Topic
Mathematics for Land Surveyors

Presenters

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Course Outline:

An Outline including Proposed Time Increment (8:00AM - 12:30PM)

1. Why mathematics is important for land surveying profession: **20 Minutes**

2. Which portion of mathematics knowledge is critical for land surveyors: **20 Minutes**

3. Basic surveying mathematics that applies to land surveying profession: **90 Minutes**

Overview of basic undergraduate surveying mathematics **emphasize mathematical concepts and principles rather than computation.**

BREAK- 15 Minutes

4. Advanced surveying mathematics that applies to land surveying profession: **90 Minutes**

Overview of advanced mathematics for land surveyors **emphasize mathematical concepts and principles rather than computation**.

5. How to visualize immediate preliminary solution to complex mathematical problems : **20 Minutes**

6. Open Discussion and Q & A: **15 Minutes**

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Mr. Avinash Prasad is a Licensed Professional Engineer, Land Surveyor with more than 25 years of professional experience in civil engineering & management field since his graduation in engineering. He is a Doctor of Philosophy (candidate) at New York University. His ongoing doctorate degree at NYU major is Construction Management and minors are Structural Engineering & Technology Management. He has double Masters of Science Degrees in Civil Engineering and Engineering Management from New Jersey institute of technology, NJ and BS Degree in Civil Engineering. He has more than 1000 hours of continuous professional education as instructor/participating professional (1990-2017) excluding formal education. He is a (NJ) state certified emergency medical technician (EMT), emergency medical responder (EMR), fire fighter (FF). He is also a certified CPR, AED administer. He is a FELLOW of American Society of Civil Engineers and also an active member of several professional organizations such as: AREMA, PMI, AISC, NYSAPLS, IPWE, IRT and IIBE. His research papers have been accepted and published by several technical magazines including Railway Track & structures. His technical papers were accepted in AREMA, ASCE & NYSAPLS conference proceedings for presentation and or publication multiple times.

Ms. Purnima Prasad

Research NYU Engineering Student, M.ASCE, M.AICHE, M.AREMA, M.NYSAPLS, SWEM, M.ASQ

Currently, Purnima Prasad is an undergraduate engineering student at NYU's Tandon School of Engineering, and she is an active member of organizations such as ASCE (American Society of Civil Engineers), AREMA (American Railway Engineering and Maintenance), AIChE (American Institute of Chemical Engineers), SWE (Society of Women Engineers), NYSAPLS (New York State Association of Professional Land Surveyors) and ASQ (American Society for Quality). She has been working as an engineering intern for P&P Consulting Engineers, LLC, located in Hasbrouck Heights, New Jersey for over 4 years, and she has also been working for Arco Engineering, PC, located in Lyndhurst, New Jersey for the past 2 years as an engineering assistant. Her expertise in the engineering field is being recognized by industries such as Railway Age, American Society of Civil Engineers, and New York State Association of Professional Land Surveyors. In April 2017, she presented her topic of Railroad Tunnel Life Cost Analysis at the Railway Age Light Rail Conference. In addition to these accomplishments, a paper, Railroad accidents causes & innovative prevention techniques is scheduled to be published in the April 2018 edition of Railway Track & Structure magazine.

Author’s (Avinash / Purnima) Salient Publications/Presentations

- Science Fair Presentation in Hudson County 2015 on “Suspension Bridges” and 2016 on “Maglev as Hybrid Rail Transportation System”

- Railway Age. (2017). Light Rail 2017 + Rail Transit Finance Forum - Agenda. [online] Available at: http://www.railwayage.com/index.php/conference_details/light-rail-2017-agenda.html [Accessed 1 Apr. 2017].

- Prasad, A. (2016). Various Cost-Effective Maintenance Practices for Conventional Track Structures. Railway Track & Structures, [online] pp.29-31. Available at: http://www.rtands.com/index.php/track-maintenance/on-track-maintenance/various-cost-effective-maintenance-practices-for-conventional-track-structures.html?device=desktop [Accessed 10 Jul. 2017].

- Prasad, A. (2013). Suspension Bridges: Concepts and Various Innovative Techniques of Structural Evaluation. RT&S, [online] pp.33-35. Available at: http://www.rtands.com/index.php/track-structure/bridge-retaining-walls/suspension-bridges-concepts-and-various-innovative-techniques-of-structural-evaluation.html [Accessed 12 Mar. 2017].

- Prasad, A. (2011). Higher Diverging Speed Turnout Design in the Same Footprint. Railway Track and Structures, [online] pp.39-44. Available at: https://www.highbeam.com/doc/1G1-319803497.html [Accessed 11 Jun. 2017].

- Prasad, A. (2016). Various Cost Effective Maintenance Practices for Conventional Track Structure. [PDF] AREMA. Available at: https://www.arema.org/proceedings/proceedings_2016.aspx [Accessed 4 May 2017].

- Prasad, A. (2011). Turnout Design: Higher Diverging Speed in The Same Footprint. [PDF] AREMA. Available at: https://www.arema.org/proceedings/proceedings_2011.aspx [Accessed 6 Jul. 2017].

- Marzzoengineering.com. (n.d.). Marzzo Engineering, PLLC - All Instructor Profiles. [online] Available at: http://www.marzzoengineering.com/instructor_profile.html [Accessed 13 May 2017].

- Iceclasses.com. (n.d.). Professional Engineer | Product Categories | ICE Classes. [online] Available at: https://www.iceclasses.com/product-category/professional-engineer/ [Accessed 4 Jul. 2017].

- Presented topics of “Railroad Tunnel Life Cost Analysis” as paper presentation and “Transition from Conventional Railroad to Hybrid Maglev Technology” as poster presentation at ASCE Conference at Duluth, MN in Sept, 2017.

- Presentation and Publication of multiple technical topics at multiple ASCE conferences to be held in USA and abroad in 2017 and 2018(scheduled)

- Proposed Publication in RT & S April 2018 Edition - Topic: “Rail Accidents: Causes and Innovative Preventive Techniques”

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Presentation Highlights

- This course will consist of basic and advanced mathematics for land surveyors. The purpose of this course is to present basic and advanced math concepts and principles useful to survey computations.

- Basic survey mathematics generally consists of applications of formulas and equations that have been adapted to work toward the specific needs of the surveyor. **Overview of basic undergraduate mathematics for land surveyors emphasize mathematical concepts and principles rather than computation.** This will include overview of General surveying: geometry, trigonometry, pre-calculus, error theory, and analysis; Boundary surveying: geometry, trigonometry, error theory and analysis; Cartography: pre-calculus; map projections: linear algebra, advanced calculus; GIS: trigonometry, pre-calculus, statistics, error theory and analysis; remote sensing: trigonometry, pre-calculus, statistics; Photogrammetry: linear algebra, error theory, and analysis; Geