vibration for the structure under consideration. Two additional vertical lines are shown that represent the periods for Modes 112 and 118, at which the basement walls and grade-level slab become dynamically effective. Three important points are noted from Figures 4.1-18:

- The match for the lower few modes ($T > 1.0$ sec) is good for the “00” components, but not as good for the “90” components. In particular, the ground motion coordinates for motions A90 and B90 are considerably less than those for the target spectrum.
- Higher mode responses ($T < 1.0$ sec) will be significantly greater in Earthquake C than in Earthquake A or B. In Modes 10 through 12, the response for Earthquake A is several times greater than for Earthquake B.
- In Modes 112 and 118, the response for Earthquake A is approximately three times that for the code spectrum.

The impact of these points on the computed response of the structure will be discussed in some detail later in this example.
Figure 4.1-14 Unscaled SRSS spectra and target spectrum

Figure 4.1-15 Average S1 scaled SRSS spectrum and target spectrum
Figure 4.1-16  Ratio of target spectrum to average S1 scaled SRSS spectrum

Figure 4.1-17  SS scaled average SRSS spectrum and target spectrum
Another detail not directly specified by Chapter 16 of ASCE 7-10 is how ground motions should be oriented when applied. In the analysis presented herein, 12 dynamic analyses were performed with scaled ground motions applied only in one direction, as follows:

- A00-X: SS scaled component A00 applied in X direction
- A00-Y: SS scaled component A00 applied in Y direction
- A90-X: SS scaled component A90 applied in X direction
- A90-Y: SS scaled component A90 applied in Y direction
- B00-X: SS scaled component B00 applied in X direction
- B00-Y: SS scaled component B00 applied in Y direction
- B90-X: SS scaled component B90 applied in X direction
- B90-Y: SS scaled component B90 applied in Y direction
- C00-X: SS scaled component C00 applied in X direction
- C00-Y: SS scaled component C00 applied in Y direction
- C90-X: SS scaled component C90 applied in X direction
- C90-Y: SS scaled component C90 applied in Y direction

The scaled motions, without the \((I/R)\) factor, were applied at the base of the basement walls. Accidental torsion effects are included in a separate static analysis, as described later. All 12 individual response history analyses were carried out using SAP2000. As with the response spectrum analysis, 12 modes were used in the analysis. Five percent of critical damping was used in each mode. The integration time-step used in all analyses was equal to the digitization interval of the ground motion used (see Table 4.1-20a). The results from the analyses are summarized Tables 4.1-21.

A summary of base shear and roof displacement results from the analyses using the SS scaled ground motions is provided in Table 4.1-21. As may be observed, the base shears range from a low of 1,392 kips for analysis A90-Y to a high of 5,075 kips for analysis C90-Y. Roof displacements range from a low of 5.16 inches for analysis A90-Y to a high of 20.28 inches for analysis A00-X. This is a remarkable range of behavior when one considers that the ground motions were scaled for consistency with the design spectrum.

| Table 4.1-21 Result Maxima from Response History Analysis Using SS Scaled Ground Motions |
|--------------------------------------|-----------------|-----------------|-----------------|
| Analysis    | Maximum base shear (kips) | Time of maximum shear (sec.) | Maximum roof displacement (in.) | Time of maximum displacement (sec.) |
| A00-X       | 3507             | 11.29            | 20.28            | 11.38            |
| A00-Y       | 3573             | 11.27            | 14.25            | 11.28            |
| A90-X       | 1588             | 12.22            | 7.32             | 12.70            |
| A90-Y       | 1392             | 13.56            | 5.16             | 10.80            |
| B00-X       | 3009             | 8.28             | 12.85            | 9.39             |
| B00-Y       | 3130             | 9.37             | 11.20            | 10.49            |
| B90-X       | 2919             | 8.85             | 11.99            | 7.11             |
| B90-Y       | 3460             | 7.06             | 11.12            | 8.20             |
| C00-X       | 3130             | 13.5             | 9.77             | 13.54            |
| C00-Y       | 2407             | 4.64             | 6.76             | 8.58             |
| C90-X       | 3229             | 6.92             | 15.61            | 6.98             |
| C90-Y       | 5075             | 6.88             | 14.31            | 7.80             |

1.0 in. = 25.4 mm, 1.0 kip = 4.45 kN.
The analysis was performed without the \((I/R)\) factor, so in conformance with Section 16.1.4 of ASCE 7-10, all force quantities produced from the analysis were multiplied by this factor. All displacements from the analysis were multiplied by the factor \(C_d/R\).

Additionally, the 2010 version of the Standard requires that forces be scaled by the factor \(0.85V/V_i\) where the base shears from the response history analysis, \(V_i\), are less than 0.85 times the base shears, \(V\), produced by the ELF method when either Equation 12.8-5 or 12.8-6 controls the seismic base shear. The displacements must be scaled by the same factor only if Equation 12.8-6 controls when computing the seismic base shear. (It is noted that these requirements are similar to the scaling requirements provided for modal response spectrum analysis [Sections 12.9.4.1 and 12.9.4.2] except that forces from modal response spectrum analysis would be scaled if the shear from the response spectrum analysis is less than \(0.85V_i\), regardless of the \(C_s\) equation which controls \(V\).)

The base shears from the SS scaled motions with the \(I/R = 1/8\) scaling are provided in the first column of Table 4.1-22. These forces are all significantly less than 0.85 times the ELF base shear, which is \(0.85(112.5) = 956\) kips. The required scale factors to bring the base shears up to the 85 percent requirement are shown in Column 2 of Table 4.2-22.

Before proceeding, it is important to remind the reader that three separate sets of scale factors apply to the response history analysis of this structure when member design forces are being obtained:

1. The ground motion SS scale factors
2. The \(I/R\) scale factor
3. The \(0.85V/V_i\) factor because the base shear from modal response history analysis (including scale factors 1 and 2 above) is less than 85 percent of that determined from ELF when ELF is governed by Equation 12.8-5 or 12.8-6.

<table>
<thead>
<tr>
<th>Analysis</th>
<th>((I/R)) times maximum base shear from analysis (kips)</th>
<th>Required additional scale factor for (V = 0.85V_{ELF} = 956) kips</th>
</tr>
</thead>
<tbody>
<tr>
<td>A00-X</td>
<td>438.4</td>
<td>2.18</td>
</tr>
<tr>
<td>A00-Y</td>
<td>446.7</td>
<td>2.14</td>
</tr>
<tr>
<td>A90-X</td>
<td>198.5</td>
<td>4.81</td>
</tr>
<tr>
<td>A90-Y</td>
<td>173.9</td>
<td>5.49</td>
</tr>
<tr>
<td>B00-X</td>
<td>376.1</td>
<td>2.54</td>
</tr>
<tr>
<td>B00-Y</td>
<td>391.2</td>
<td>2.44</td>
</tr>
<tr>
<td>B90-X</td>
<td>364.8</td>
<td>2.62</td>
</tr>
<tr>
<td>B90-Y</td>
<td>432.5</td>
<td>2.21</td>
</tr>
<tr>
<td>C00-X</td>
<td>391.2</td>
<td>2.44</td>
</tr>
<tr>
<td>C00-Y</td>
<td>300.9</td>
<td>3.18</td>
</tr>
<tr>
<td>C90-X</td>
<td>403.6</td>
<td>2.37</td>
</tr>
<tr>
<td>C90-Y</td>
<td>634.4</td>
<td>1.51</td>
</tr>
</tbody>
</table>
4.1.7.2 Drift and P-delta effects. Only two scale factors are required for displacement and drift because Equation 12.8-6 did not control the base shear for this structure:

- The ground motion scale factors SS
- The \( C_d/R \) scale factor

Drift is checked for each individual component of motion acting in the X direction and the envelope values of drift are taken as the design drift values. The procedure is repeated for motions applied in the Y direction. As with the ELF and Modal Response Spectrum analyses, drifts are taken as the difference between the displacement at the center of mass of one level and the displacement at the projection of this point on the level below. The results of the analysis, shown in Table 4.1-23 for the X direction only, indicate that the allowable drift is not exceeded at any level of the structure. Similar results were obtained for Y direction loading.

### Table 4.1-23 Response History Drift for Building Responding in X Direction for All of the Ground Motions in the X Directions

<table>
<thead>
<tr>
<th>Level</th>
<th>Envelope of drift (in.) for each ground motion</th>
<th>Envelope of drift for all the ground motions</th>
<th>Envelope of drift ( \times \frac{C_d}{R} )</th>
<th>Allowable drift (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>1.17 0.49 0.95 0.81 0.91 1.23</td>
<td>1.23</td>
<td>0.85</td>
<td>3.00</td>
</tr>
<tr>
<td>12</td>
<td>1.64 0.66 1.22 0.95 1.16 1.27</td>
<td>1.64</td>
<td>1.13</td>
<td>3.00</td>
</tr>
<tr>
<td>11</td>
<td>1.97 0.78 1.32 0.99 1.25 1.52</td>
<td>1.97</td>
<td>1.35</td>
<td>3.00</td>
</tr>
<tr>
<td>10</td>
<td>2.05 0.86 1.42 1.04 1.20 1.68</td>
<td>2.05</td>
<td>1.41</td>
<td>3.00</td>
</tr>
<tr>
<td>9</td>
<td>1.79 0.82 1.26 1.25 0.99 1.41</td>
<td>1.79</td>
<td>1.23</td>
<td>3.00</td>
</tr>
<tr>
<td>8</td>
<td>1.83 0.87 1.22 1.42 1.23 1.50</td>
<td>1.83</td>
<td>1.26</td>
<td>3.00</td>
</tr>
<tr>
<td>7</td>
<td>1.82 0.83 1.27 1.36 1.21 1.67</td>
<td>1.82</td>
<td>1.25</td>
<td>3.00</td>
</tr>
<tr>
<td>6</td>
<td>1.77 0.74 1.36 1.35 1.06 1.94</td>
<td>1.94</td>
<td>1.33</td>
<td>3.00</td>
</tr>
<tr>
<td>5</td>
<td>1.50 0.59 1.19 1.21 1.09 1.81</td>
<td>1.81</td>
<td>1.24</td>
<td>3.00</td>
</tr>
<tr>
<td>4</td>
<td>1.55 0.62 1.22 1.32 1.23 1.76</td>
<td>1.76</td>
<td>1.21</td>
<td>3.00</td>
</tr>
<tr>
<td>3</td>
<td>1.56 0.64 1.24 1.30 1.33 1.60</td>
<td>1.60</td>
<td>1.10</td>
<td>3.00</td>
</tr>
<tr>
<td>2</td>
<td>1.97 0.86 1.64 1.58 1.73 1.85</td>
<td>1.97</td>
<td>1.35</td>
<td>4.32</td>
</tr>
</tbody>
</table>

1.0 in. = 25.4 mm.

ASCE 7-10 does not provide information on how P-delta effects should be addressed in response history analysis. It would appear reasonable to use the same procedure as specified for ASCE 7-05 and ASCE 7-10 for Modal Response Spectrum Analysis (Sec. 12.9.6), where it is stated that the Equivalent Lateral Force method of analysis be used. Such an analysis was performed in Section 4.1.5.3.2 of this example, with results provided in Table 4.1-11. These results indicate that allowable stability ratios are marginally exceeded at Levels 2, 3 and 4, but that rigorous analysis with \( \beta \) less than 1.0 would show that the allowable stability ratios are not exceeded.

4.1.7.3 Torsion, orthogonal loading and member design forces. As with ELF or response spectrum analysis, it is necessary to add the effects of accidental torsion and orthogonal loading into the analysis.
Accidental torsion is applied separately with a static analysis in exactly the same manner as done for the response spectrum approach. Member shears for this torsion-only analysis are shown separately in Figure 4.1-9b. These shears must be multiplied by 0.85 before adding to the scaled shears produced by the dynamic response history analysis.

Orthogonal loading is automatically accounted for by applying the 100 percent of the ground motions in the X and Y direction simultaneously. For each ground motion pair, these forces are applied in the orientations shown in Figure 4.1-19. The figure also shows the scale factor that was used in each analysis.

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Load Combination</th>
<th>Loading X Direction</th>
<th>Loading Y Direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>A00-X</td>
<td>2.18</td>
<td>A00-Y</td>
</tr>
<tr>
<td>2</td>
<td>A90-X</td>
<td>-4.81</td>
<td>A90-Y</td>
</tr>
<tr>
<td>3</td>
<td>A00-X</td>
<td>-2.18</td>
<td>A00-Y</td>
</tr>
<tr>
<td>4</td>
<td>A90-X</td>
<td>4.81</td>
<td>A90-Y</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>B00-X</td>
<td>2.54</td>
<td>B00-Y</td>
</tr>
<tr>
<td>6</td>
<td>B90-X</td>
<td>-2.62</td>
<td>B90-Y</td>
</tr>
<tr>
<td>7</td>
<td>B00-X</td>
<td>-2.54</td>
<td>B00-Y</td>
</tr>
<tr>
<td>8</td>
<td>B90-X</td>
<td>2.62</td>
<td>B90-Y</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>C00-X</td>
<td>2.44</td>
<td>C00-Y</td>
</tr>
<tr>
<td>10</td>
<td>C90-X</td>
<td>-2.36</td>
<td>C90-Y</td>
</tr>
<tr>
<td>11</td>
<td>C00-X</td>
<td>-2.44</td>
<td>C00-Y</td>
</tr>
<tr>
<td>12</td>
<td>C90-X</td>
<td>2.36</td>
<td>C90-Y</td>
</tr>
</tbody>
</table>

**Figure 4.1-19** Orthogonal Loading in Response History Analysis
Using the load combinations described above, the individual beam shear maxima developed in Frame 1 were computed for each load combination. Envelope values from all combinations are shown in Figure 4.1-20.

<table>
<thead>
<tr>
<th></th>
<th>14.15</th>
<th>12.82</th>
<th>14.17</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-12</td>
<td>21.5</td>
<td>20.6</td>
<td>21.5</td>
</tr>
<tr>
<td>12-11</td>
<td>29.5</td>
<td>29.4</td>
<td>30.6</td>
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<tr>
<td>11-10</td>
<td>33.7</td>
<td>33.2</td>
<td>35.5</td>
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<tr>
<td>10-9</td>
<td>32.9</td>
<td>32.0</td>
<td>29.5</td>
</tr>
<tr>
<td>9-8</td>
<td>33.6</td>
<td>32.3</td>
<td>30.7</td>
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<td>8-7</td>
<td>36.3</td>
<td>34.5</td>
<td>33.2</td>
</tr>
<tr>
<td>7-6</td>
<td>39.0</td>
<td>35.3</td>
<td>34.5</td>
</tr>
<tr>
<td>6-5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5-4</td>
<td>15.1</td>
<td>32.9</td>
<td>33.9</td>
</tr>
<tr>
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<td>33.6</td>
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<td>35.7</td>
<td>33.1</td>
</tr>
<tr>
<td>2 - G</td>
<td>21.6</td>
<td>34.3</td>
<td>32.3</td>
</tr>
</tbody>
</table>

**Figure 4.1-20** Envelope of seismic shears in girders (kips) as computed using response history analysis (analysis includes orthogonal loading but excludes accidental torsion)

### 4.1.8 Comparison of Results from Various Methods of Analysis

A summary of the results from all of the analyses is provided in Tables 4.1-24 through 4.1-28.

**4.1.8.1 Comparison of base shear and story shear.** The maximum story shears are shown in Table 4.1-24. For the response history analysis, the shears are the envelope values of story shears for all twelve individual analyses. Note that the modal response spectrum and modal response history shears for the lowest level are both equal to 956 kips, which is 0.85 times the ELF base shear.

The story shear is basically of the same character—lower values in upper stories, larger values in lower stories. It appears, however, that the maximum shears from the modal response history analysis occur at stories 2, 3 and 4. This must be due to the amplified energy in the higher modes in the actual ground motions (when compared to the design spectrum).
4.1.8.2 Comparison of drift. Table 4.1-25 summarizes the drifts computed from each of the analyses. The modal response history drifts are the envelopes among all analyses. The ELF drifts are significantly greater than those determined using modal response spectrum analysis. The drifts from the modal response history analysis are slightly greater than those from the response spectrum analysis.

4.1.8.3 Comparison member forces. The shears developed in Bay D-E of Frame 1 are compared in Table 4.1-26. The shears from the response history analysis are envelope values among all analyses, including torsion and orthogonal load effects. The response history approach produced beam shears similar to those from ELF analysis and somewhat greater than those produced by response spectrum analysis.

**Table 4.1-24** Summary of Results of Various Methods of Analysis: Story Shear

<table>
<thead>
<tr>
<th>Level</th>
<th>ELF</th>
<th>Modal response spectrum</th>
<th>Modal response history</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>187</td>
<td>180</td>
<td>295</td>
</tr>
<tr>
<td>12</td>
<td>341</td>
<td>286</td>
<td>349</td>
</tr>
<tr>
<td>11</td>
<td>471</td>
<td>357</td>
<td>462</td>
</tr>
<tr>
<td>10</td>
<td>578</td>
<td>418</td>
<td>537</td>
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<tr>
<td>9</td>
<td>765</td>
<td>524</td>
<td>672</td>
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<tr>
<td>8</td>
<td>866</td>
<td>587</td>
<td>741</td>
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<tr>
<td>7</td>
<td>943</td>
<td>639</td>
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<td>999</td>
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</tr>
<tr>
<td>5</td>
<td>1,070</td>
<td>784</td>
<td>1,135</td>
</tr>
<tr>
<td>4</td>
<td>1,102</td>
<td>840</td>
<td>1,099</td>
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<tr>
<td>3</td>
<td>1,118</td>
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<td>1,008</td>
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<tr>
<td>2</td>
<td>1,124</td>
<td>956</td>
<td>956</td>
</tr>
</tbody>
</table>

**Table 4.1-25** Summary of Results from Various Methods of Analysis: Story Drifts

<table>
<thead>
<tr>
<th>Level</th>
<th>X Direction Drift (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ELF</td>
</tr>
<tr>
<td>R</td>
<td>0.99</td>
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<td>1.41</td>
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<tr>
<td>11</td>
<td>1.75</td>
</tr>
<tr>
<td>10</td>
<td>1.92</td>
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<tr>
<td>9</td>
<td>1.82</td>
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<td>8</td>
<td>1.97</td>
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<td>7</td>
<td>2.01</td>
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<tr>
<td>6</td>
<td>1.97</td>
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<tr>
<td>5</td>
<td>1.67</td>
</tr>
</tbody>
</table>
Table 4.1-25 Summary of Results from Various Methods of Analysis: Story Drifts

<table>
<thead>
<tr>
<th>Level</th>
<th>X Direction Drift (in.)</th>
<th>ELF</th>
<th>Modal response spectrum</th>
<th>Modal response history</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1.69</td>
<td>1.02</td>
<td>1.21</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1.65</td>
<td>1.05</td>
<td>1.10</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2.00</td>
<td>1.34</td>
<td>1.35</td>
<td></td>
</tr>
</tbody>
</table>

1.0 in. = 25.4 mm.

Table 4.1-26 Summary of Results from Various Methods of Analysis: Beam Shear

<table>
<thead>
<tr>
<th>Level</th>
<th>Beam Shear Force in Bay D-E of Frame 1 (kips)</th>
<th>ELF</th>
<th>Modal response spectrum</th>
<th>Modal response history</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>10.27</td>
<td>8.72</td>
<td>12.82</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>18.91</td>
<td>15.61</td>
<td>20.61</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>28.12</td>
<td>21.61</td>
<td>29.45</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>33.15</td>
<td>24.02</td>
<td>33.22</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>34.69</td>
<td>23.32</td>
<td>32.02</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>35.92</td>
<td>24.73</td>
<td>32.30</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>40.10</td>
<td>26.15</td>
<td>34.53</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>40.58</td>
<td>26.76</td>
<td>35.29</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>36.52</td>
<td>25.29</td>
<td>35.82</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>34.58</td>
<td>24.93</td>
<td>35.65</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>35.08</td>
<td>26.60</td>
<td>34.27</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>35.28</td>
<td>28.25</td>
<td>33.07</td>
<td></td>
</tr>
</tbody>
</table>

1.0 kip = 4.45 kN.

4.1.8.4 Which analysis method is best? In this example, an analysis of an irregular steel moment frame was performed using three different techniques: equivalent lateral force analysis, modal response spectrum analysis and modal response history analysis. Each analysis was performed using a linear elastic model of the structure even though it is recognized that the structure will repeatedly yield during the earthquake. Hence, each analysis has significant shortcomings with respect to providing a reliable prediction of the actual response of the structure during an earthquake.

The purpose of analysis, however, is not to predict response but rather to provide information that an engineer can use to proportion members and to estimate whether or not the structure has sufficient stiffness to limit deformations and avoid overall instability. In short, the analysis only has to be “good enough for design.” If, on the basis of any of the above analyses, the elements are properly designed for strength, the stiffness requirements are met and the elements and connections of the structure are detailed for inelastic response according to the requirements of ASCE 7 and AISC 341, the structure will likely survive an earthquake consistent with the MCE ground motion. The exception would be if a highly
irregular structure were analyzed using the ELF procedure. Fortunately, ASCE 7 safeguards against this by requiring three-dimensional dynamic analysis for highly irregular structures.

For the structure analyzed in this example, the irregularities were probably not so extreme such that the ELF procedure would produce a “bad design.” However, where computer programs that can perform modal response spectrum analysis with only marginally increased effort over that required for ELF are available (e.g., SAP2000 and ETABS), the modal analysis should always be used for final design in lieu of ELF (even if ELF is allowed by the Provisions). As mentioned in the example, this does not negate the need for or importance of ELF analysis because such an analysis is useful for preliminary design and several components of the ELF analysis are necessary for application of accidental torsion.

Modal response history analysis is of limited practical use where applied to a linear elastic model of the structure. The amount of additional effort required to select and scale the ground motions, perform the modal response history analysis, scale the results and determine envelope values for use in design simply is not warranted where compared to the effort required for modal response spectrum analysis. This might change in the future where “standard” suites of ground motions are developed and are made available to the earthquake engineering community. Also, significant improvement is needed in the software available for the preprocessing and, particularly, for the post-processing of the huge amounts of information that produced by the analysis.

Scaling the ground motions used for modal response history analysis is also an issue. The Standard requires that the selected motions be consistent with the magnitude, distance and source mechanism of the MCE expected at the site. If the ground motions satisfy this criterion, then why scale at all? Distant earthquakes may have a lower peak acceleration but contain a frequency content that is more significant. Near-source earthquakes may display single damaging pulses. Scaling these two earthquakes to the Standard design spectrum seems to eliminate some of the most important characteristics of the ground motions. The fact that there is a degree of freedom in the ASCE 7 scaling requirements compensates for this effect, but only for very knowledgeable users.

The main benefit of modal response history analysis is in the nonlinear dynamic analysis of structures or in the analysis of non-proportionally damped linear systems. This type of analysis is the subject of Example 4.2.

4.1.9 Consideration of Higher Modes in Analysis

All of the computed results for the modal response spectrum and modal response history methods of analysis were based on the first 12 modes of the model with the basement level explicitly modeled. Recall that the basement walls were modeled with 1.0-foot shell elements, that the grade-level diaphragm was modeled using 6.0-inch-thick shell elements and that the grade level was not laterally restrained. The weight associated with the basement-level walls and grade-level slab is 6,526 kips, which is approximately 15 percent of the total weight of the structure (see Table 4.1-3).

The accumulated effective modal mass for the first 12 modes (see Table 4.1-14a) is in the neighborhood of 82 percent of the total mass of the structure, which is less than the 90 percent required by Section 12.9-1 of the Standard. However, the first 12 modes capture more than 90 percent of the mass above grade, so it was deemed sufficient to run the analysis with only 12 modes. If the requirement of Section 12.9-1 were satisfied for the structure as modeled, it would have taken 119 modes to capture more than 90 percent of the effective mass of the entire system (see Table 4.1-14b).

In the analysis presented so far, all of the seismic base shears were computed at the base of the first story above grade, not the base of the entire structure (the base of the basement walls). It is of some interest to
examine how the results of the analysis would change if 120 modes were to be used in the analysis. This would definitely satisfy the requirements of Section 12.9-1 for the full structure.

4.1.9.1 Modal response spectrum analysis with higher modes. Table 4.1-27a provides the seismic shears through the basement level and through the first floor above grade for the analysis run with 12, 18, 120 and 200 modes. In this part of the table, the “modes” are the natural mode shapes from an eigenvalue analysis. As may be seen, the shear through the first story above grade is unchanged as the number of modes increases above 12 modes. However, the shears through the basement level are substantially increased when 120 or more modes are used. In the X direction, for example, the ratio of the basement-level shear for 120 modes to that for 12 modes is 630/439 = 1.44. Thus, in terms of the shear at the base of the structure, the activation of the higher modes increases the shears 44 percent, while the added weight associated with the basement level is only 15 percent.

This increase in shear was rather unexpected, so the analysis was re-run using Ritz vectors in lieu of the natural mode shapes. Ritz vectors automatically include the “static corrections” that are sometimes needed for very high frequency modes. As may be seen from Table 4.1-27b, the results using Ritz Vectors are virtually identical to those obtained using the natural mode shapes.

4.1.9.2 Modal response history analysis with higher modes. The comparison of shears using modal response history analysis with 12, 18, 120 and 200 modes are presented in Table 4.1-28. The results are based on the use of natural mode shapes. For brevity, results are given only for motions A00, B00 and C00 applied in the X and Y directions. The analyses include SS ground motion scaling, I/R scaling, but not the 85 percent scaling.

As may be observed from Table 4.1-27, the use of the higher modes produces virtually no change in the shears through the first level above grade. However, very significant increases in shear are developed through the basement. The most extreme increase in shears is for ground motion A00, wherein the shears in the basement increase from 439 kips to 744 kips for loading in the X direction and increase from 440 kips to 862 kips for loading in the Y direction. These increases in shear are not unexpected because of the spectral amplitudes of the ground motions at periods associated with modes 112 and 118 (see Figure 4.1-18).

4.1.9.3 Discussion on use of higher modes. Many structures have stiff lower stories or have one or more levels of basement. If the basement is modeled explicitly and if full lateral restraint is not provided at the top of the basement or at subgrade slab levels, the phenomena described herein will likely result.

Based on the results presented above, there is some question as to which results should be used for the 85 percent scaling requirements of Standard Section 12.9.4. If the basement level were included in the ELF analysis, the computed period would not significantly change, and the base shear would increase 15 percent to accommodate the added weight associated with the basement walls and grade-level slab. However, the scale factors required to bring the modal response spectrum or modal response history shears up to 85 percent of the ELF shears (with 15 percent increase) could be significantly less than those obtained when the basement level is not included in the model. The net result would be significantly reduced for design shears in the upper levels of the structure.

Given these results, it is recommended that scaling always be based on the shears determined at the first level above grade. The question of how many modes to use in the analysis is not as easy to answer. Certainly, a sufficient number of modes must be used to capture at least 90 percent of the above-grade mass. In cases where it is desired to explicitly determine the shears at the base of unrestrained basements, enough modes should be used to capture 90 percent of the mass of the entire structure.
### Table 4.1-27 Comparison of Modal Response Spectrum Shears Using 12, 18, 120 and 200 Modes

(a) Using Natural Mode Shapes  
(values from SAP2000 without 85% scaling)

<table>
<thead>
<tr>
<th>Shear Location</th>
<th>Load Case</th>
<th>Shear (kips) for number of modes =</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>12</td>
</tr>
<tr>
<td>Base of 1st story</td>
<td>Code spectrum X direction</td>
<td>438</td>
</tr>
<tr>
<td>Base of structure</td>
<td>Code spectrum X direction</td>
<td>439</td>
</tr>
<tr>
<td>Base of 1st story</td>
<td>Code spectrum Y direction</td>
<td>492</td>
</tr>
<tr>
<td>Base of structure</td>
<td>Code spectrum Y direction</td>
<td>493</td>
</tr>
</tbody>
</table>

(b) Using Ritz Vectors  
(values from SAP2000 without 85% scaling)

<table>
<thead>
<tr>
<th>Shear Location</th>
<th>Load Case</th>
<th>Shear (kips) for number of modes =</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>12</td>
</tr>
<tr>
<td>Base of 1st story</td>
<td>Code spectrum X direction</td>
<td>435</td>
</tr>
<tr>
<td>Base of structure</td>
<td>Code spectrum X direction</td>
<td>435</td>
</tr>
<tr>
<td>Base of 1st story</td>
<td>Code spectrum Y direction</td>
<td>485</td>
</tr>
<tr>
<td>Base of structure</td>
<td>Code spectrum Y direction</td>
<td>485</td>
</tr>
</tbody>
</table>

### Table 4.1-28 Comparison of Modal Response History Shears Using 12, 18, 120 and 200 Modes

Using Natural Mode Shapes  
(values from SAP2000 without 85% scaling)

<table>
<thead>
<tr>
<th>Shear Location</th>
<th>Load Case</th>
<th>Shear (kips) for number of modes =</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>12</td>
</tr>
<tr>
<td>Base of 1st story</td>
<td>A00 in X direction</td>
<td>438</td>
</tr>
<tr>
<td>Base of structure</td>
<td>A00 in X direction</td>
<td>439</td>
</tr>
<tr>
<td>Base of 1st story</td>
<td>B00 in X direction</td>
<td>376</td>
</tr>
<tr>
<td>Base of structure</td>
<td>B00 in X direction</td>
<td>377</td>
</tr>
<tr>
<td>Base of 1st story</td>
<td>C00 in X direction</td>
<td>391</td>
</tr>
<tr>
<td>Base of structure</td>
<td>C00 in X direction</td>
<td>392</td>
</tr>
<tr>
<td>Base of 1st Story</td>
<td>A00 in Y direction</td>
<td>447</td>
</tr>
<tr>
<td>Base of Structure</td>
<td>A00 in Y direction</td>
<td>440</td>
</tr>
<tr>
<td>Base of 1st Story</td>
<td>B00 in Y direction</td>
<td>391</td>
</tr>
<tr>
<td>Base of Structure</td>
<td>B00 in Y direction</td>
<td>397</td>
</tr>
<tr>
<td>Base of 1st Story</td>
<td>C00 in Y direction</td>
<td>301</td>
</tr>
<tr>
<td>Base of Structure</td>
<td>C00 in Y direction</td>
<td>307</td>
</tr>
</tbody>
</table>
4.1.10 Commentary on the ASCE 7 Requirements for Analysis

As mentioned in this example, ASCE 7-05 contained several inconsistencies in scaling requirements for modal response spectrum analysis and for modal response history analysis. The main source of problems was in Chapter 16 of ASCE 7-05 and fortunately, most of these problems have been eliminated in ASCE 7-10.

There are still a few issues that need to be clarified. Some of these are listed below:

- **Accidental torsion:** The *Standard* needs to be more specific on how accidental torsion should be applied where used with modal response spectrum and modal response history analyses. The method suggested herein, to apply such torsions as part of a static loading, is easy to implement. However, “automatic” methods based on shifting center of mass need to be explored and, if effective, standardized.

- **Amplification of accidental torsion:** Currently, accidental torsion need be amplified only for torsionally irregular structures in SDC C and higher (Sec. 12.8.4.3). However, the torsion need not be amplified if a “dynamic” analysis is performed (Sec. 12.9.5). This implies that the amplification of torsion is a dynamic phenomenon, but the author has found no published technical basis for such amplification. Indeed, most references to amplification are based on problems associated with uneven yielding of lateral load-resisting components. This issue needs to be clarified and resolved.

- **P-Delta effects:** It appears that the most efficient method for handling P-delta effects is to perform the analysis without such effects, use a separate ELF analysis to determine if such effects are significant and if so, magnify forces and displacements to include such effects. It would be much more reasonable to include such effects in the analysis directly and establish procedures to determine if such effects are excessive. Comparison of analysis results with and without P-delta effects included is an effective means to assess the significance of the effects.

- **Computing drift:** When three-dimensional analysis is performed, drift should be checked at the corners of the building, not the center of mass. Consideration should be given to eliminating the use of story drift in favor of computing shear strain in damageable components. Such calculations can be easily automated.

- **Scaling ground motions for linear response history analysis:** The need to scale ground motions over a period range of $0.2T$ to $1.5T$ is not appropriate for elastic analysis because the effective period in any mode will never exceed $1.0T$. Additionally, placing equal weight on scaling spectral ordinates at higher modes does not seem rational. In some cases a high mode that is only barely contributing to response can dominate the scaling process.

- **Development of standard ground motion histories:** The requirement that analysts scan through thousands of ground motion records to find appropriate suites for analysis is unnecessarily burdensome. The *Standard* should provide tables of ground motion suites that are appropriate to simple parameters such as magnitude, site class and distance.

Finally, it is suggested that requirements for linear response history analysis be removed from Chapter 16 and placed in Chapter 12 (as Section 12.10, for example). Requirements for performing such analysis should be as consistent as possible with those of modal response spectrum analysis.
4.2 SIX-­STORY STEEL FRAME BUILDING, SEATTLE, WASHINGTON

In this example, the behavior of a simple, six-story structural steel moment-resisting frame is investigated using a variety of analytical techniques. The structure was initially proportioned using a preliminary analysis and it is this preliminary design that is investigated. The analysis will show that the structure falls short of several performance expectations. In an attempt to improve performance, viscous fluid dampers are considered for use in the structural system. Analysis associated with the added dampers is performed in a very preliminary manner.

The following analytical techniques are employed:

- Linear static analysis
- Plastic strength analysis (using virtual work)
- Nonlinear static (pushover) analysis
- Linear dynamic (response history) analysis
- Nonlinear dynamic (response history) analysis

The primary purpose of this example is to highlight some of the more advanced analytical techniques; hence, more detail is provided on these methods. It is also noted that the linear dynamic analysis was performed only as a precursor and check on the analytical model used for nonlinear dynamic analysis and is not discussed in the example.

The 2005 and 2010 versions of the Standard do not provide any guidance on pushover analysis because it is not a permitted method of analysis in Table 12.6-1. Some guidance for pushover analysis is provided in Resource Paper 2 in Part 3 of the Provisions. More detailed information on pushover analysis is provided in FEMA 440 and in ASCE 41. The procedures outlined in ASCE 41 are used in this example.

Chapter 16 of the Standard provides some guidance and requirements for linear and nonlinear response history analysis. Certain aspects of these requirements are clarified in ASCE 7-10, but the basic methodology is unchanged. More detailed requirements for response history analysis are provided in Resource Paper 3 of the Provisions. This example follows the recommendations in Resource Paper 3, with certain exceptions, which are noted as the example proceeds.

4.2.1 Description of Structure

The structure analyzed for this example is a six-story office building in Seattle, Washington. According to the descriptions in Standard Table 1-1, the building is assigned to Occupancy Category II. From Standard Table 11.5-1, the importance factor (I) is 1.0. A plan and elevation of the building are shown in Figures 4.2-1 and 4.2-2, respectively. The lateral load-resisting system consists of steel moment-resisting frames on the perimeter of the building. There are five bays at 28 feet on center in the north-south (N-S) direction and six bays at 30 feet on center in the east-west (E-W) direction. The typical story height is 12 feet-6 inches with the exception of the first story, which has a height of 15 feet. There is a 5-foot-tall perimeter parapet at the roof and one basement level that extends 15 feet below grade. For this example, it is assumed that the columns of the moment-resisting frames are embedded into pilasters formed into the reinforced concrete basement wall.
For the moment-resisting frames in the N-S direction (Frames A and G), all of the columns bend about their strong axes and the girders are attached with fully welded moment-resisting connections. The expected plastic hinge regions of the girders have reduced flange sections, detailed in accordance with AISC 341.

For the frames in the E-W direction (Frames 1 and 6), moment-resisting connections are used only at the interior columns. At the exterior bays, the E-W girders are connected to the weak axis of the exterior (corner) columns using non-moment-resisting connections. All interior columns are gravity columns and are not intended to resist lateral loads. A few of these columns, however, would be engaged as part of the added damping system described in the last part of this example. With minor exceptions, all of the analyses in this example are for lateral loads acting in the N-S direction. Analysis for lateral loads acting in the E-W direction would be performed in a similar manner.

Figure 4.2-1 Plan of structural system
Prior to analyzing the structure, a preliminary design was performed in accordance with AISC 341. All members, including miscellaneous plates, were designed using steel with a nominal yield stress of 50 ksi and expected yield strength of 55 ksi. Detailed calculations for the design are beyond the scope of this example. Table 4.2-1 summarizes the members selected for the preliminary design.¹

Table 4.2-1 Member Sizes Used in N-S Moment Frames

<table>
<thead>
<tr>
<th>Member supporting level</th>
<th>Column</th>
<th>Girder</th>
<th>Doubler plate thickness (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>W21x122</td>
<td>W24x84</td>
<td>1.00</td>
</tr>
<tr>
<td>6</td>
<td>W21x122</td>
<td>W24x84</td>
<td>1.00</td>
</tr>
<tr>
<td>5</td>
<td>W21x147</td>
<td>W27x94</td>
<td>1.00</td>
</tr>
<tr>
<td>4</td>
<td>W21x147</td>
<td>W27x94</td>
<td>1.00</td>
</tr>
<tr>
<td>3</td>
<td>W21x201</td>
<td>W27x94</td>
<td>0.875</td>
</tr>
<tr>
<td>2</td>
<td>W21x201</td>
<td>W27x94</td>
<td>0.875</td>
</tr>
</tbody>
</table>

¹The term *Level* is used in this example to designate a horizontal plane at the same elevation as the centerline of a girder. The top level, Level R, is at the roof elevation; Level 2 is the first level above grade; and Level 1 is at grade. The term *Story* represents the distance between adjacent levels. The story designation is the same as the designation of the level at the bottom of the story. Hence, Story 1 is the lowest story (between Levels 2 and 1) and Story 6 is the uppermost story (between Levels R and 6).
The sections shown in Table 4.2-1 meet the width-to-thickness requirements for special moment frames and the size of the column relative to the girders should ensure that plastic hinges initially will form in the girders. Due to strain hardening, plastic hinges will eventually form in the columns. However, these form under lateral displacements that are in excess of those allowed under the Design Basis Earthquake (DBE). Doubler plates of 0.875 inch thick are used at each of the interior columns at Levels 2 and 3 and 1.00 inch thick plates are used at the interior columns at Levels 4, 5, 6 and R. Doubler plates were not used in the exterior columns.

### 4.2.2 Loads

#### 4.2.2.1 Gravity loads.

It is assumed that the floor system of the building consists of a normal-weight composite concrete slab formed on metal deck. The slab is supported by floor beams that span in the N-S direction. These floor beams have a span of 28 feet and are spaced 10 feet on center.

The dead weight of the structural floor system is estimated at 70 psf. Adding 15 psf for ceiling and mechanical units, 10 psf for partitions at Levels 2 through 6 and 10 psf for roofing at Level R, the total dead load at each level is 95 psf. The cladding system is assumed to weigh 15 psf.

A basic live load of 50 psf is used at Levels 2 through 6. The roof live load is 20 psf and (based on calculations not shown here) the roof snow load is 25 psf. The reduced floor loads are taken as 0.4(50), or 20 psf. Only half of this load is required in seismic load combinations (see Standard Section 2.3), so the design live loads for the floor is 0.5(20) = 10 psf. The roof live load is not reducible, but never appears in seismic load combinations. The snow load for seismic load combinations is 0.2(25) = 5 psf, which is half of the floor live load.

Based on these loads, the total dead load, live or snow load and dead plus live or snow load applied to each level of the entire building are given in Table 4.2-2. The slight difference in dead loads at Levels R and 2 is due to the parapet and the tall first story, respectively.

Tributary areas for columns and girders as well as individual element gravity loads used in the analysis are illustrated in Figure 4.2-3. These loads are based on a total dead load of 95 psf, a cladding weight of 15 psf and a live load of 10 psf.

<table>
<thead>
<tr>
<th>Level</th>
<th>Dead load (kips)</th>
<th>Reduced live or snow load (kips)</th>
<th>Total load (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Story</td>
<td>Accumulated</td>
<td>Story</td>
</tr>
<tr>
<td>R</td>
<td>2,596</td>
<td>2,596</td>
<td>131</td>
</tr>
<tr>
<td>6</td>
<td>2,608</td>
<td>5,204</td>
<td>262</td>
</tr>
<tr>
<td>5</td>
<td>2,608</td>
<td>7,813</td>
<td>262</td>
</tr>
<tr>
<td>4</td>
<td>2,608</td>
<td>10,421</td>
<td>262</td>
</tr>
<tr>
<td>3</td>
<td>2,608</td>
<td>13,029</td>
<td>262</td>
</tr>
<tr>
<td>2</td>
<td>2,621</td>
<td>15,650</td>
<td>262</td>
</tr>
</tbody>
</table>

*Loads are for the entire building.*
4.2.2.2 **Equivalent static earthquake loads.** Although the main analysis in this example is nonlinear, equivalent static forces are computed in accordance with *Standard* Section 12.8. These forces are used in a preliminary static analysis to determine whether the structure, as designed, conforms to the drift requirements limitations imposed by *Standard* Section 12.2.

The structure is situated in Seattle, Washington. The short period and the 1-second mapped spectral acceleration parameters for the site are as follows:

- \( S_S = 1.63 \)
- \( S_I = 0.57 \)

The structure is situated on Site Class C materials. From *Standard* Tables 11.4-1 and 11.4-2:

- \( F_a = 1.00 \)
- \( F_v = 1.30 \)

From *Standard* Equations 11.4-1 and 11.4-2, the maximum considered spectral acceleration parameters are as follows:

\[
S_{MS} = F_a S_S = 1.00(1.63) = 1.63
\]
\[
S_{MI} = F_v S_I = 1.30(0.57) = 0.741
\]

And from *Standard* Equations 11.4-3 and 11.4-4, the design acceleration parameters are as follows:

\[
S_{DS} = (2/3)S_{MI} = (2/3)1.63 = 1.09
\]
\[
S_{DI} = (2/3)S_{MI} = (2/3)0.741 = 0.494
\]
Figure 4.2-3  Element loads used in analysis
Based on the above coefficients and on Standard Tables 11.6-1 and 11.6-2, the structure is assigned to Seismic Design Category D. For the purpose of analysis, it is assumed that the structure complies with the requirements for a special moment frame, which, according to Standard Table 12.2-1, has the following design values:

- $R = 8$
- $C_d = 5.5$
- $\Omega_0 = 3.0$

Note that the overstrength factor, $\Omega_0$, is not needed for the analysis presented herein.

**4.2.2.2.1 Approximate period of vibration.** Standard Equation 12.8-7 is used to estimate the building period:

$$T_a = C_t h_n^x$$

where, from Standard Table 12.8-2, $C_t = 0.028$ and $x = 0.8$ for a steel moment frame. Using $h_n$ (the total building height above grade) = 77.5 feet, $T_a = 0.028(77.5)^{0.8} = 0.91$ sec/cycle.\(^5\)

Where the period is determined from a properly substantiated analysis, the Standard requires that the period used for computing base shear not exceed $C_u T_a$, where, from Standard Table 12.8-1 (using $S_{DI} = 0.494$), $C_u = 1.4$. For the structure under consideration, $C_u T_a = 1.4(0.91) = 1.27$ seconds. This period is used for base shear calculation as it is expected that the period computed for the actual structure will be greater than 1.27 seconds.

**4.2.2.2.2 Computation of base shear.** Using Standard Equation 12.8-1, the total seismic base shear is:

$$V = C_S W$$

where $W$ is the total seismic weight of the structure. From Standard Equation 12.8-2, the maximum (constant acceleration region) seismic response coefficient is:

$$C_{S_{\text{max}}} = \frac{S_{DS}}{(R/I)} = \frac{1.09}{(8/1)} = 0.136$$

Equation 12.8-3 controls in the constant velocity region:

$$C_S = \frac{S_{\text{DI}}}{T(R/I)} = \frac{0.494}{1.27(8/1)} = 0.0485$$

The seismic response coefficient, however, must not be less than that given by Equation 12.8-5:

$$C_{S_{\text{max}}} = 0.044 S_{DS} = 0.044(1)(1.09) = 0.0480$$

---

\(^5\) The correct computational units for period of vibration is “seconds per cycle”. However, the traditional units of “seconds” are used in the remainder of this example.
Thus, the value from Equation 12.8-3 controls for this building. Using \( W = 15,650 \) kips, \( V = 0.0485(15,650) = 759 \) kips.

### 4.2.2.2 Vertical distribution of forces

The seismic base shear is distributed along the height of the building using Standard Equations 12.8-11 and 12.8-12:

\[
F_x = C_{vx}V \quad \text{and} \quad C_{sx} = \frac{\sum w_i h_i^k}{\sum w_i h_i^k}
\]

where \( k = 0.75 + 0.5T = 0.75 + 0.5(1.27) = 1.385 \). The lateral forces acting at each level and the story shears acting at the bottom of the story below the indicated level are summarized in Table 4.2-3. These are the forces acting on the whole building. For analysis of a single frame, one-half of the tabulated values are used.

<table>
<thead>
<tr>
<th>Level ( x )</th>
<th>( w_x ) (kips)</th>
<th>( h_x ) (ft)</th>
<th>( w_x h_x^k )</th>
<th>( C_{sx} )</th>
<th>( F_x ) (kips)</th>
<th>( V_x ) (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>2,596</td>
<td>77.5</td>
<td>1,080,327</td>
<td>0.321</td>
<td>243.6</td>
<td>243.6</td>
</tr>
<tr>
<td>6</td>
<td>2,608</td>
<td>65.0</td>
<td>850,539</td>
<td>0.253</td>
<td>191.8</td>
<td>435.4</td>
</tr>
<tr>
<td>5</td>
<td>2,608</td>
<td>52.5</td>
<td>632,564</td>
<td>0.188</td>
<td>142.6</td>
<td>578.0</td>
</tr>
<tr>
<td>4</td>
<td>2,608</td>
<td>40.0</td>
<td>433,888</td>
<td>0.129</td>
<td>97.8</td>
<td>675.9</td>
</tr>
<tr>
<td>3</td>
<td>2,608</td>
<td>27.5</td>
<td>258,095</td>
<td>0.077</td>
<td>58.2</td>
<td>734.1</td>
</tr>
<tr>
<td>2</td>
<td>2,621</td>
<td>15.0</td>
<td>111,909</td>
<td>0.033</td>
<td>25.2</td>
<td>759.3</td>
</tr>
<tr>
<td>( \Sigma )</td>
<td>15,650</td>
<td>3,367,323</td>
<td>1.000</td>
<td></td>
<td>759.3</td>
<td></td>
</tr>
</tbody>
</table>

### 4.2.3 Preliminaries to Main Structural Analysis

Performing a nonlinear analysis of a structure is an incremental process. The analyst should first perform a linear analysis to obtain some basic information on expected behavior and to serve later as a form of verification for the more advanced analysis. Once the linear behavior is understood (and extrapolated to expected nonlinear behavior), the anticipated nonlinearities are introduced. If more than one type of nonlinear behavior is expected to be of significance, it is advisable to perform a preliminary analysis with each nonlinearity considered separately and then to perform the final analysis with all nonlinearities considered. This is the approach employed in this example.

### 4.2.3.1 The computer programs NONLIN-Pro and DRAIN 2Dx

The computer program NONLIN-Pro was used for all of the analyses described in this example. This program is basically a pre- and post-processor to DRAIN 2Dx (Prakash et al., 1993). While DRAIN is not the most robust program currently available for performing nonlinear response history analysis, it was used because many of the details of the analysis (e.g., panel zone modeling) must be done explicitly. This detail provides insight into the modeling process which is not available when using the automated features of the more robust software. Note that a full version of NONLIN-Pro, as well as input files used for this example, is provided on the CD.
DRAIN has several shortcomings that are related specifically to the example at hand. These shortcomings are listed below. Also provided is a brief explanation of the influence the shortcoming may have on the analysis.

- It is not possible to model strength loss when using the ASCE 41 model for girder plastic hinges. However, as discussed later in the example, this loss of strength generally occurs at plastic hinge rotations well beyond the rotational demands produced under the DBE ground motions. Maximum plastic rotation angles of plastic hinges were checked with the values in Table 5-6 of ASCE 41-06.

- The DRAIN model for axial-flexural interaction in columns is not particularly accurate. This is of some concern in this example because hinges form at the base of the columns in all of the analyses and in some of the upper columns during analysis with MCE level ground motions.

- Only two-dimensional analysis may be performed. Such an analysis is reasonable for the structure considered in this example because of its regular shape and because full moment connections are provided only in the N-S direction for the corner columns (see Fig. 4.2-1).

As with any finite element analysis program, DRAIN models the structure as an assembly of nodes and elements. While a variety of element types is available, only three element types were used in the analysis:

- Type 1 inelastic bar (truss) element

- Type 2 beam-column element

- Type 4 connection element

Two models of the structure were prepared for DRAIN. The first model, used for preliminary analysis and for verification of the second (more advanced) model, consisted only of Type 2 elements for the main structure and Type 1 elements for modeling P-delta effects. All analyses carried out using this model were linear.

For the second, more detailed model, Type 1 elements were used for modeling P-delta effects and the dampers in the damped system. It was assumed that these elements would remain linear elastic throughout the response. Type 2 elements were used to model the beams, the columns and the braces in the damped system, as well as the rigid links associated with the panel zones. Plastic hinges were allowed to form in all columns. The column hinges form through the mechanism provided in DRAIN’s Type 2 element. Plastic behavior in girders and in the panel zone region of the structure was modeled explicitly through the use of Type 4 connection elements. Girder yielding was forced to occur in the Type 4 elements (in lieu of the main span represented by the Type 2 elements) to provide more control in hinge location and modeling. A complete description of the implementation of these elements is provided later.

4.2.3.2 Description of preliminary model and summary of preliminary results

The preliminary DRAIN model is shown in Figure 4.2-4. Important characteristics of the model are as follows:

- Only a single frame (Frame A or G) is modeled. Hence one-half of the loads shown in Tables 4.2-2 and 4.2-3 are applied.
- Columns are fixed at their base (at grade level; the basement is not modeled).

- Each beam or column element is modeled using a Type 2 element. For the columns, axial, flexural and shear deformations are included. For the girders, flexural and shear deformations are included but, because of diaphragm slaving, axial deformation is not included. Composite action in the floor slab is ignored for all analysis.

- All members are modeled using centerline dimensions without rigid end offsets. This approach allows for the effects of panel zone deformation to be included in an approximate but reasonably accurate manner. Note that this model does not provide any increase in beam-column joint stiffness due to the presence of doubler plates. The stiffness of the girders was decreased by 7 percent (in preliminary analyses) to account for the reduced flange sections. Moment rotation properties of the reduced flange sections are used in the detailed analyses.

P-delta effects are modeled using the leaner “ghost” column shown in Figure 4.2-4 at the right of the main frame. This column is modeled with an axially rigid truss element. P-delta effects are activated for this column only (P-delta effects are turned off for the columns of the main frame). The lateral degree of freedom at each level of the P-delta column is slaved to the floor diaphragm at the matching elevation. Where P-delta effects are included in the analysis, a special initial load case was created and executed. This special load case consists of a vertical force equal to one-half of the total story weight (dead load plus 50 percent of the fully reduced live load) applied to the appropriate node of the P-delta column. When P-delta effects are included, modal analysis should be performed after the P-delta load case is applied so that stiffness modification of P-delta effects will increase the period of the structure. P-delta effects are modeled in this manner to provide true column axial forces for assessing strength.

![Frame A or G and P-Δ column](image)

**Figure 4.2-4** Simple wire frame model used for preliminary analysis

### 4.2.3.2.1 Results of preliminary analysis: period of vibration and drift

The computed periods for the first three natural modes of vibration are shown in Table 4.2-4. As expected, the period including P-delta effects is slightly larger than that produced by the analysis without such effects. More significant is the fact that the first mode period is considerably longer than that predicted from *Standard*...
Equation 12.8-7. Recall from previous calculations that this period \( T_a \) is 0.91 seconds and the upper limit on the computed period \( C_u T_a \) is \( 1.4(0.91) = 1.27 \) seconds. Where doubler plate effects are included in the detailed analysis, the period will decrease slightly, but it remains obvious that the structure is quite flexible.

<table>
<thead>
<tr>
<th>Mode</th>
<th>P-delta excluded</th>
<th>P-delta included</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.054</td>
<td>2.130</td>
</tr>
<tr>
<td>2</td>
<td>0.682</td>
<td>0.698</td>
</tr>
<tr>
<td>3</td>
<td>0.373</td>
<td>0.379</td>
</tr>
</tbody>
</table>

The results of the preliminary analysis for drift are shown in Tables 4.2-5 and 4.2-6 for the computations excluding and including P-delta effects, respectively. In each table, the deflection amplification factor \( C_d \) equals 5.5 and the acceptable story drift (story drift limit) is taken as 2 percent of the story height, which is the limit provided by Standard Table 12.12-1. In the Standard it is permitted to determine the elastic drifts using seismic design forces based on the computed fundamental period of the structure without the upper limit \( C_u T_a \). Thus a new set of lateral loads based on the computed period of the actual structure is applied to the structure to calculate the elastic drifts.

Where P-delta effects are not included, the computed story drift is less than the allowable story drift at each level of the structure. The largest magnified story drift, including \( C_d = 5.5 \), is 2.26 inches in Stories 2 and 3. As a preliminary estimate of the importance of P-delta effects, story stability coefficients, \( \theta \), were computed in accordance with Standard Section 12.8-7. These are shown in the last column of Table 4.2-5. At Story 2, the stability coefficient is 0.0862. According to the Standard, P-delta effects may be ignored where the stability coefficient is less than 0.10. For this example, however, analyses are performed with and without P-delta effects.

When P-delta effects are included (Table 4.2-6), the drifts can also be estimated as the drifts without P-delta times the quantity \( 1/(1-\theta) \), where \( \theta \) is the stability coefficient for the story. As can be seen in Table 4.2-6, drifts calculated in this manner are consistent with the results obtained by running the analyses with P-delta effects. The difference is always less than 2 percent.

<table>
<thead>
<tr>
<th>Story</th>
<th>Total drift (in.)</th>
<th>Story drift (in.)</th>
<th>Magnified story drift (in.)</th>
<th>Drift limit (in.)</th>
<th>Story stability ratio, ( \theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>2.08</td>
<td>0.22</td>
<td>1.21</td>
<td>3.00</td>
<td>0.0278</td>
</tr>
<tr>
<td>5</td>
<td>1.86</td>
<td>0.32</td>
<td>1.76</td>
<td>3.00</td>
<td>0.0453</td>
</tr>
<tr>
<td>4</td>
<td>1.54</td>
<td>0.38</td>
<td>2.09</td>
<td>3.00</td>
<td>0.0608</td>
</tr>
<tr>
<td>3</td>
<td>1.16</td>
<td>0.41</td>
<td>2.26</td>
<td>3.00</td>
<td>0.0749</td>
</tr>
<tr>
<td>2</td>
<td>0.75</td>
<td>0.41</td>
<td>2.26</td>
<td>3.00</td>
<td>0.0862</td>
</tr>
<tr>
<td>1</td>
<td>0.34</td>
<td>0.34</td>
<td>1.87</td>
<td>3.60</td>
<td>0.0691</td>
</tr>
</tbody>
</table>
### Table 4.2-6 Results of Preliminary Analysis Including P-delta Effects

<table>
<thead>
<tr>
<th>Story</th>
<th>Total drift (in.)</th>
<th>Story drift (in.)</th>
<th>Magnified story drift (in.)</th>
<th>Drift from $\theta$ (in.)</th>
<th>Drift limit (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>2.23</td>
<td>0.23</td>
<td>1.27</td>
<td>1.24</td>
<td>3.00</td>
</tr>
<tr>
<td>5</td>
<td>2.00</td>
<td>0.34</td>
<td>1.87</td>
<td>1.84</td>
<td>3.00</td>
</tr>
<tr>
<td>4</td>
<td>1.66</td>
<td>0.40</td>
<td>2.20</td>
<td>2.23</td>
<td>3.00</td>
</tr>
<tr>
<td>3</td>
<td>1.26</td>
<td>0.45</td>
<td>2.48</td>
<td>2.44</td>
<td>3.00</td>
</tr>
<tr>
<td>2</td>
<td>0.81</td>
<td>0.45</td>
<td>2.48</td>
<td>2.47</td>
<td>3.00</td>
</tr>
<tr>
<td>1</td>
<td>0.36</td>
<td>0.36</td>
<td>1.98</td>
<td>2.01</td>
<td>3.60</td>
</tr>
</tbody>
</table>

### 4.2.3.2.2 Results of preliminary analysis: demand-to-capacity ratios.

To determine the likelihood and possible order of yielding, demand-to-capacity ratios (DCR) are computed for each element. The results are shown in Figure 4.2-5. For this analysis, the structure is subjected to full dead load plus 0.5 times the fully reduced live load, followed by equivalent lateral forces computed without the $R$ factor. P-delta effects are included. Figure 4.2-5(a) displays the DCR of columns and girders and Figure 4.2-5(b) displays the DCR of panel zones with and without doubler plates. In Figure 4.2-5b, the values in parentheses represent the DCRs without doubler plates. Since the DCRs in Figure 4.2-5 are found from preliminary analyses, in which the centerline model is used, doubler plates aren’t added into the model. Thus, the demand values of Figure 4.2-5(b) are the same with and without doubler plates. However, since the capacity of the panel zone increases with added doubler plates, the DCRs decrease at the interior beam column joints as the doubler plates are used only at the interior joints. As may be seen in Figure 4.2-5(b), the DCR at the exterior joints are the same with and without doubler plates added.

For girders, the DCR is simply the maximum moment in the member divided by the member’s plastic moment capacity where the plastic capacity is $ZeF_{ye}$. $Ze$ is the plastic section modulus at center of reduced beam section and $F_{ye}$ is the expected yield strength. For columns, the ratio is similar except that the plastic flexural capacity is estimated to be $Z_{col}(F_{ye} - P_u/A_{col})$ where $P_u$ is the total axial force in the column. The ratios are computed at the center of the reduced section for beams and at the face of the girder for columns.

To find the shear demand at the panel zones, the total moment in the girders (at the left and right sides of the joint) is divided by the effective beam depth to produce the panel shear due to beam flange forces. Then the column shear at above or below the panel zone joint was subtracted from the beam flange shears and the panel zone shear force is obtained. This force is divided by the shear strength capacity, $R_v$ (which is discussed in Section 4.2.4.2) to determine the DCR of the panel zones.

Several observations are made regarding the likely inelastic behavior of the frame:

- The structure has considerable overstrength, particularly at the upper levels.

- The sequence of yielding will progress from the lower-level girders to the upper-level girders. Because of relatively low live load, the DCRs in the girders are almost uniform at each level. Hence, all the hinges in the girders in a level will form almost simultaneously.

- With the possible exception of the first level, the girders should yield before the columns. While not shown in the table, the DCRs for the lower-story columns are controlled by the moment at the base of the column. It is usually very difficult to prevent yielding of the base of the first-story
columns in moment frames and this frame is no exception. The column on the leeward (right) side of the building will yield first because of the additional axial compressive force arising from the seismic effects.

- The maximum DCR of the columns and girders is 3.475, while the maximum DCR for the panel zones without doubler plates is 4.339. Thus, if doubler plates aren’t used, the first yield in the structure is in the panel zones. However, with doubler plates added, the first yield is at the girders as the maximum DCR of the panel zones reduces to 2.405.
4.2.3.2.3 Results of preliminary analysis: overall system strength. The last step in the preliminary analysis is to estimate the total lateral strength (collapse load) of the frame using virtual work. In the analysis, it is assumed that plastic hinges are perfectly plastic. Girders hinge at a value \( \frac{Z_e F_{y_e}}{e} \) and the hinges form at the center of the reduced section (approximately 15 inches from the face of the column).
Columns hinge only at the base and the plastic moment capacity is assumed to be \( Z_{col}(F_{ye} - P_u/A_{col}) \). The fully plastic mechanism for the system is illustrated in Figure 4.2-6. The inset to the figure shows how the angle modification term, \( \sigma \), was computed. The strength, \( V \), for the total structure is computed from the following relationships (see Figure 4.2-6 for nomenclature):

- Internal Work = External Work
- Internal Work = \( 2[20\sigma \theta M_{PA} + 40\sigma \theta M_{PB} + \theta(M_{PC} + 4M_{PD} + M_{PE})] \)
- External Work = \( V \theta \sum_{i=1}^{nLevels} F_i H_i \) where \( \sum_{i=1}^{nLevels} F_i = 1 \)

Three lateral force patterns are used: uniform, upper triangular and Standard (where the Standard pattern is consistent with the vertical force distribution of Table 4.2-3 in this volume of design examples). The results of the analysis are shown in Table 4.2-7. As expected, the strength under uniform load is significantly greater than under triangular or Standard load. The closeness of the Standard and triangular load strengths results from the vertical-load-distributing parameter \( (k = 1.385) \) being close to 1.0.

The ELF base shear, 759 kips (see Table 4.2-3), when divided by the Standard pattern capacity, 2,616 kips, is 0.29. This is reasonably consistent with the DCRs shown in Figure 4.2-5.

<table>
<thead>
<tr>
<th>Lateral Load Pattern</th>
<th>Lateral strength for entire structure (kips)</th>
<th>Lateral strength single frame (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>3,332</td>
<td>1,666</td>
</tr>
<tr>
<td>Upper Triangular</td>
<td>2,747</td>
<td>1,373</td>
</tr>
<tr>
<td>Standard</td>
<td>2,616</td>
<td>1,308</td>
</tr>
</tbody>
</table>
Three important points concerning the virtual work analysis are as follows:

- The rigid-plastic analysis does not include strain hardening, which is an additional source of overstrength.

- The rigid-plastic analysis does not consider the true behavior of the panel zone region of the beam-column joint. Yielding in this area can have a significant effect on system strength.

- Slightly more than 15 percent of the system strength comes from plastic hinges that form in the columns. If the strength of the column is taken simply as $M_p$ (without the influence of axial force), the difference in total strength is less than 2 percent.

### 4.2.4 Description of Model Used for Detailed Structural Analysis

Nonlinear static and nonlinear dynamic analyses require a much more detailed model than was used in the linear analysis. The primary reason for the difference is the need to explicitly represent yielding in the girders, columns and panel zone region of the beam-column joints.
The DRAIN model used for the nonlinear analysis is shown in Figure 4.2-7. A detail of a girder and its connection to two interior columns is shown in Figure 4.2-8. The detail illustrates the two main features of the model: an explicit representation of the panel zone region and the use of concentrated plastic hinges in the girders.

In Figure 4.2-7, the column shown to the right of the structure is used to represent P-delta effects. See Section 4.2.3.2 for details.

Figure 4.2-7 Detailed analytical model of six-story frame

Figure 4.2-8 Model of girder and panel zone region

The development of the numerical properties used for panel zone and girder hinge modeling is not straightforward. For this reason, the following theoretical development is provided before proceeding with the example.

4.2.4.1 Plastic hinge modeling and compound nodes. In the analysis described below, much use is made of compound nodes. These nodes are used to model plastic hinges in girders and deformations in the panel zone region of beam-column joints.
A compound node typically consists of a pair of single nodes with each node sharing the same point in space. The X and Y degrees of freedom of the first node of the pair (the slave node) are constrained to be equal to the X and Y degrees of freedom of the second node of the pair (the master node), respectively. Hence, the compound node has four degrees of freedom: an X displacement, a Y displacement and two independent rotations.

In most cases, one or more rotational spring connection elements (DRAIN element Type 4) are placed between the two single nodes of the compound node and these springs develop bending moment in resistance to the relative rotation between the two single nodes. If no spring elements are placed between the two single nodes, the compound node acts as a moment-free hinge. A typical compound node with a single rotational spring is shown in Figure 4.2-9. The figure also shows the assumed bilinear, inelastic moment-rotation behavior for the spring.
4.2.4.2 Modeling of beam-column joint regions. A very significant portion of the total story drift of a moment-resisting frame is due to deformations that occur in the panel zone region of the beam-column joint. In this example, panel zones are modeled explicitly using an approach developed by Krawinkler (1978) and described in more detail in Charney and Marshall (2006). Only a brief overview is presented here.

This model, illustrated in Figure 4.2-10, represents the panel zone stiffness and strength by an assemblage of four rigid links and two rotational springs. The links form the boundary of the panel and the springs are used to provide the desired inelastic behavior. The model has the advantage of being conceptually
simple, yet robust. The disadvantage of the model is that the number of degrees of freedom required to model a structure is significantly increased.\textsuperscript{4}

![Figure 4.2-10 Krawinkler beam-column joint model](image)

The Krawinkler model assumes that the panel zone has two resistance mechanisms acting in parallel:

- Shear resistance of the web of the column, including doubler plates
- Flexural resistance of the flanges of the column

These two resistance mechanisms, apparent in AISC 360 Section J10-11, are used for determining panel zone shear strength:

\[
R_v = 0.6F_yd_c t_p \left[ 1 + \frac{3b_{cf}t_{cf}^2}{d_b d_c t_p} \right]
\]

The equation can be rewritten as:

\[
R_v = 0.6F_yd_c t_p + 1.8 \frac{F_yb_{cf}t_{cf}^2}{d_b} = V_{\text{Panel}} + 1.8V_{\text{Flanges}}
\]

In ASCE 41, the first term of the above equation is taken as \(0.55F_yd_c t_p\) and the second term is neglected conservatively. In this study, the following equation—in which the first term is taken as the same as in

\textsuperscript{4} The numbers of degrees of freedom in the Krawinkler model may be reduced to only four if the rigid links around the perimeter of the model are represented by mathematical constraints instead of stiff elements. Most commercial programs employ this approach for the Krawinkler model.
ASCE 41 and the second term is taken from AISC 360, with the exception of replacing nominal yield stress with expected yield strength (for consistency)—is used to calculate the panel zone shear strength:

\[ R_v = 0.55F_{ye} d_c t_p + 1.8 \frac{F_{ye} b_{cf} t_{cf}}{d_b} = V_{\text{Panel}} + 1.8V_{\text{Flanges}} \]

where the first term is the panel shear resistance and the second term is the plastic flexural resistance of the column flange. The terms in the equations are defined as follows:

- \( F_{ye} \) = expected yield strength of the column and the doubler plate
- \( d_c \) = total depth of column
- \( t_p \) = thickness of panel zone region = column web thickness plus doubler plate thickness
- \( b_{cf} \) = width of column flange
- \( t_{cf} \) = thickness of column flange
- \( d_b \) = total depth of girder

Additional terms used in the subsequent discussion are:

- \( t_{gf} \) = girder flange thickness
- \( G \) = shear modulus of steel
The panel zone shear resistance, $V_{Panel}$, is simply the effective shear area of the panel, $d_c f_p$, multiplied by the yield stress in shear, assumed as $0.55 F_{ye}$. (The 0.55 factor is a simplification of the Von Mises yield criterion that gives the yield stress in shear as $1/\sqrt{3} = 0.577$ times the strength in tension.) The additional plastic flexural resistance provided by yielding in the column flange is neglected in ASCE 41-06 but is included herein.

The second term, $1.8 V_{Flanges}$ is based on experimental observation. Testing of simple beam-column subassemblies show that a “kink” forms in the column flanges as shown in Figure 4.2-11(a). If it can be assumed that the kink is represented by a plastic hinge with a plastic moment capacity of $M_p = F_{ye} Z = F_{ye} b_c A^f / 4$, it follows from virtual work (see Figure 4.2-11b) that the equivalent shear strength of the column flanges is:

$$ V_{Flanges} = \frac{4M_p}{d_b} $$
and by simple substitution for $M_p$:

$$V_{Flanges} = \frac{F_{ye} b_f t_{cf}^2}{d_b}$$

This value does not include the 1.8 multiplier that appears in the AISC equation. This multiplier is based on calibration of experimental results. It should be noted that the flange component of strength is small compared to the panel component unless the column has very thick flanges.

The shear stiffness of the panel is derived as shown in Figure 4.2-12:

$$K_{Panel,\gamma} = \frac{V_{Panel}}{\gamma} = \frac{V_{Panel}}{\delta/d_b}$$

noting that the displacement $\delta$ can be written as follows:

$$\delta = \frac{V_{Panel} d_b}{Gt_p d_c}$$

$$K_{Panel,\gamma} = \frac{V_{Panel}}{Gt_p d_c} = \frac{1}{d_b}$$

![Figure 4.2-12](image)

**Figure 4.2-12** Column web component of panel zone resistance

Krawinkler assumes that the column flange component yields at four times the yield deformation of the panel component, where the panel yield deformation is:

$$\gamma_y = \frac{V_{Panel}}{K_{Panel,\gamma}} = \frac{0.55F_{ye}d_c t_p}{Gd_c t_p} = \frac{0.55F_{ye}}{G}$$

At this deformation, the panel zone strength is $V_{Panel} + 0.25 V_{flanges}$; at four times this deformation, the strength is $V_{Panel} + V_{Flanges}$. The inelastic force-deformation behavior of the panel is illustrated in
Figure 4.2-13. This figure applies also to exterior joints (girder on one side only), roof joints (girders on both sides, column below only) and corner joints (girder on one side only, column below only).

The actual Krawinkler model is shown in Figure 4.2-10. This model consists of four rigid links, connected at the corners by compound nodes. The columns and girders frame into the links at right angles at Points I through L. These are moment-resisting connections. Rotational springs are used at the upper left (Point A) and lower right (Point D) compound nodes. These springs are used to represent the panel resistance mechanisms described earlier. The upper right and lower left corners (Points B and C), without rotational springs, act as real hinges.

The finite element model of the joint requires 12 individual nodes: one node each at Points I through L and two nodes (compound node pairs) at Points A through D. It is left to the reader to verify that the total number of degrees of freedom in the model is 28 (if the only constraints are associated with the corner compound nodes).

The rotational spring properties are related to the panel shear resistance mechanisms by a simple transformation, as shown in Figure 4.2-14. From the figure it may be seen that the moment in the rotational spring is equal to the applied shear times the beam depth. Using this transformation, the properties of the rotational spring representing the panel shear component of resistance are as follows:

\[ M_{\text{Panel}} = V_{\text{Panel}}d_h = 0.55F_{\gamma}d_cd_bt_p \]

\[ K_{\text{Panel,}\theta} = K_{\text{Panel,}\gamma}d_b = Gd_cd_bt_p \]
It is interesting to note that the shear strength in terms of the rotation spring is simply $0.55F_{ye}$ times the volume of the panel and the shear stiffness in terms of the rotational spring is equal to $G$ times the panel volume.

The flange component of strength in terms of the rotational spring is determined in a similar manner:

$$M_{Flanges} = 1.8V_{Flanges}d_b = 1.8F_{ye} b_{cf} t_{cf}^2$$

Because of the equivalence of rotation and shear deformation, the yield rotation of the panel is the same as the yield strain in shear:

**Figure 4.2-14** Transforming shear deformation to rotational deformation in the Krawinkler model
To determine the initial stiffness of the flange spring, it is assumed that this spring yields at four times the yield deformation of the panel spring. Hence:

\[ K_{\text{Flanges}, \theta} = \frac{M_{\text{Flanges}}}{4\theta_y} = 0.82Gb_{cf}I_{cf}^2 \]

The complete resistance mechanism, in terms of rotational spring properties, is shown in Figure 4.2-13. This trilinear behavior is represented by two elastic-perfectly plastic springs at the opposing corners of the joint assemblage.

If desired, strain-hardening may be added to the system. ASCE 41 suggests use of a strain hardening stiffness equal to 6 percent of the initial stiffness of the joint. In this analysis, the strain-hardening component was simply added to both the panel and the flange components:

\[ K_{SH, \theta} = 0.06(K_{\text{Panel}, \theta} + K_{\text{Flanges}, \theta}) \]

Before continuing, one minor adjustment is made to the above derivations. Instead of using the nominal total beam and girder depths in the calculations, the distance between the center of the flanges was used as the effective depth. Hence:

\[ d_c = d_{c,\text{nom}} - t_{cf} \]

where the \textit{nom} part of the subscript indicates the property listed as the total depth in the AISC Manual.

The Krawinkler properties are now computed for a typical interior subassembly of the six-story frame. A summary of the properties used for all connections is shown in Table 4.2-8.

| Table 4.2-8 Properties for the Krawinkler Beam-Column Joint Model |
|---|---|---|---|---|---|---|
| Connection | Girder | Column | Doubler plate (in.) | \( M_{\text{panel}, \theta} \) (in.-k) | \( K_{\text{panel}, \theta} \) (in.-k/rad) | \( M_{\text{flanges}, \theta} \) (in.-k) | \( K_{\text{flanges}, \theta} \) (in.-k/rad) |
| A | W24x84 | W21x122 | – | 8,782 | 3,251,567 | 1,131 | 104,721 |
| B | W24x84 | W21x122 | 1.00 | 23,419 | 8,670,846 | 1,131 | 104,721 |
| C | W27x94 | W21x147 | – | 11,934 | 4,418,647 | 1,637 | 151,486 |
| D | W27x94 | W21x147 | 1.00 | 28,510 | 10,555,656 | 1,637 | 151,486 |
| E | W27x94 | W21x201 | – | 15,386 | 5,696,639 | 3,314 | 306,771 |
| F | W27x94 | W21x201 | 0.875 | 30,180 | 11,174,176 | 3,314 | 306,771 |

Example calculations shown for row in \textbf{bold} type.

The sample calculations below are for Connection D in Table 4.2-8.

- Material Properties:
F_{ye} = 55.0 ksi (girder, column and doubler plate)

G = 11,200 ksi

- **Girder:**
  
  W27x94
  
  $d_{b, nom} = 26.90$ in.
  
  $t_{bf} = 0.745$ in.
  
  $d_b = 26.16$ in.

- **Column:**
  
  W21x147
  
  $d_{c, nom} = 22.10$ in.
  
  $t_w = 0.72$ in.
  
  $t_{cf} = 1.150$ in.
  
  $d_c = 20.95$ in.
  
  $b_{cf} = 12.50$ in.

- **Doubler plate:** 1.00 in.

  Total panel zone thickness = $t_p = 0.72 + 1.00 = 1.72$ in.

  $V_{Panel} = 0.55F_{ye}d_c t_p = 0.55(55)(20.95)(1.72) = 1,090$ kips

  $V_{Flanges} = 1.8 \frac{F_{yw} d_f t_f^2}{d_b} = 1.8 \frac{55(12.50)(1.15^3)}{26.16} = 62.6$ kips

  $K_{Panel, y} = G t_p d_c = 11,200(1.72)(20.95) = 403,581$ kips/unit shear strain

  $\gamma_y = \frac{0.55F_{yw}}{G} = \frac{0.55(55)}{11,200} = 0.0027$

  $M_{Panel} = V_{Panel} d_b = 1,090(26.16) = 28,510$ in.-kips

  $K_{Panel, \theta} = K_{Panel, y} d_b = 403,581(26.16) = 10,555,656$ in.-kips/radian
Flanges Flanges bM Vd == 1,637 in.-kips

\[
M_{\text{Flanges}} = V_{\text{Flanges}} d_b = 62.6(26.16) = 1,637 \text{ in.-kips}
\]

\[
K_{\text{Flanges,}_\theta} = \frac{M_{\text{Flanges}}}{4\gamma_y} = \frac{1,637}{4(0.0027)} = 151,486 \text{ in.-kips/radian}
\]

4.2.4.3 Modeling girders. Because this structure is designed in accordance with the strong-column/weak-beam principle, it is anticipated that the girders will yield in flexure. Although DRAIN provides special yielding beam elements (Type 2 elements), more control over behavior is obtained through the use of the Type 4 connection element.

The modeling of a typical girder is shown in Figure 4.2-8. This figure shows an interior girder, together with the panel zones at the ends. The portion of the girder between the panel zones is modeled as four segments with one simple node at mid-span and one compound node near each end. The mid-span node is used to enhance the deflected shape of the structure.\(^5\) The compound nodes are used to represent inelastic behavior in the hinging region.

The following information is required to model each plastic hinge:

- The initial stiffness (moment per unit rotation)
- The effective yield moment
- The secondary stiffness
- The location of the hinge with respect to the face of the column

AISC SDM recommends design practices to force the plastic hinge forming in the beam away from the face of the column. There are two methods used to move the plastic hinges of the beam away from the column face. The first one aims to reduce the cross-sectional properties of the beam at a specific location away from the column, and the second one focuses on special detailing of the beam-column connection to provide adequate strength and toughness in the connection so that inelasticity will be forced into the beam adjacent to the column face. In this study the reduced beam section (RBS) was used.

A side view of the reduced beam sections is shown in Figure 4.2-15. The distance between the column face and the edge of the reduced beam section was chosen as \(a = 0.625b\), and the reduced section length was assumed as \(b = 0.75d_b\). Both of these values are just at the middle of the limits stated in AISC 358. Plastic hinges of the beams are modeled at the center of the reduced section length.

---

\(^5\) A graphic post-processor was used to display the deflected shape of the structure. The program represents each element as a straight line. Although the computational results are unaffected, a better graphical representation is obtained by subdividing the member.
To determine the plastic hinge capacities of the girder cross section, a moment-curvature analysis, which is dependent on the stress-strain curve of the steel, was implemented. The idealized stress-strain curve is shown in Figure 4.2-16. This curve does not display a yield plateau, which is consistent with the assumption that the section has yielded in previous cycles, with the Bauschinger effect eliminating any trace of the yield plateau. The strain hardening ratio is taken as 3 percent of the initial stiffness and the curvature ductility limit used is 20.

To compute the moment-curvature relationship, the girder is divided into 50 slices through its depth, with 10 slices in each flange and 30 slices at the web. By gradually increasing the rotation, fiber strains, fiber stresses, fiber forces and then the resisting moment are found consecutively. Figure 4.2-17 shows the top view of the assumed reduced beam section in this study. The reduced beam length is divided into seven equal sections and flange widths of each section are calculated using the radius of the cut. The radius of the cut, \( R \), is calculated using the formulas in AISC 358.

\[
R = \frac{(4c^2 + b^2)}{8c}
\]

\[c = 0.175b_f\]

\[b = 0.75d_b\]

\[a = 0.625b_f\]

where:

\[c = \text{depth of cut at center of the RBS, in.}\]
\( b_f = \) width of beam flange, in.

\( b = \) length of RBS cut, in.

\( d_b = \) beam depth, in.

\( a = \) distance from face of the column to start of RBS cut, in.

\( R = \) radius of cut, in.

**Figure 4.2-16** Assumed stress-strain curve for modeling girders

**Figure 4.2-17** Top view of RBS

Figure 4.2-18 shows the moment-curvature diagram for the W27x94 girder. As may be seen in the figure, the moment-curvature relationship is different at each segment of the reduced length. The locations of the different reduced beam sections used in Figure 4.2-18, named as “\( b_{f1} \)”, “\( b_{f2} \)” and “\( b_{f3} \)”, can be seen in
Figure 4.2-17. Because of the closely adjacent locations chosen for “0.65bf” and “bf3” (see Figure 4.2-17), their moment-curvature plots are nearly indistinguishable from each other in Figure 4.2-18.

![Figure 4.2-18 Moment-curvature diagram for W27x94 girder](image)

A tip loaded cantilever beam analysis using half of the clear span length is used to generate the moment-rotation relationship for the inelastic hinges. For regular beams, where a cantilever beam is tip loaded, the moment diagram is linear and the curvature diagram is also linear as long as the moment along the beam remains in the elastic region (see Figure 4.2-19). If the moment along the beam exceeds the yield moment, the curvature along the beam will be as shown in Figure 4.2-20.

![Figure 4.2-19 Tip loaded cantilever beam and moment diagram for cantilever beam](image)
Because a RBS is used in this study, the curvature diagrams are different from those for regular beams. As may be seen in Figure 4.2-21, the curvatures in the reduced flange region of the beam have a distinctive “bump”. Because the moment diagram of the tip loaded cantilever beam is always linear, the moment values can be found easily at the different sections of the reduced flange, and then the corresponding curvature values can be assigned from the moment curvature diagram (Figure 4.2-18) to the curvature diagram along half of the clear span length (Figure 4.2-21).

Figure 4.2-21 shows the curvature diagram when the curvature ductility reaches 20. The curvature difference (the bump at the center of RBS in Figure 4.2-21) section is less prominent when the ductility is smaller. Given the curvature distribution along the cantilever beam length, the deflections at the point of load (tip deflections) can be found using the moment area method. Figure 4.2-22 illustrates the force-displacement relationship at the end of the half span cantilever for the W27x94 with the reduced flange section.
To convert the force-tip displacement diagram into moment-rotation of the plastic hinge, the following procedure is followed:

1. Using the trilinear force displacement relationship shown in Figure 4.2-22, find the moment at the plastic hinge for $P_1$, $P_2$ and $P_3$ load levels and name them $M_1$, $M_2$ and $M_3$. To find the moments, the tip forces ($P_1$, $P_2$ and $P_3$) are multiplied by the distance from the center of the reduced section to the tip of the cantilever.

2. Calculate the change in moment for each added load (for example: $dM_1 = M_2 - M_1$).

3. Find the flexural rigidity ($EI$) of the beam given a tip displacement of 1 inch under the first load ($P_1$ in Figure 4.2-22).

4. Calculate the required rotational stiffness of the hinge between $M_1$ and $M_2$ and then $M_2$ and $M_3$.

5. Calculate the change in rotation from $M_1$ to $M_2$ and from $M_2$ to $M_3$, by dividing the change in moment found at Step 2 by the required rotational stiffness values calculated at Step 4.

6. Find the specific rotations at $M_1$, $M_2$ and $M_3$ using the change in rotation values found in Step 5. Note that the rotation is zero at $M_1$.

7. Plot a moment-rotation diagram of the plastic hinge using the values calculated at Step 1 and Step 6.

Figure 4.2-23 shows the moment-rotation diagrams for the plastic hinges of both of the girders used in the models. Note that two bilinear springs (Components 1 and 2) are needed to represent the trilinear behavior shown in the figure.
The properties for the W24x84 and W27x94 girder are shown in Table 4.2-9. Note that the first yield of the model is the yield moment from Component 1.

### Table 4.2-9 Girder Properties as Modeled in DRAIN

<table>
<thead>
<tr>
<th>Property</th>
<th>W24x84</th>
<th>W27x94</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic Properties</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Moment of Inertia (in. (^4))</td>
<td>2,370</td>
<td>3,270</td>
</tr>
<tr>
<td>Shear Area (in. (^2))</td>
<td>11.3</td>
<td>13.2</td>
</tr>
<tr>
<td>Inelastic Component 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yield Moment (in.-kip)</td>
<td>8,422</td>
<td>10,458</td>
</tr>
<tr>
<td>Initial Stiffness (in.-kip/radian)</td>
<td>(1 \times 10^{10})</td>
<td>(1 \times 10^{10})</td>
</tr>
<tr>
<td>S.H. Ratio</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Inelastic Component 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yield Moment (in.-kip)</td>
<td>2,075</td>
<td>2,615</td>
</tr>
<tr>
<td>Initial Stiffness (in.-kip/radian)</td>
<td>287,550</td>
<td>337,020</td>
</tr>
<tr>
<td>S.H. Ratio</td>
<td>0.217</td>
<td>0.232</td>
</tr>
<tr>
<td>Comparative Property</td>
<td>Plastic Moment = (Z_c F_{yw})</td>
<td>9,200</td>
</tr>
</tbody>
</table>

### 4.2.4.4 Modeling columns. All columns in the analysis are modeled in DRAIN with Type-2 elements. Preliminary analysis indicated that columns should not yield, except at the base of the first story. Subsequent analysis shows that the columns will yield in the upper portion of the structure as well. For this reason, column yielding must be activated in all of the Type-2 column elements. The columns are modeled using the built-in yielding functionality of the DRAIN program, wherein the yield moment is a function of the axial force in the column. The yield surfaces used by DRAIN for all the columns in the model are shown in Figure 4.2-24.
The rules employed by DRAIN to model column yielding are adequate for event-to-event nonlinear static pushover analysis, but leave much to be desired where dynamic analysis is performed. The greatest difficulty in the dynamic analysis is adequate treatment of the column when unloading and reloading. An assessment of the effect of these potential problems is beyond the scope of this example.

4.2.4.5 Results of detailed analysis.

4.2.4.5.1 Period of vibration. Table 4.2-10 tabulates the first three natural modes of vibration for models with and without doubler plates. While the P-delta effects increase the period, the doubler plates decrease the period because the model becomes stiffer with doubler plates. As may be seen, different period values are obtained from preliminary and detailed analyses (see Table 4.2-4). The detailed model results in a slightly stiffer structure than the preliminary model especially when doubler plates are added.

<table>
<thead>
<tr>
<th>Model</th>
<th>Mode</th>
<th>P-delta excluded</th>
<th>P-delta included</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong Panel with Doubler Plates</td>
<td>1</td>
<td>1.912</td>
<td>1.973</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.627</td>
<td>0.639</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.334</td>
<td>0.339</td>
</tr>
<tr>
<td>Weak Panel without Doubler Plates</td>
<td>1</td>
<td>2.000</td>
<td>2.069</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.654</td>
<td>0.668</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.344</td>
<td>0.349</td>
</tr>
</tbody>
</table>

4.2.4.5.2 Demand-to-capacity ratios. DCRs are found for the detailed analyses with the same load combination used for the preliminary analyses. The main reason for repeating the DCR for the detailed
model is to make a comparison with the DCR of the preliminary model. The detailed analyses include the advanced panel zone, girder and column modeling discussed in Section 4.2.4. Figures 4.2-25(a) and 4.2-25(b) illustrate the DCR of the beams with columns and panel zones of the detailed model respectively. In both figures, the values in the parentheses represent the DCR with no doubler plates added to the structure.

The DCR values of the detailed analyses are similar to those of the preliminary analysis displayed in Figure 4.2-5. The girders of the first and fifth bays at the third level have the maximum DCR in both the preliminary and detailed analyses. Similar trends are also observed for the DCR of the columns in both analyses. Note that the flexural stiffness of the girders is decreased by 7 percent in the preliminary analyses to compensate for the effect of reduced beam sections (with 35 percent flange reduction) which are included in the detailed analyses.

Similar to the preliminary DCR, the panel zone DCR increases significantly when doubler plates aren’t used. Since the doubler plates are used only at the interior columns, that is where the difference of the DCRs changes significantly with and without doubler plates. See Figure 4.2-5(b) and Figure 4.2-25 (b).
### Chapter 4: Structural Analysis

#### (a) DCRs of columns and girders with and without doubler plates

(DCR values in parentheses are for without doubler plates)

<table>
<thead>
<tr>
<th>Level</th>
<th>0.999 (1.152)</th>
<th>1.013 (1.023)</th>
<th>1.009 (1.014)</th>
<th>1.013 (1.016)</th>
<th>1.067 (1.217)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level R</td>
<td>0.581 (0.665)</td>
<td>1.113 (1.159)</td>
<td>1.125 (1.143)</td>
<td>1.125 (1.143)</td>
<td>1.119 (1.155)</td>
</tr>
<tr>
<td></td>
<td>1.782 (1.945)</td>
<td>1.848 (1.772)</td>
<td>1.848 (1.772)</td>
<td>1.859 (1.772)</td>
<td>1.837 (2.032)</td>
</tr>
<tr>
<td>Level 6</td>
<td>0.969 (1.064)</td>
<td>1.507 (1.546)</td>
<td>1.520 (1.514)</td>
<td>1.519 (1.513)</td>
<td>1.513 (1.532)</td>
</tr>
<tr>
<td></td>
<td>2.427 (2.669)</td>
<td>2.375 (2.305)</td>
<td>2.375 (2.305)</td>
<td>2.375 (2.279)</td>
<td>2.496 (2.747)</td>
</tr>
<tr>
<td>Level 5</td>
<td>1.027 (1.134)</td>
<td>1.731 (1.765)</td>
<td>1.729 (1.697)</td>
<td>1.728 (1.697)</td>
<td>1.734 (1.739)</td>
</tr>
<tr>
<td></td>
<td>2.869 (3.120)</td>
<td>2.791 (2.669)</td>
<td>2.791 (2.678)</td>
<td>2.791 (2.643)</td>
<td>2.938 (3.181)</td>
</tr>
<tr>
<td>Level 4</td>
<td>1.214 (1.292)</td>
<td>1.903 (1.888)</td>
<td>1.885 (1.793)</td>
<td>1.884 (1.795)</td>
<td>1.894 (1.835)</td>
</tr>
<tr>
<td></td>
<td>3.302 (3.527)</td>
<td>3.181 (3.111)</td>
<td>3.181 (3.120)</td>
<td>3.181 (3.094)</td>
<td>3.371 (3.596)</td>
</tr>
<tr>
<td>Level 3</td>
<td>0.935 (1.063)</td>
<td>1.572 (1.654)</td>
<td>1.540 (1.558)</td>
<td>1.539 (1.559)</td>
<td>1.561 (1.606)</td>
</tr>
<tr>
<td></td>
<td>3.189 (3.337)</td>
<td>2.973 (2.851)</td>
<td>2.981 (2.869)</td>
<td>2.955 (2.825)</td>
<td>3.250 (3.397)</td>
</tr>
<tr>
<td>Level 2</td>
<td>3.189 (3.405)</td>
<td>2.777 (2.980)</td>
<td>2.737 (2.839)</td>
<td>2.737 (2.844)</td>
<td>2.743 (2.853)</td>
</tr>
</tbody>
</table>

(b) DCRs of panel zones with and without doubler plates

(DCR values in parentheses are for without doubler plates)

### Figure 4.2-25 DCRs for elements from detailed analysis with P-delta effects included
4.2.5 Nonlinear Static Analysis

As mentioned in the introduction to this example, nonlinear static (pushover) analysis is not an allowed analysis procedure in the Standard, nor does it appear in ASCE 7-10. The method is allowed in analysis related to rehabilitation of existing buildings and guidance for that use is provided in ASCE 41.

The Provisions makes at least two references to pushover analysis. In Section 12.8.7 of Part 1 pushover analysis is used to determine if structures with stability ratios (see Equation 12.8-16) greater than 0.1 are allowed. Such systems have a potential for dynamic instability and the pushover curve is used to determine if the slope of the pushover curve is continuously positive up to the target displacement. If the slope is positive, the system is deemed acceptable. If not, it must be redesigned such that either the stability ratio is less than 0.1, or the slope stays positive. The analysis carried out for this purpose must be performed according to the requirements of ASCE 41.

Pushover analysis is also mentioned in Provisions Part 3 Resource Paper 2. The intent of the procedure outlined there is to “determine whether lateral strength is nominally less than that required by the ELF procedure.” The use of nonlinear static analysis for this purpose is limited to structures with a height of less than 40 feet. The building under consideration has a height of 77.5 feet and violates this limit.

In this example, pushover analysis is used simply to establish an estimate of the inelastic behavior of the structure under gravity and lateral loads. Of particular interest is the sequence of yielding in the beams, columns and panel zones; the lateral strength of the structure; the expected inelastic displacement; and the basic shape of the pushover curve. In the authors’ opinion, such analysis should always be used as a precursor to nonlinear response history analysis. Without pushover analysis as a precursor, it is difficult to determine if the response history analysis is producing reasonable results.

The nonlinear static analysis illustrated in this example follows the recommendations of ASCE 41. The reader is also referred to FEMA 440.

The pushover curve obtained from a nonlinear static analysis is a function of both modeling and load application. For this example, the structure is subjected to the full dead load plus 50 percent of the fully reduced live load, followed by the lateral loads.

The Provisions states that the lateral load pattern should follow the shape of the first mode. In this example, three different load patterns are initially considered:

- **UL =** uniform load (equal force at each level)
- **ML =** modal load (lateral loads proportional to first mode shape)
- **BL =** Provisions load distribution (using the forces indicated in Table 4.2-3)

Relative values of these load patterns are summarized in Table 4.2-11. The loads have been normalized to a value of 15 kips at Level 2.

DRAIN analyses are run with P-delta effects included and, for comparison purposes, with such effects excluded. This effect is represented through linearized geometric stiffness, which is the basis of the outrigger column shown in Figure 4.2-4. Consistent geometric stiffness, which may be used to represent the influence of axial forces on the flexural flexibility of individual columns, may not be used directly in
DRAIN. Such effects may be approximated in DRAIN by subdividing columns into several segments and activating the linearized geometric stiffness on a column-by-column basis.\(^6\)

<table>
<thead>
<tr>
<th>Level</th>
<th>Uniform load, UL (kips)</th>
<th>Modal load, ML (kips)</th>
<th>Provisions load, BL (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>15.0</td>
<td>85.1</td>
<td>144.8</td>
</tr>
<tr>
<td>6</td>
<td>15.0</td>
<td>77.3</td>
<td>114.0</td>
</tr>
<tr>
<td>5</td>
<td>15.0</td>
<td>64.8</td>
<td>84.8</td>
</tr>
<tr>
<td>4</td>
<td>15.0</td>
<td>49.5</td>
<td>58.2</td>
</tr>
<tr>
<td>3</td>
<td>15.0</td>
<td>32.2</td>
<td>34.6</td>
</tr>
<tr>
<td>2</td>
<td>15.0</td>
<td>15.0</td>
<td>15.0</td>
</tr>
</tbody>
</table>

As described later, the pushover analysis indicates most of the yielding in the structure occurs in the clear span of the girders and columns. Panel zone hinging occurs only at the exterior columns where doubler plates weren’t used. To see the effect of doubler plates, the ML analysis is repeated for a structure without doubler plates. These structures are referred to as the strong panel (SP) and weak panel (WP) structures, respectively.

The analyses are carried out using the DRAIN-2Dx computer program. Using DRAIN, an analysis may be performed under “load control” or under “displacement control.” Under load control, the structure is subjected to gradually increasing lateral loads. If, at any load step, the tangent stiffness matrix of the structure has a negative on the diagonal, the analysis is terminated. Consequently, loss of strength due to P-delta effects cannot be tracked. Using displacement control, one particular point of the structure (the control point) is forced to undergo a monotonically increasing lateral displacement, and the lateral forces are constrained to follow the desired pattern. In this type of analysis, the structure can display loss of strength because the displacement control algorithm adds artificial stiffness along the diagonal to overcome the stability problem. This approach is meaningful because structures subjected to dynamic loading can display strength loss and remain stable incrementally. It is for this reason that the post-strength-loss realm of the pushover response is of interest.

Where performing a displacement controlled pushover analysis in DRAIN with P-delta effects included, one must be careful to recover the base-shear forces properly.\(^7\) At any displacement step in the analysis, the true base shear in the system consists of two parts:

\[
V = \sum_{i=1}^{n} V_{C,i} - \frac{P_1 \Delta_1}{h_i}
\]

where the first term represents the sum of all the column shears in the first story, and the second term represents the destabilizing P-delta shear in the first story. The P-delta effects for this structure are

\(^6\) DRAIN uses the axial forces at the end of the gravity load analysis to set geometric stiffness for the structure. This is reasonably accurate where consistent geometric stiffness is used, but is questionable where linearized geometric stiffness is used.

\(^7\) If P-delta effects have been included, this procedure needs to be used where recovering base shear from column shear forces. This is true for displacement controlled static analysis, force controlled static analysis and dynamic response history analysis.
included through the use of the outrigger column shown at the right of Figure 4.2-4. Figure 4.2-26 plots two base shear components of the pushover response for the SP structure subjected to the ML loading. Also shown is the total response. The kink in the line representing P-delta forces occurs because these forces are based on first-story displacement, which, for an inelastic system, generally will not be proportional to the roof displacement.

For all of the pushover analyses reported in this example, the structure is pushed to a displacement of 37.5 inches at the roof level. This value is approximately 4 percent of the total height.

![Figure 4.2-26 Two base shear components of pushover response](image)

**Figure 4.2-26** Two base shear components of pushover response

### 4.2.5.1 Pushover response of strong panel structure.

Figure 4.2-27 shows the pushover response of the SP structure to all three lateral load patterns where P-delta effects are excluded. In each case, gravity loads are applied first, and then the lateral loads are applied using the displacement control algorithm. Figure 4.2-28 shows the response curves if P-delta effects are included. In Figure 4.2-29, the response of the structure under ML loading with and without P-delta effects is illustrated. Clearly, P-delta effects are an extremely important aspect of the response of this structure and the influence grows in significance after yielding. This is particularly interesting in the light of the *Standard*, which ignores P-delta effects in elastic analysis if the maximum stability ratio is less than 0.10 (see Sec. 12.8-7). For this structure, the maximum computed stability ratio is 0.0862 (see Table 4.2-5), which is less than 0.10 and is also less than the upper limit of 0.0909. The upper limit is computed according to *Standard* Equation 12.8-17 and is based on the very conservative assumption that $\beta = 1.0$. While the *Standard* allows the analyst to exclude P-delta effects in an elastic analysis, this clearly should not be done in the pushover analysis (or in response history analysis). (In the *Provisions* the upper limit for the stability ratio is eliminated. Where the calculated $\theta$ is greater than 0.10, a pushover analysis must be performed in accordance with ASCE 41, and it must be shown that that the slope of the pushover curve is positive up to the target displacement. The pushover analysis must be based on the MCE spectral acceleration and must include P-delta effects [and loss of strength, as appropriate]. If the slope of the pushover curve is negative at displacements less than the target displacement, the structure must be redesigned such that $\theta$ is less than 0.10 or the pushover slope is positive up to the target displacement.)
Figure 4.2-27 Response of strong panel model to three load patterns, excluding P-delta effects

Figure 4.2-28 Response of strong panel model to three load patterns, including P-delta effects
In Figure 4.2-30, a plot of the tangent stiffness versus roof displacement is shown for the SP structure with ML loading and with P-delta effects excluded or included. This plot, which represents the slope of the pushover curve at each displacement value, is more effective than the pushover plot in determining when yielding occurs. As Figure 4.2-30 illustrates, the first significant yield occurs at a roof displacement of approximately 6.5 inches and that most of the structure’s original stiffness is exhausted by the time the roof displacement reaches 13 inches.
For the case with P-delta effects excluded, the final tangent stiffness shown in Figure 4.2-30 is approximately 10.2 kips/in., compared to an original value of 139 kips/in. Hence, the strain-hardening stiffness of the structure is 0.073 times the initial stiffness. This is somewhat greater than the 0.03 (3.0 percent) strain hardening ratio used in the development of the model because the entire structure does not yield simultaneously.

Where P-delta effects are included, the final tangent stiffness is -1.6 kips per inch. The structure attains this negative tangent stiffness at a displacement of approximately 23 inches.

**4.2.5.1.1 Sequence and pattern of plastic hinging.** The sequence of yielding in the structure with ML loading and with P-delta effects included is shown in Figure 4.2-31. Part (a) of the figure shows an elevation of the structure with numbers that indicate the sequence of plastic hinge formation. For example, the numeral “1” indicates that this was the first hinge to form. Part (b) of the figure shows a pushover curve with several hinge formation events indicated. These events correspond to numbers shown in Part (a) of the figure. The pushover curve only shows selected events because an illustration showing all events would be difficult to read. Comparing Figure 4.2-31(a) with Figures 4.2-5 and 4.2-25, it can be seen how the DCRs indicate the plastic hinge formation sequence. The highest ratios in Figure 4.2-5 are observed at the girders of the third and the second levels beginning from the bays at the leeward (right) side. As may be seen from Figure 4.2-31(a), first plastic hinges form at the same locations of the building. Similarly, the first panel zone hinge forms at the beam-column joint of the sixth column at the fourth level, and this is where the highest DCR values are obtained for the panel zones in both preliminary and detailed DCR analyses.

![Diagram](a)
Several important observations are made from Figure 4.2-31:

- There is no hinging in Levels 6 and R.
- There is panel zone hinging only at the exterior columns at Levels 4 and 5. Panel zone hinges do not form at the interior joints where doubler plates are used.
- Hinges form at the base of all the Level 1 columns.
- Plastic hinges form in all columns on Level 3 and all the interior columns on Level 4.
- Both ends of all the girders at Levels 2 through 5 yield.

It appears the structure is somewhat weak in the middle two stories and is relatively strong at the upper stories. The doubler plates added to the interior columns prevent panel zone yielding.

The presence of column hinging at Levels 3 and 4 is a bit troublesome because the structure is designed as a strong-column/weak-beam system. This design philosophy, however, is intended to prevent the formation of complete story mechanisms, not to prevent individual column hinging. While hinges do form at the top of each column in the third story, hinges do not form at the bottom of these columns and a complete story mechanism is avoided.

Even though the pattern of hinging is interesting and useful as an evaluation tool, the performance of the structure in the context of various acceptance criteria cannot be assessed until the expected inelastic displacement can be determined. This is done below in Section 4.2.5.3.
4.2.5.1.2 **Comparison with strength from plastic analysis.** It is interesting to compare the strength of the structure from pushover analysis with that obtained from the rigid-collapse analysis performed using virtual work. These values are summarized in Table 4.2-12. The strength from the case with P-delta excluded was estimated from the curves shown in Figure 4.2-27 and is taken as the strength at the principal bend in the curve (the estimated yield from a bilinear representation of the pushover curve). Consistent with the upper bound theorem of plastic analysis, the strength from virtual work is greater than that from pushover analysis. The reason for the difference in predicted strengths is related to the pattern of yielding that actually formed in the structure, compared to that assumed in the rigid-plastic analysis.

<table>
<thead>
<tr>
<th>Pattern</th>
<th>Lateral Strength (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P-delta Excluded</td>
</tr>
<tr>
<td>Uniform</td>
<td>1,340</td>
</tr>
<tr>
<td>Modal (Triangular)</td>
<td>1,200</td>
</tr>
<tr>
<td>BSSC</td>
<td>1,170</td>
</tr>
</tbody>
</table>

4.2.5.2 **Pushover response of weak panel structure.** Before continuing, the structure should be re-analyzed without panel zone reinforcing, and the behavior compared with that determined from the analysis described above. For this exercise, only the modal load pattern is considered, but the analysis is performed with and without P-delta effects.

The pushover curves for the structure under modal loading and with weak panels are shown in Figure 4.2-32. Curves for the analyses run with and without P-delta effects are included. Figures 4.2-33 and 4.2-34 are more informative because they compare the response of the structures with and without panel zone reinforcement. Figure 4.2-35 shows the tangent stiffness history comparison for the structures with and without doubler plates. In both cases P-delta effects have been included.

Figures 4.2-32 through 4.2-35 show that the doubler plates, which represent approximately 2.0 percent of the volume of the structure, increase the strength and initial stiffness by approximately 10 percent.
Figure 4.2-32  Weak panel zone model under ML load

Figure 4.2-33  Comparison of weak panel zone model with strong panel zone model, excluding P-delta effects
The difference between the behavior of the structures with and without doubler plates is attributed to the yielding of the panel zones in the structure without panel zone reinforcement. The sequence of hinging is illustrated in Figure 4.2-36. Part (a) of this figure indicates that panel zone yielding occurs early. (Panel zone yielding is indicated by a numeric sequence label in the corner of the panel zone.) In fact, the first yielding in the structure is due to yielding of a panel zone at the fourth level of the structure, which is consistent with panel zone DCR calculated before where no doubler plates were added to the structure.
Under very large displacements the flange component of the panel zone yields. Girder and column hinging also occurs, but the column hinging appears relatively late in the response. It is also significant that the upper two levels of the structure display yielding in several of the panel zones.

Aside from the relatively marginal loss in stiffness and strength due to removal of the doubler plates, it appears that the structure without panel zone reinforcement behaves adequately. Of course, actual performance cannot be evaluated without predicting the maximum inelastic panel shear strain and assessing the stability of the panel zones under these strains.
4.2.5.3 Target displacement. In this section, the only loading pattern considered is the modal load pattern discussed earlier. This is consistent with the requirements of ASCE 41 and FEMA 440. The structures with strong and weak panel zones are analyzed including P-delta effects.

ASCE 41 uses the coefficient method for calculating target displacement. The target displacement is computed as follows:

\[
\delta_t = C_0 C_1 C_2 S_a \frac{T_e^2}{4\pi^2} g
\]

where:

- \( C_0 = \phi_{1,r} \Gamma_1 \) = modification factor to relate roof displacement of a multiple degree of freedom building system to the spectral displacement of an equivalent single degree of freedom system
- \( \phi_{1,r} \) = the ordinate of mode shape 1 at the roof (control node)
- \( \Gamma_1 \) = the first mode participation factor
- \( C_1 \) = modification factor to relate expected maximum inelastic displacements to displacements calculated for linear elastic response

Figure 4.2-36 Patterns of plastic hinge formation: weak panel zone model under ML load, including P-delta effects
\( C_2 = \) modification factor to represent the effect of pinched hysteresis shape, cyclic stiffness degradation and strength deterioration on maximum displacement response

\( S_a = \) response spectrum acceleration, at the effective fundamental period and damping ratio of the building in the direction under consideration

\( T_c = T_i \frac{K_i}{K_e} = \) effective fundamental period of the building in the direction under consideration

\( T_i = \) elastic fundamental period in the direction under consideration calculated by elastic dynamic analysis

\( K_i = \) elastic lateral stiffness of the building in the direction under consideration

\( K_e = \) effective lateral stiffness of the building in the direction under consideration

\( g = \) acceleration due to gravity

To find the coefficient \( S_a \), the general horizontal response spectrum defined in ASCE 41 Section 1.6.1.5 is used. The damping of the spectrum is chosen as 2 percent for this study. The same damping ratio is used in the dynamic analysis. The parameters \( S_{XS} \) and \( S_{X1} \) are chosen as the same values as \( S_{MS} \) and \( S_{M1} \) which are defined in Section 4.2.2.2 of this study. Note that these are the MCE spectral acceleration parameters. Figure 4.2-37 shows the horizontal response spectrum obtained from ASCE 41. The parameters of this spectrum are discussed further with the dynamic analyses. This spectrum is for the Basic Safety Earthquake 2 (BSE-2) hazard level which has a 2 percent probability of exceedance in 50 years.

Coefficient \( C_0 \) is found using the first mode shape of the model with the mass matrix. For both strong and weak panel models, the \( C_0 \) coefficient is found a bit higher than 1.3, which is the value provided in Table 3.2 of ASCE 41 for the shear buildings with triangular load pattern.

\( C_1 \) and \( C_2 \) are equal to 1.0 for periods greater than 1.0 second and 0.7 second respectively. Since the first mode periods of the strong and weak panel models are both approximately 2 seconds, these coefficients are taken as 1.0.
To find the target displacement, the procedure described in ASCE 41 is followed. The nonlinear force-displacement relationship between the base shear and displacement of the control node are replaced with an idealized force-displacement curve. The effective lateral stiffness and the effective period depend on the idealized force-displacement curve. The idealized force-displacement curve is developed using an iterative graphical procedure where the areas below the actual and idealized curves are balanced approximately up to a displacement value of $\Delta_d$. $\Delta_d$ is the displacement at the end of second line segment of the idealized curve and $V_d$ is the base shear at the same displacement. $(\Delta_d, V_d)$ should be a point on the actual force-displacement curve either at the calculated target displacement or at the displacement corresponding to the maximum base shear, whichever is the least. The first line segment of the idealized force-displacement curve should begin at the origin and finish at $(\Delta_y, V_y)$, where $V_y$ is the effective yield strength and $\Delta_y$ is the yield displacement of idealized curve. The slope of the first line segment is equal to the effective lateral stiffness, $K_e$, which should be taken as the secant stiffness calculated at a base shear force equal to 60 percent of the effective yield strength of the structure. See Figures 4.2-38 and 4.2-39 for the actual and idealized force-displacement curves of strong and weak panel models which are under ML loading and both include P-delta effects.
Figure 4.2-38  Actual and idealized force displacement curves for strong panel model, under ML load, including P-delta effects

Figure 4.2-39  Actual and idealized force displacement curves for weak panel model, under ML load, including P-delta effects

Table 4.2-13 shows the target displacement values of SP and WP models. Story drifts are also shown at the load level of target displacement for both models.

Table 4.2-13  Target Displacement for Strong and Weak Panel Models

<table>
<thead>
<tr>
<th></th>
<th>Strong Panel</th>
<th>Weak Panel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_0$</td>
<td>1.303</td>
<td>1.310</td>
</tr>
<tr>
<td>$C_1$</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>
Table 4.2-13 Target Displacement for Strong and Weak Panel Models

<table>
<thead>
<tr>
<th></th>
<th>Strong Panel</th>
<th>Weak Panel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_2$</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>$S_a$ (g)</td>
<td>0.461</td>
<td>0.439</td>
</tr>
<tr>
<td>$T_e$ (sec)</td>
<td>1.973</td>
<td>2.069</td>
</tr>
<tr>
<td>$\delta_i$ (in.) at Roof Level</td>
<td><strong>22.9</strong></td>
<td><strong>24.1</strong></td>
</tr>
<tr>
<td>Drift R-6 (in.)</td>
<td>0.96</td>
<td>1.46</td>
</tr>
<tr>
<td>Drift 6-5 (in.)</td>
<td>1.76</td>
<td>2.59</td>
</tr>
<tr>
<td>Drift 5-4 (in.)</td>
<td>2.87</td>
<td>3.73</td>
</tr>
<tr>
<td>Drift 4-3 (in.)</td>
<td>4.84</td>
<td>4.84</td>
</tr>
<tr>
<td>Drift 3-2 (in.)</td>
<td>5.74</td>
<td>5.35</td>
</tr>
<tr>
<td>Drift 2-1 (in.)</td>
<td>6.73</td>
<td>6.12</td>
</tr>
</tbody>
</table>

Negative tangent stiffness starts at 22.9 inches and 29.3 inches for strong and weak panel models, respectively. Thus negative tangent stiffness starts after target displacements for both models. Again note that these displacements are computed from the 2 percent-damped MCE horizontal response spectrum of ASCE 41.

4.2.6 Response History Analysis

The response history analysis method, with three ground motions, is used to estimate the inelastic deformation demands for the structure. While an analysis with seven or more ground motions generally is preferable, that was not done here due to time and space limitations.

The analysis did consider a number of parameters, as follows:

- Scaling of ground motions to the DBE and MCE level
- Analysis with and without P-delta effects
- Two percent and five percent inherent damping
- Added linear viscous damping

All of the models analyzed have “Strong Panels” (with doubler plates included in the interior beam-column joints).

4.2.6.1 Modeling and analysis procedure.

The DRAIN-2Dx program is used for each of the response history analyses. With the exception of requirements for including inherent damping, the structural model is identical to that used in the nonlinear static analysis. Second-order effects are included through the use of the leaning column element shown to the right of the actual frame in Figure 4.2-4. Only one-half of the building (a single frame in the N-S direction) is modeled.

Inelastic hysteretic behavior is represented through the use of a bilinear model. This model exhibits neither a loss of stiffness nor a loss of strength and for this reason, it will generally have the effect of overestimating the hysteretic energy dissipation in the yielding elements. Fortunately, the error produced by such a model will not be of great concern for this structure because the hysteretic behavior of panel
zones and flexural plastic hinges should be very robust where inelastic rotations are less than about 0.03 radians.

Rayleigh proportional damping was used to represent viscous energy dissipation in the structure. The mass and stiffness proportional damping factors are set initially to produce 2.0 percent damping in the first and third modes. It is generally recognized that this level of damping (in lieu of the 5 percent damping that is traditionally used in elastic analysis) is appropriate for nonlinear response history analysis. Two percent damping is also consistent with that used in the pushover analysis (see Section 4.2.5 of this example).

In Rayleigh proportional damping, the damping matrix, $C$, is a linear combination of the mass matrix, $M$ and the initial stiffness matrix, $K$:

$$ C = \alpha M + \beta K $$

where $\alpha$ and $\beta$ are mass and stiffness proportionality factors, respectively. If the first and third mode frequencies, $\omega_1$ and $\omega_3$, are known, the proportionality factors may be computed from the following expression (Clough & Penzien):

$$ \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \frac{2 \xi}{w_1 + w_3} \begin{bmatrix} w_1 & w_3 \\ 1 & 1 \end{bmatrix} $$

Note that $\alpha$ and $\beta$ are directly proportional to $\xi$. To increase the target damping from 2 percent to 5 percent of critical, all that is required is a multiplying factor of 2.5 on $\alpha$ and $\beta$.

The targeted structural frequencies and the resulting damping proportionality factors are shown in Table 4.2-14. The frequencies shown in the table are computed from the detailed model shown in Figure 4.2-7.

<table>
<thead>
<tr>
<th>Model/Damping Parameters</th>
<th>$\omega_1$ (rad/sec)</th>
<th>$\omega_3$ (rad/sec)</th>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong Panel with P-delta</td>
<td>3.184</td>
<td>18.55</td>
<td>0.109</td>
<td>0.00184</td>
</tr>
<tr>
<td>Strong Panel without P-delta</td>
<td>3.285</td>
<td>18.81</td>
<td>0.112</td>
<td>0.00181</td>
</tr>
</tbody>
</table>

The stiffness proportional damping factor must not be included in the Type 4 elements used to represent rotational plastic hinges in the structure. These hinges, particularly those in the girders, have a very high initial stiffness. Before the hinge yields, there is virtually no rotational velocity in the hinge. After yielding, the rotational velocity is significant. If a stiffness proportional damping factor is used for the hinge, a viscous moment will develop in the hinge. This artificial viscous moment—the product of the rotational velocity, the initial rotational stiffness of the hinge and the stiffness proportional damping factor—can be quite large. In fact, the viscous moment may even exceed the intended plastic capacity of the hinge. These viscous moments occur in phase with the plastic rotation; hence, the plastic moment and the viscous moments are additive. These large moments transfer to the rest of the structure, affecting the sequence of hinging in the rest of the structure and produce artificially high base shears. The use of
stiffness proportional damping in discrete plastic hinges can produce a totally inaccurate analysis result. See Charney (2008) for details.

The structure is subjected to dead load and half of the fully reduced live load, followed by ground acceleration. The incremental differential equations of motion are solved in a step-by-step manner using the Newmark constant average acceleration approach. Time steps and other integration parameters are carefully controlled to minimize errors. The minimum time step used for analysis is as small as 0.0005 second for the first earthquake and 0.001 second for the second and third earthquakes. A smaller integration time step is required for the first earthquake because of its impulsive nature.

4.2.6.2 Development of ground motion records. The ground motion acceleration histories used in the analysis are developed specifically for the site. Basic information for the records is shown in Table 4.1-20a.

Ground acceleration histories and 2- and 5-percent-damped pseudoacceleration spectra for each of the motions are shown in Figures 4.2-40 through 4.2-42. For these two-dimensional analyses performed using DRAIN, single ground motion components are applied one at a time. For this example, the component that produces the larger spectral acceleration at the structure’s fundamental period (A00, B90 and C90) is used. A complete analysis would require consideration of both components of ground motions and possibly of a rotated set of components.

When analyzing structures in two dimensions, Section 16.1.3.1 of the Standard (as well as ASCE 7-10) gives the following instructions for scaling:

The ground motions shall be scaled such that the average value of the 5 percent damped response spectra for the suite of motions is not less than the design response spectrum for the site for periods ranging from 0.2\(T\) to 1.5\(T\) where \(T\) is the natural period of the structure in the fundamental mode for the direction of response being analyzed.

The scaling requirements in Provisions Part 3 Resource Paper 3 are similar, except that the target spectrum for scaling is the MCE\(R\) spectrum. In this example, the only adjustment is made for scaling when the inherent damping is taken as 2 percent of critical. In this case, the ground motion spectra are based on 2 percent damping and the DBE or MCE spectrum is adjusted from 5 percent damping to 2 percent damping using the modification factors given in ASCE 41.

The scaling procedure described above has a “degree of freedom” in that there are an infinite number of scaling factors that can fit the criterion. To avoid this, a two-step scaling process is used wherein each spectrum is initially scaled to match the target spectrum at the structure’s fundamental period and then the average of the scaled spectra are re-scaled such that no ordinate of the scaled average spectrum falls below the target spectrum in the range of periods between 0.2\(T\) and 1.5\(T\). The final scale factor for each motion consists of the product of the initial scale factor and the second scale factor.

The initial scale factors, referred to as \(S1_i\) (for each ground motion, \(i\)), are different for the three ground motions. The second scale factor, \(S2\), is the same for each ground motion. The scale factors used in the response history analyses are shown in Table 4.2-15. Factors are determined for 2 percent and 5 percent damping and for the DBE and MCE motions. The 2 percent damped target MCE spectrum corresponds to ASCE 41-06 spectrum used in the pushover analysis. If a scale factor of 1.367 is used for the structure with 2 percent damping, Figure 4.2-43 indicates that the scaling criteria specified by the Standard are met for all periods in the range 0.2(1.973) = 0.4 second to 1.5(1.973) = 3.0 seconds. 1.973 seconds is the period of the SP model with P-delta effects included (See Table 4.2-10).
Figure 4.2-40  Ground acceleration histories and response spectra for Record A
Figure 4.2-41 Ground acceleration histories and response spectra for Record B
Figure 4.2-42  Ground acceleration histories and response spectra for Record C
Table 4.2-15  Ground Motion Scale Factors Used in the Analyses

<table>
<thead>
<tr>
<th>Scale Factor</th>
<th>2% Damped DBE</th>
<th>2% Damped MCE</th>
<th>5% Damped DBE</th>
<th>5% Damped MCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motion A00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S1</td>
<td>0.919</td>
<td>1.380</td>
<td>0.765</td>
<td>1.147</td>
</tr>
<tr>
<td>S2</td>
<td>1.367</td>
<td>1.367</td>
<td>1.428</td>
<td>1.428</td>
</tr>
<tr>
<td>SS</td>
<td><strong>1.257</strong></td>
<td><strong>1.886</strong></td>
<td><strong>1.092</strong></td>
<td><strong>1.638</strong></td>
</tr>
<tr>
<td>Motion B90</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S1</td>
<td>1.495</td>
<td>2.245</td>
<td>1.439</td>
<td>2.159</td>
</tr>
<tr>
<td>S2</td>
<td>1.367</td>
<td>1.367</td>
<td>1.428</td>
<td>1.428</td>
</tr>
<tr>
<td>SS</td>
<td><strong>2.045</strong></td>
<td><strong>3.068</strong></td>
<td><strong>2.056</strong></td>
<td><strong>3.084</strong></td>
</tr>
<tr>
<td>Motion C90</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S1</td>
<td>1.332</td>
<td>2.000</td>
<td>1.359</td>
<td>2.039</td>
</tr>
<tr>
<td>S2</td>
<td>1.367</td>
<td>1.367</td>
<td>1.428</td>
<td>1.428</td>
</tr>
<tr>
<td>SS</td>
<td><strong>1.822</strong></td>
<td><strong>2.734</strong></td>
<td><strong>1.941</strong></td>
<td><strong>2.911</strong></td>
</tr>
</tbody>
</table>

(a) Comparison of average of scaled spectra and target spectrum (SF = 1.367)

(b) Ratio of average of scaled spectra to target spectrum (SF = 1.367)

Figure 4.2-43  Ground motion scaling parameters
4.2.6.3 Results of response history analysis. The following parameters are varied to determine the sensitivity of the response to that parameter:

- Analyses are run with and without P-delta effects for all three ground motions.
- Analyses are run with 2 percent and 5 percent inherent damping. The ground motion scale factors are correlated with the corresponding inherent damping of the structure.
- Added dampers are used for the structure with 2 percent inherent damping. Various added damper configurations are used. These analyses are performed to assess the potential benefit of added viscous fluid damping devices. The SP model with P-delta effects included is used for this analysis and only Ground Motions A00 and B90 are used.

The results from the first series of analyses, all run with 2 or 5 percent of critical damping with and without P-delta effects, are summarized in Tables 4.2-16 through 4.2-23. Selected time history traces are shown in Figures 4.2-44 through 4.2-48.

The tabulated shears in the tables are for the single frame analyzed and should be doubled to obtain the total shear in the structure. The tables of story shear also provide two values for each ground motion. The first value is the maximum total elastic column story shear, including P-delta effects if applicable. The second value represents the maximum total inertial force for the structure. The inertial base shear, which is not necessarily concurrent with the column shears, was obtained as a sum of the products of the total horizontal accelerations and nodal mass of each joint. For a system with no damping, the story shears obtained from the two methods should be identical. For a system with damping, the base shear obtained from column forces generally will be less than the shear from inertial forces because the viscous component of column shear is not included. Additionally, the force absorbed by the mass proportional component of damping will be lost (as this is not directly recoverable in DRAIN).

The total roof displacement and the story drifts listed in the tables are peak (envelope) values and are not necessarily concurrent.

4.2.6.3.1 Response of structure with 2 and 5 percent of critical damping. Tables 4.2-16 and 4.2-17 summarize the results of the DBE analyses with 2 percent inherent damping, including and excluding P-delta effects. Part (a) of each table provides the maximum base shears, computed either as the sum of column forces (including P-delta effects as applicable), or as the sum of the products of the total acceleration and mass at each level. In each case, the shears computed using the two methods are similar, which serves as a check on the accuracy of the analysis. Had the analysis been run without damping, the shears computed by the two methods should be identical. As expected base shears decrease when P-delta effects are included.

The maximum story drifts are shown in the (b) parts of each table. The drift limits in the table, equal to 2 percent of the story height, are the same as provided in Standard Table 12.12-1. Standard Section 16.2.4.3 provides for the allowable drift to be increased by 25 percent where nonlinear response history analysis is used; these limits are shown in the tables in parentheses. Provisions Part 2 states that the increase in drift limit is attributed to “the more accurate analysis and the fact that drifts are computed explicitly.” Drifts that exceed the increased limits are shown in bold text in the tables.

When a SP frame with 2 percent inherent damping is analyzed under MCE spectrum scaled motions excluding P-delta effects, earthquake A00 results in 62.40-inch displacement at the roof level and approximately between 15- to 20-inch drifts at the first three stories of the structure. These story drifts are well above the limits. When P-delta effects are included with the same level of motion, roof
displacement increases to 101.69 inches with approximately 20- to 40-inch displacement at the first three stories.

It is clear from Part (b) of Tables 4.2-16 and 4.2-17 that Ground Motion A00 is much more demanding with respect to drift than are the other two motions. The drifts produced by Ground Motion A00 are particularly large at the lower levels, with the more liberal drift limits being exceeded in the lower four stories of the building. When P-delta effects are included, the drifts produced by Ground Motion A00 increase significantly; drifts produced by Ground Motions B90 and C90 change only slightly.

Tables 4.2-18 and 4.2-19 provide result summaries for the structure analyzed with the MCE ground motions. Damping is still set at 2 percent of critical and analysis is run with and without P-delta effects. The drift limits listed in the (b) parts of Tables 4.2-18 and 4.2-19 are based on Provisions Part 3 Resource Paper 3 Section 16.4.5. These limits are 1.5 times those allowed by Standard Section 12.2.1. The 50 percent increase in drift limits is consistent with the increase in ground motion intensity when moving from DBE to MCE ground motions. If all of the increase in drift limit is attributed to the DBE-MCE scaling, there is no apparent adjustment related to “the more accurate analysis and explicit computation of drift”.

When P-delta effects are included maximum story shears decrease and the drifts in lower stories increase for all motions. The drifts predicted for Ground Motion A00 (as much as 40 inches) indicate probable collapse of the structure. Loss of strength associated with such large drifts is not included in the analytical model (since DRAIN does not provide a mechanism for decreasing moment capacity under large plastic rotations). It is highly likely, however, that collapse would be predicted by more accurate modeling.

Similar trends in response are produced when the inherent damping is increased from 2 percent critical to 5 percent. The results for the 5 percent damped analysis are provided in Tables 4.2-20 through 4.2-23. The first two of these tables, Tables 4.1-20 and 4.1-21, are for the analysis using the DBE ground motions. When compared to the results using 2 percent damping, it is seen that both the base shears and the story drifts decrease significantly. DBE-level drifts at lower stories due to Ground Motion A00 exceed the drift limit but may not indicate collapse. MCE-level drifts produced by the A00 ground motion indicate likely collapse.

<table>
<thead>
<tr>
<th>Table 4.2-16</th>
<th>DBE Results for 2% Damped Strong Panel Model with P- Delta Excluded</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Maximum Base Shear (kips)</td>
<td></td>
</tr>
<tr>
<td>Level</td>
<td>Motion A00</td>
</tr>
<tr>
<td>--------------</td>
<td>------------</td>
</tr>
<tr>
<td>Column forces</td>
<td>1,780</td>
</tr>
<tr>
<td>Inertial forces</td>
<td>1,848</td>
</tr>
<tr>
<td>(b) Maximum Displacement and Story Drift (in.)</td>
<td></td>
</tr>
<tr>
<td>Level</td>
<td>Motion A00</td>
</tr>
<tr>
<td>--------------</td>
<td>------------</td>
</tr>
<tr>
<td>Roof displacement</td>
<td>26.80</td>
</tr>
<tr>
<td>Drift R-6</td>
<td>1.85</td>
</tr>
<tr>
<td>Drift 6-5</td>
<td>2.51</td>
</tr>
<tr>
<td>Drift 5-4</td>
<td><strong>3.75</strong></td>
</tr>
</tbody>
</table>
(b) Maximum Displacement and Story Drift (in.)

<table>
<thead>
<tr>
<th>Level</th>
<th>Motion A00</th>
<th>Motion B90</th>
<th>Motion C90</th>
<th>Limit*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drift 4-3</td>
<td>5.62</td>
<td>2.98</td>
<td>3.03</td>
<td>3.00 (3.75)</td>
</tr>
<tr>
<td>Drift 3-2</td>
<td>6.61</td>
<td>3.58</td>
<td>2.82</td>
<td>3.00 (3.75)</td>
</tr>
<tr>
<td>Drift 2-G</td>
<td>8.09</td>
<td>4.68</td>
<td>3.29</td>
<td>3.60 (4.50)</td>
</tr>
</tbody>
</table>

*Values in ( ) reflect increased drift limits provided by Standard Sec. 16.2.4.3.

Table 4.2-17 DBE Results for 2% Damped Strong Panel Model with P-Delta Included

(a) Maximum Base Shear (kips)

<table>
<thead>
<tr>
<th>Level</th>
<th>Motion A00</th>
<th>Motion B90</th>
<th>Motion C90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column forces</td>
<td>1,467</td>
<td>1,458</td>
<td>1,417</td>
</tr>
<tr>
<td>Inertial forces</td>
<td>1,558</td>
<td>1,481</td>
<td>1,419</td>
</tr>
</tbody>
</table>

(b) Maximum Displacement and Story Drift (in.)

<table>
<thead>
<tr>
<th>Level</th>
<th>Motion A00</th>
<th>Motion B90</th>
<th>Motion C90</th>
<th>Limit*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roof displacement</td>
<td>32.65</td>
<td>14.50</td>
<td>14.75</td>
<td>NA</td>
</tr>
<tr>
<td>Drift R-6</td>
<td>1.86</td>
<td>1.82</td>
<td>1.70</td>
<td>3.00 (3.75)</td>
</tr>
<tr>
<td>Drift 6-5</td>
<td>2.64</td>
<td>2.50</td>
<td>2.41</td>
<td>3.00 (3.75)</td>
</tr>
<tr>
<td>Drift 5-4</td>
<td>4.08</td>
<td>2.81</td>
<td>3.19</td>
<td>3.00 (3.75)</td>
</tr>
<tr>
<td>Drift 4-3</td>
<td>6.87</td>
<td>3.21</td>
<td>3.33</td>
<td>3.00 (3.75)</td>
</tr>
<tr>
<td>Drift 3-2</td>
<td>8.19</td>
<td>3.40</td>
<td>2.90</td>
<td>3.00 (3.75)</td>
</tr>
<tr>
<td>Drift 2-G</td>
<td>10.40</td>
<td>4.69</td>
<td>3.44</td>
<td>3.60 (4.50)</td>
</tr>
</tbody>
</table>

*Values in ( ) reflect increased drift limits provided by Standard Sec. 16.2.4.3.

Table 4.2-18 MCE Results for 2% Damped Strong Panel Model with P-Delta Excluded

(a) Maximum Base Shear (kips)

<table>
<thead>
<tr>
<th>Level</th>
<th>Motion A00</th>
<th>Motion B90</th>
<th>Motion C90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column forces</td>
<td>2,181</td>
<td>1,851</td>
<td>1,723</td>
</tr>
<tr>
<td>Inertial forces</td>
<td>2,261</td>
<td>1,893</td>
<td>1,725</td>
</tr>
</tbody>
</table>

(b) Maximum Displacement and Story Drift (in.)

<table>
<thead>
<tr>
<th>Level</th>
<th>Motion A00</th>
<th>Motion B90</th>
<th>Motion C90</th>
<th>Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roof displacement</td>
<td>62.40</td>
<td>22.45</td>
<td>20.41</td>
<td>NA</td>
</tr>
<tr>
<td>Drift R-6</td>
<td>1.98</td>
<td>2.30</td>
<td>3.05</td>
<td>4.5</td>
</tr>
<tr>
<td>Drift 6-5</td>
<td>3.57</td>
<td>2.77</td>
<td>3.69</td>
<td>4.5</td>
</tr>
<tr>
<td>Drift 5-4</td>
<td>7.36</td>
<td>3.33</td>
<td>4.43</td>
<td>4.5</td>
</tr>
<tr>
<td>Drift 4-3</td>
<td>14.61</td>
<td>4.61</td>
<td>4.45</td>
<td>4.5</td>
</tr>
<tr>
<td>Drift 3-2</td>
<td>16.29</td>
<td>5.21</td>
<td>3.97</td>
<td>4.5</td>
</tr>
<tr>
<td>Drift 2-G</td>
<td>19.76</td>
<td>6.60</td>
<td>5.11</td>
<td>5.4</td>
</tr>
</tbody>
</table>
### Table 4.2-19 MCE Results for 2% Damped Strong Panel Model with P-Delta Included

**(a) Maximum Base Shear (kips)**

<table>
<thead>
<tr>
<th>Level</th>
<th>Motion A00</th>
<th>Motion B90</th>
<th>Motion C90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column Forces</td>
<td>1,675</td>
<td>1,584</td>
<td>1,507</td>
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<tr>
<td>Inertial Forces</td>
<td>1,854</td>
<td>1,633</td>
<td>1,515</td>
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</table>

**(b) Maximum Story Drifts (in.)**

<table>
<thead>
<tr>
<th>Level</th>
<th>Motion A00</th>
<th>Motion B90</th>
<th>Motion C90</th>
<th>Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Roof</td>
<td>101.69</td>
<td>26.10</td>
<td>20.50</td>
<td>NA</td>
</tr>
<tr>
<td>R-6</td>
<td>1.95</td>
<td>2.32</td>
<td>2.93</td>
<td>4.5</td>
</tr>
<tr>
<td>6-5</td>
<td>2.97</td>
<td>2.60</td>
<td>3.49</td>
<td>4.5</td>
</tr>
<tr>
<td>5-4</td>
<td>6.41</td>
<td>3.62</td>
<td>4.32</td>
<td>4.5</td>
</tr>
<tr>
<td>4-3</td>
<td>20.69</td>
<td>5.61</td>
<td>4.63</td>
<td>4.5</td>
</tr>
<tr>
<td>3-2</td>
<td>31.65</td>
<td>6.32</td>
<td>4.18</td>
<td>4.5</td>
</tr>
<tr>
<td>2-G</td>
<td>40.13</td>
<td>7.03</td>
<td>5.11</td>
<td>5.4</td>
</tr>
</tbody>
</table>

### Table 4.2-20 DBE Results for 5% Damped Strong Panel Model with P-Delta Excluded

**(a) Maximum Base Shear (kips)**

<table>
<thead>
<tr>
<th>Level</th>
<th>Motion A00</th>
<th>Motion B90</th>
<th>Motion C90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column Forces</td>
<td>1,622</td>
<td>1,568</td>
<td>1,483</td>
</tr>
<tr>
<td>Inertial Forces</td>
<td>1,773</td>
<td>1,576</td>
<td>1,482</td>
</tr>
</tbody>
</table>

**(b) Maximum Story Drifts (in.)**

<table>
<thead>
<tr>
<th>Level</th>
<th>Motion A00</th>
<th>Motion B90</th>
<th>Motion C90</th>
<th>*Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Roof</td>
<td>19.17</td>
<td>14.09</td>
<td>13.14</td>
<td>NA</td>
</tr>
<tr>
<td>R-6</td>
<td>1.33</td>
<td>1.73</td>
<td>1.77</td>
<td>3.00 (3.75)</td>
</tr>
<tr>
<td>6-5</td>
<td>2.18</td>
<td>2.52</td>
<td>2.32</td>
<td>3.00 (3.75)</td>
</tr>
<tr>
<td>5-4</td>
<td>3.06</td>
<td>2.98</td>
<td>2.89</td>
<td>3.00 (3.75)</td>
</tr>
<tr>
<td>4-3</td>
<td>3.97</td>
<td>2.86</td>
<td>2.78</td>
<td>3.00 (3.75)</td>
</tr>
<tr>
<td>3-2</td>
<td>5.02</td>
<td>3.19</td>
<td>2.72</td>
<td>3.00 (3.75)</td>
</tr>
<tr>
<td>2-G</td>
<td>6.13</td>
<td>4.05</td>
<td>3.01</td>
<td>3.60 (4.50)</td>
</tr>
</tbody>
</table>

*Values in ( ) reflect increased drift limits provided by Standard Sec. 16.2.4.3.*
### Table 4.2-21 DBE Results for 5% Damped Strong Panel Model with P-Delta Included

<table>
<thead>
<tr>
<th>(a) Maximum Base Shear (kips)</th>
<th>Level</th>
<th>Motion A00</th>
<th>Motion B90</th>
<th>Motion C90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column forces</td>
<td></td>
<td>1,374</td>
<td>1,419</td>
<td>1,355</td>
</tr>
<tr>
<td>Inertial forces</td>
<td></td>
<td>1,524</td>
<td>1,448</td>
<td>1,361</td>
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</table>

<table>
<thead>
<tr>
<th>(b) Maximum Displacement and Story Drift (in.)</th>
<th>Level</th>
<th>Motion A00</th>
<th>Motion B90</th>
<th>Motion C90</th>
<th>*Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roof displacement</td>
<td>21.76</td>
<td>14.07</td>
<td>14.16</td>
<td>NA</td>
<td></td>
</tr>
<tr>
<td>Drift R-6</td>
<td>1.40</td>
<td>1.56</td>
<td>1.73</td>
<td>3.00 (3.75)</td>
<td></td>
</tr>
<tr>
<td>Drift 6-5</td>
<td>2.25</td>
<td>2.42</td>
<td>2.33</td>
<td>3.00 (3.75)</td>
<td></td>
</tr>
<tr>
<td>Drift 5-4</td>
<td>3.23</td>
<td>2.80</td>
<td>3.00</td>
<td>3.00 (3.75)</td>
<td></td>
</tr>
<tr>
<td>Drift 4-3</td>
<td>4.38</td>
<td>3.04</td>
<td>3.09</td>
<td>3.00 (3.75)</td>
<td></td>
</tr>
<tr>
<td>Drift 3-2</td>
<td>5.60</td>
<td>3.28</td>
<td>2.77</td>
<td>3.00 (3.75)</td>
<td></td>
</tr>
<tr>
<td>Drift 2-G</td>
<td>7.12</td>
<td>4.33</td>
<td>3.15</td>
<td>3.60 (4.50)</td>
<td></td>
</tr>
</tbody>
</table>

*Values in () reflect increased drift limits provided by Standard Sec. 16.2.4.3.

### Table 4.2-22 MCE Results for 5% Damped Strong Panel Model with P-Delta Excluded

<table>
<thead>
<tr>
<th>(a) Maximum Base Shear (kips)</th>
<th>Level</th>
<th>Motion A00</th>
<th>Motion B90</th>
<th>Motion C90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column forces</td>
<td></td>
<td>1,918</td>
<td>1,760</td>
<td>1,630</td>
</tr>
<tr>
<td>Inertial forces</td>
<td></td>
<td>2,139</td>
<td>1,861</td>
<td>1,633</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(b) Maximum Displacement and Story Drift (in.)</th>
<th>Level</th>
<th>Motion A00</th>
<th>Motion B90</th>
<th>Motion C90</th>
<th>Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roof displacement</td>
<td>40.84</td>
<td>20.17</td>
<td>21.10</td>
<td>NA</td>
<td></td>
</tr>
<tr>
<td>Drift R-6</td>
<td>1.68</td>
<td>1.94</td>
<td>2.97</td>
<td>4.5</td>
<td></td>
</tr>
<tr>
<td>Drift 6-5</td>
<td>2.91</td>
<td>2.61</td>
<td>3.75</td>
<td>4.5</td>
<td></td>
</tr>
<tr>
<td>Drift 5-4</td>
<td>4.86</td>
<td>3.12</td>
<td>4.50</td>
<td>4.5</td>
<td></td>
</tr>
<tr>
<td>Drift 4-3</td>
<td>9.04</td>
<td>4.18</td>
<td>4.43</td>
<td>4.5</td>
<td></td>
</tr>
<tr>
<td>Drift 3-2</td>
<td>10.48</td>
<td>4.77</td>
<td>3.98</td>
<td>4.5</td>
<td></td>
</tr>
<tr>
<td>Drift 2-G</td>
<td>13.04</td>
<td>6.09</td>
<td>4.93</td>
<td>5.4</td>
<td></td>
</tr>
</tbody>
</table>

### Table 4.2-23 MCE Results for 5% Damped Strong Panel Model with P-Delta Included

<table>
<thead>
<tr>
<th>(a) Maximum Base Shear (kips)</th>
<th>Level</th>
<th>Motion A00</th>
<th>Motion B90</th>
<th>Motion C90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column forces</td>
<td></td>
<td>1,451</td>
<td>1,486</td>
<td>1,413</td>
</tr>
<tr>
<td>Inertial forces</td>
<td></td>
<td>1,798</td>
<td>1,607</td>
<td>1,419</td>
</tr>
</tbody>
</table>
### 4.2.6.3.2 Discussion of response history analyses.

The computed structural response to Ground Motion A00 is clearly quite different from that for Ground Motions B90 and C90. This difference in behavior occurs even though the records are all scaled to produce exactly the same spectral acceleration at the structure’s fundamental period. A casual inspection of the ground acceleration histories and response spectra (Figures 4.2-40 through 4.2-42) does not reveal the underlying reason for this difference in behavior.

Figure 4.2-44 shows response histories of roof displacement and first story drift for the 2 percent damped SP model subjected to the DBE-scaled A00 ground motion. Two trends are readily apparent. First, the vast majority of the roof displacement results in residual deformation in the first story. Second, the P-delta effect increases residual deformations by about 50 percent. Such extreme differences in behavior do not appear in plots of base shear, as provided in Figure 4.2-45.

The residual deformations shown in Figure 4.2-44 may be real (due to actual system behavior) or may reflect accumulated numerical errors in the analysis. Numerical errors are unlikely because the shears computed from member forces and from inertial forces are similar. The energy response history can provide further validation. Figure 4.2-46 shows the energy response history for the 2 percent damped DBE analysis with P-delta effects included. If the analysis is accurate, the input energy will coincide with the total energy (sum of kinetic, damping and structural energy). DRAIN 2D produces individual energy values as well as the input energy. See the article by Uang and Bertero for background on computing energy curves.

As evident from Figure 4.2-46, the total and input energy curves coincide, so the analysis is numerically accurate. Where this accuracy is in doubt, the analysis should be re-run using a smaller integration time step. A time step of 0.0005 second is required to produce the energy balance shown in Figure 4.2-46. A time step of 0.001 second is sufficient for analyses with Ground Motions B90 and C90.

The trends observed for the DBE analysis are even more extreme when the MCE ground motion is used. Figure 4.2-47 shows the displacement histories for the 2 percent damped structure under the MCE scaled A00 ground motion. As may be seen, residual deformations again dominate and in this case the total residual roof displacement with P-delta effects included is five times that without P-delta effects. This behavior indicates dynamic instability and eventual collapse.

It is interesting to compare the response computed for Ground Motion B90 with that obtained for ground motion A00. Displacements occurring for the 2 percent damped model under the MCE-scaled B90 ground motions are shown in Figure 4.2-48. While there is some small residual deformation in this

<table>
<thead>
<tr>
<th>Level</th>
<th>Motion A00</th>
<th>Motion B90</th>
<th>Motion C90</th>
<th>Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roof displacement</td>
<td>54.33</td>
<td>23.12</td>
<td>21.83</td>
<td>NA</td>
</tr>
<tr>
<td>Drift R-6</td>
<td>1.66</td>
<td>2.01</td>
<td>2.88</td>
<td>4.5</td>
</tr>
<tr>
<td>Drift 6-5</td>
<td>2.65</td>
<td>2.38</td>
<td>3.64</td>
<td>4.5</td>
</tr>
<tr>
<td>Drift 5-4</td>
<td>4.88</td>
<td>3.31</td>
<td>4.49</td>
<td>4.5</td>
</tr>
<tr>
<td>Drift 4-3</td>
<td>12.63</td>
<td>5.09</td>
<td>4.72</td>
<td>4.5</td>
</tr>
<tr>
<td>Drift 3-2</td>
<td>15.27</td>
<td>5.66</td>
<td>4.28</td>
<td>4.5</td>
</tr>
<tr>
<td>Drift 2-G</td>
<td>19.31</td>
<td>6.14</td>
<td>5.07</td>
<td>5.4</td>
</tr>
</tbody>
</table>
system, it is not extreme, and it appears that the structure is not in danger of collapse. (The corresponding plastic rotations are less than those that would be associated with significant strength loss.)

The characteristic of the ground motion that produces the residual deformations shown in Figures 4.2-44 and 4.2-48 (the DBE and MCE scaled A00 ground motions, respectively) is not evident from the ground acceleration history or from the acceleration response spectrum. The source of the behavior is quite obvious from plots of the ground velocity and ground displacement histories, shown in Figure 4.2-49(a) and (b), respectively. The ground velocity history shows that a very large velocity pulse occurs approximately 10 seconds into the earthquake. This leads to a surge in ground displacement, also occurring approximately 10 seconds into the response. The surge in ground displacement is more than 8 feet, which is somewhat unusual. Recall from Table 4.1-20(a) that the distance between the epicenter and the recording site for this ground motion is 44 kilometers; so, the motion would not be considered as near-field. The unusual characteristics of Ground Motion A00 may be seen in Figure 4.2-49 (c), which is a tripartite spectrum.

![Figure 4.2-44](image1.png)  
**Figure 4.2-44** Response history of roof and first-story displacement, Ground Motion A00 (DBE)

![Figure 4.2-45](image2.png)  
**Figure 4.2-45** Response history of total base shear, Ground Motion A00 (DBE)
Figure 4.2-46  Energy response history, Ground Motion A00 (DBE), including P-delta effects

Figure 4.2-47  Response history of roof and first-story displacement, Ground Motion A00 (MCE)
Figure 4.2-48  Response history of roof and first-story displacement, Ground Motion B90 (MCE)
Figure 4.2-49  Ground velocity and displacement histories and tripartite spectrum of Ground Motion A00 (unscaled)

(c) Tripartite spectrum

Figure 4.2-50 shows the pattern of yielding in the structure subjected to a 2 percent damped MCE-scaled Ground Motion B90 including P-delta effects. Recall that the model incorporates panel zone reinforcement at the interior beam-column joints. The circles on the figure represent yielding at any time during the response; consequently, yielding does not necessarily occur at all locations simultaneously. The circles shown at the upper left corner of the beam-column joint region indicate yielding in the rotational spring, which represents the web component of panel zone behavior. There is no yielding in the flange component of the panel zones, as seen in Figure 4.2-50.

Yielding patterns for the other ground motions and for analyses run with and without P-delta effects are similar but are not shown here. As expected, there is more yielding in the columns when the structure is subjected to the A00 ground motion.

Figure 4.2-50 shows that yielding occurs at both ends of each of the girders at Levels 2, 3, 4 and 5. Yielding occurs at the bottom of all the first-story columns as well as at the top of the interior columns at the third and fourth stories and at bottom of the fifth-story interior columns. The panel zones at the exterior joints of Levels 4 and 5 also yield. The maximum plastic hinge rotations are shown where they occur for the columns, girders and panel zones; values are shown in Table 4.2-24. The maximum plastic shear strain in the web of the panel zone is identical to the computed hinge rotation in the panel zone spring. For the DBE-scaled B90 ground motion, the maximum rotations occurring at the plastic hinges are less than 0.02 radians.
Figure 4.2-50 Yielding locations for structure with strong panels subjected to MCE-scaled B90 motion, including P-delta effects

4.2.6.3.3 Comparison with results from other analyses. Table 4.2-24 compares the results from the response history analysis with those from the ELF and the nonlinear static analyses. Base shears in the table are half of the total shear. The nonlinear static analysis results are for the 2 percent damped MCE target displacement so, for consistency, the tabulated dynamic analysis results are for the 2 percent damped MCE-scaled B90 ground motion. In addition, the lateral forces used to find the ELF drifts in Table 4.2-6 are multiplied by 1.5 for consistency with MCE-level shaking; the ELF analysis drift values include the deflection amplification factor of 5.5. The results show some similarities and some striking differences, as follows:

- The base shear from nonlinear dynamic analysis is approximately three times the value from ELF analysis. The predicted displacements and story drifts are similar at the top three stories but are significantly different at the bottom three stories. Due to the highly empirical nature of the ELF approach, it is difficult to explain these differences. The ELF method also has no mechanism to include the overstrength that will occur in the structure, although it is represented explicitly in the static and dynamic nonlinear analyses.

- The nonlinear static analysis predicts base shears and story displacements that are less than those obtained from response history analysis. Excessive drift occurred at the bottom three stories as a result of both pushover and response history analyses.
<table>
<thead>
<tr>
<th>Response Quantity</th>
<th>Analysis Method</th>
<th>Equivalent</th>
<th>Nonlinear Static</th>
<th>Nonlinear Dynamic</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lateral Forces</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Base shear (kips)</td>
<td>569</td>
<td>1,208</td>
<td>1,633</td>
<td></td>
</tr>
<tr>
<td>Roof disp. (in.)</td>
<td>18.4</td>
<td>22.9</td>
<td>26.1</td>
<td></td>
</tr>
<tr>
<td>Drift R-6 (in.)</td>
<td>1.86</td>
<td>0.96</td>
<td>2.32</td>
<td></td>
</tr>
<tr>
<td>Drift 6-5 (in.)</td>
<td>2.78</td>
<td>1.76</td>
<td>2.60</td>
<td></td>
</tr>
<tr>
<td>Drift 5-4 (in.)</td>
<td>3.34</td>
<td>2.87</td>
<td>3.62</td>
<td></td>
</tr>
<tr>
<td>Drift 4-3 (in.)</td>
<td>3.73</td>
<td>4.84</td>
<td>5.61</td>
<td></td>
</tr>
<tr>
<td>Drift 3-2 (in.)</td>
<td>3.67</td>
<td>5.74</td>
<td>6.32</td>
<td></td>
</tr>
<tr>
<td>Drift 2-1 (in.)</td>
<td>2.98</td>
<td>6.73</td>
<td>7.03</td>
<td></td>
</tr>
<tr>
<td>Girder hinge rot. (rad)</td>
<td>NA</td>
<td>0.03304</td>
<td>0.03609</td>
<td></td>
</tr>
<tr>
<td>Column hinge rot. (rad)</td>
<td>NA</td>
<td>0.02875</td>
<td>0.02993</td>
<td></td>
</tr>
<tr>
<td>Panel hinge rot. (rad)</td>
<td>NA</td>
<td>0.00335</td>
<td>0.00411</td>
<td></td>
</tr>
<tr>
<td>Panel plastic shear strain</td>
<td>NA</td>
<td>0.00335</td>
<td>0.00411</td>
<td></td>
</tr>
</tbody>
</table>

Note: Shears are for half of total structure.

Some of the difference between pushover and nonlinear response history results is due to the scale factor (1.367) used to satisfy ground motion scaling requirements for the nonlinear response history analysis, but most of the difference is due to higher mode effects. Figure 4.2-51 shows the inertial forces from the nonlinear response history analyses at the time of peak base shear and the loads applied to the nonlinear static analysis model at the target displacement. The higher mode effects apparent in Figure 4.2-51 likely are the cause of the different hinging patterns and certainly are the reason for the very high base shear developed in the response history analysis. (If the inertial forces were constrained to follow the first mode response, the maximum base shear that could be developed in the system would be in the range of 1200 kips. See, for example, Figure 4.2-28.)
4.2.6.3.4 Effect of increased damping on response. The nonlinear response history analysis of the structure with panel zone reinforcement indicates first story drifts in excess of the allowable limits. The most cost-effective measure to enhance the performance of the structure probably would be to provide additional strength and/or stiffness at this story. However, added damping is also a viable approach.

To investigate the viability of added damping, additional analysis that treats individual dampers explicitly is required. Linear viscous damping can be modeled in DRAIN using the stiffness proportional component of Rayleigh damping. Base shear increases with added damping, so in practice added damping systems usually employ viscous fluid devices with a “softening” nonlinear relationship between the deformational velocity in the device and the force in the device, to limit base shears when deformational velocities become large.

A linear viscous fluid damping device (Figure 4.2-52) in a selected story can be modeled using a Type 1 (truss bar) element. A damping constant for the device, \( C_{device} \), is obtained as follows:

The elastic stiffness of the damper element is simply as follows:

\[
k_{device} = \frac{A_{device} E_{device}}{L_{device}}
\]

where:

\( A_{device} \) = the cross sectional area

\( E_{device} \) = the modulus of elasticity

\( L_{device} \) = the length of the Type 1 damper element
As stiffness proportional damping is used, the damping constant for the element is:

\[ C_{device} = \beta_{device} k_{device} \]

The damper elastic stiffness should be negligible, so consider \( k_{device} = 0.001 \text{ kips/in.} \). Thus:

\[ \beta_{device} = \frac{C_{device}}{0.001} = 1000C_{device} \]

Where modeling added dampers in this manner, it is convenient to consider \( E_{device} = 0.001 \) and \( A_{device} = \) the damper length \( L_{device} \).

This value of \( \beta_{device} \) is for the added damper element only. Different dampers may require different values. Also, a different (global) value of \( \beta \) is required to model the stiffness proportional component of damping in the remaining nondamper elements.

Modeling the dynamic response using Type 1 elements is exact within the typical limitations of finite element analysis. Using the modal strain energy approach, DRAIN reports a damping value in each mode. These modal damping values are approximate and may be poor estimates of actual modal damping, particularly where there is excessive flexibility in the mechanism that connects the damper to the structure.

To determine the effect of added damping on the behavior of the structure, dampers are added to the SP frame with 2 percent inherent damping, and the structure is subjected to the DBE-scaled A00 and B90 ground motions. P-delta effects are included in the analyses. Table 4.2-25 shows the base shear and story drifts of the SP frame with 2 percent inherent damping when it is subjected to DBE-scaled A00 and B90 ground motions. The results summarized in this table can also be found in the tables of Section 4.2.6.3.1.

![Figure 4.2-52 Modeling a simple damper](image-url)
Table 4.2-25 Maximum Story Drifts (in.) and Base Shear (kips) when SP Model with 2% Inherent Damping is Subjected to DBE Scaled A00 and B90 Ground Motions, including P-delta Effects

<table>
<thead>
<tr>
<th>Level</th>
<th>Motion A00</th>
<th>Motion B90</th>
<th>Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roof displacement</td>
<td>32.65</td>
<td>14.50</td>
<td>NA</td>
</tr>
<tr>
<td>Drift R-6</td>
<td>1.86</td>
<td>1.82</td>
<td>3.75</td>
</tr>
<tr>
<td>Drift 6-5</td>
<td>2.64</td>
<td>2.50</td>
<td>3.75</td>
</tr>
<tr>
<td>Drift 5-4</td>
<td><strong>4.08</strong></td>
<td>2.81</td>
<td>3.75</td>
</tr>
<tr>
<td>Drift 4-3</td>
<td><strong>6.87</strong></td>
<td>3.21</td>
<td>3.75</td>
</tr>
<tr>
<td>Drift 3-2</td>
<td><strong>8.19</strong></td>
<td>3.40</td>
<td>3.75</td>
</tr>
<tr>
<td>Drift 2-G</td>
<td><strong>10.40</strong></td>
<td><strong>4.69</strong></td>
<td>4.50</td>
</tr>
</tbody>
</table>

| Column forces  | 1467       | 1458       | NA    |
| Inertial forces| 1558       | 1481       | NA    |

As can be seen in Table 4.2-25, drift limits are exceeded at the bottom four stories for the A00 ground motion and only for the bottom story for the B90 ground motion.

Four different added damper configurations are used to assess their effect on story drifts and base shear, as summarized in Tables 4.2-26 and 4.2-27.

Table 4.2-26 Effect of Different Added Damper Configurations when SP Model is Subjected to DBE-Scaled A00 Ground Motion, including P-delta Effects

<table>
<thead>
<tr>
<th>Level</th>
<th>First Config</th>
<th>Second Config</th>
<th>Third Config</th>
<th>Fourth Config</th>
<th>Drift Limit (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Damper Coeff. (kip-sec/in.)</td>
<td>Drift (in.)</td>
<td>Damper Coeff. (kip-sec/in.)</td>
<td>Drift (in.)</td>
<td>Damper Coeff. (kip-sec/in.)</td>
</tr>
<tr>
<td>R-6</td>
<td>10.5</td>
<td>1.10</td>
<td>60</td>
<td>1.03</td>
<td>-</td>
</tr>
<tr>
<td>6-5</td>
<td>33.7</td>
<td>1.90</td>
<td>60</td>
<td>1.84</td>
<td>-</td>
</tr>
<tr>
<td>5-4</td>
<td>38.4</td>
<td>2.99</td>
<td>70</td>
<td>2.88</td>
<td>-</td>
</tr>
<tr>
<td>4-3</td>
<td>32.1</td>
<td><strong>5.46</strong></td>
<td>70</td>
<td><strong>4.42</strong></td>
<td>-</td>
</tr>
<tr>
<td>3-2</td>
<td>36.5</td>
<td><strong>6.69</strong></td>
<td>80</td>
<td><strong>5.15</strong></td>
<td>160</td>
</tr>
<tr>
<td>2-G</td>
<td>25.6</td>
<td><strong>8.39</strong></td>
<td>80</td>
<td><strong>5.87</strong></td>
<td>160</td>
</tr>
</tbody>
</table>

| Column base shear (kips) | 1,629 | 2,170 | 2,134 | 2,267 |
| Inertial base shear (kips) | 1,728 | 2,268 | 2,215 | 2,350 |
| Total damping (%)         | 10.1  | 20.4  | 20.2  | 20.4  |
Table 4.2-27  Effect of Different Added Damper Configurations when SP Model is Subjected to DBE-Scaled B90 Ground Motion, including P-delta Effects

<table>
<thead>
<tr>
<th>Level</th>
<th>First Config Damper Coeff. (kip-sec/in.)</th>
<th>Drift (in.)</th>
<th>Second Config Damper Coeff. (kip-sec/in.)</th>
<th>Drift (in.)</th>
<th>Third Config Damper Coeff. (kip-sec/in.)</th>
<th>Drift (in.)</th>
<th>Fourth Config Damper Coeff. (kip-sec/in.)</th>
<th>Drift (in.)</th>
<th>Drift Limit (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-6</td>
<td>10.5</td>
<td>1.11</td>
<td>60</td>
<td>0.86</td>
<td>-</td>
<td>1.53</td>
<td>-</td>
<td>1.31</td>
<td>3.75</td>
</tr>
<tr>
<td>6-5</td>
<td>33.7</td>
<td>1.76</td>
<td>60</td>
<td>1.35</td>
<td>-</td>
<td>2.11</td>
<td>-</td>
<td>1.83</td>
<td>3.75</td>
</tr>
<tr>
<td>5-4</td>
<td>38.4</td>
<td>2.33</td>
<td>70</td>
<td>1.75</td>
<td>-</td>
<td>2.51</td>
<td>56.25</td>
<td>2.07</td>
<td>3.75</td>
</tr>
<tr>
<td>4-3</td>
<td>32.1</td>
<td>2.67</td>
<td>70</td>
<td>2.11</td>
<td>-</td>
<td>2.37</td>
<td>56.25</td>
<td>2.16</td>
<td>3.75</td>
</tr>
<tr>
<td>3-2</td>
<td>36.5</td>
<td>2.99</td>
<td>80</td>
<td>2.25</td>
<td>160</td>
<td>2.09</td>
<td>112.5</td>
<td>2.13</td>
<td>3.75</td>
</tr>
<tr>
<td>2-G</td>
<td>25.6</td>
<td>3.49</td>
<td>80</td>
<td>1.96</td>
<td>160</td>
<td>1.87</td>
<td>112.5</td>
<td>1.82</td>
<td>4.50</td>
</tr>
<tr>
<td></td>
<td>Column base shear (kips)</td>
<td>1,481</td>
<td>1,485</td>
<td>1,697</td>
<td>1,637</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Inertial base shear (kips)</td>
<td>1,531</td>
<td>1,527</td>
<td>1,739</td>
<td>1,680</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total damping (%)</td>
<td>10.1</td>
<td>20.4</td>
<td>20.2</td>
<td>20.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

These configurations increase total damping of the structure from 2 percent (inherent) to 10 and 20 percent. In the first configuration added dampers are distributed proportionally to approximate story stiffnesses. In the second configuration, dampers are added at all six stories, with larger dampers in lower stories. Since the structure seems to be weak at the bottom stories (where it exceeds drift limits), dampers are concentrated at the bottom stories in the last two configurations. Added dampers are used only at the first and second stories in the third configuration and at the bottom four stories in the fourth configuration.

Based on this supplemental damper study, it appears to be impossible to decrease the story drifts for the A00 ground motion below the limits. This is because of the incremental velocity of Ground Motion A00 causes such significant structural damage. The drift limits could be satisfied if the total damping ratio is increased to 33.5 percent, but since that is impractical the results are not reported here. The third configuration of added dampers reduces the first-story drift from 10.40 inches to 4.40 inches.

All of the configurations easily satisfy drift limits for the B90 ground motion. While the system with 10 percent total damping is sufficient for drift limits, systems with 20 percent damping further improve performance. Although configurations 3 and 4 have the same amount of total damping as configuration 2, story drifts are higher at the top stories since dampers are added only at lower stories.

Figures 4.2-53 through 4.2-55 show the effect of added damping of roof displacement, inertial base shear and energy history for the A00 ground motion. As Figure 4.2-53 shows added dampers reduce roof displacement significantly but do not prevent residual displacement. Figure 4.2-54 shows how added damping increases peak base shear. Figure 4.2-55 is an energy response history for the structure with damping configuration 4. It should be compared to Figure 4.2-46, which is the energy history for the structure with 2 percent inherent damping but with no added damping. As should be expected, adding discrete damping reduces the hysteretic energy demand in the structure (designated as structural energy in Figure 4.2-55). A reduction in hysteretic energy demand for the system with added damping corresponds to a reduction in structural damage.
Figures 4.2-56 through 4.2-58 display the same response plots for Ground Motion B90. As for Ground Motion A00 roof displacement decreases with added damping, peak base shear increases and hysteretic energy demand (which is related to structural damage) decreases.

**Figure 4.2-53** Roof displacement response histories with added damping (20% total) and inherent damping (2%) for Ground Motion A00

**Figure 4.2-54** Inertial base shear response histories with added damping (20% total) and inherent damping (2%) for Ground Motion A00
Figure 4.2-55  Energy response history with added damping of fourth configuration (20% total damping) for Ground Motion A00

Figure 4.2-56  Roof displacement response histories with added damping (20% total) and inherent damping (2%) for Ground Motion B90
**Summary and Conclusions**

In this example, five different analytical approaches are used to estimate the deformation demands in a simple structural steel moment-resisting frame structure:

1. Linear static analysis (the equivalent lateral force method)
2. Plastic strength analysis (using virtual work)
3. Nonlinear static (pushover) analysis
4. Linear dynamic (modal response history) analysis
5. Nonlinear dynamic (response history) analysis

The nonlinear structural model includes careful representation of possible inelastic behavior in the panel-zone regions of the beam-column joints.

The results obtained from the three different analytical approaches 1, 3 and 5 are quite dissimilar. Except for preliminary design, the ELF approach should not be used in explicit performance evaluation since it cannot reflect the location and extent of yielding in the structure. Due to higher mode effects, pushover analysis, where used alone, is inadequate.

This leaves nonlinear response history analysis as the most viable approach. Given the speed and memory capacity of personal computers, nonlinear response history analysis is increasingly common in the seismic analysis of buildings. However, significant shortcomings, limitations and uncertainties in response history analysis still exist.

Among the most pressing problems is the need for a suitable suite of ground motions. All ground motions must adequately reflect site conditions and where applicable, the suite must include near-field effects. Through future research and the efforts of code writing bodies, it may be possible to develop standard suites of ground motions that could be published together with selection tools and scaling methodologies. The scaling techniques currently recommended in the Standard are a start but need improvement.

Systematic methods need to be developed for identifying uncertainties in the modeling of the structure and for quantifying the effect of such uncertainties on the response. While probabilistic methods for dealing with such uncertainties seem like a natural extension of the analytical approach, the authors believe that deterministic methods should not be abandoned entirely.

In the context of performance-based design, improved methods for assessing the effect of inelastic response and acceptance criteria based on such measures need to be developed. Methods based on explicit quantification of damage should be considered seriously.

The ideas presented above certainly are not original. They have been presented by many academics and practicing engineers. What is still lacking is a comprehensive approach to seismic-resistant design based on these principles.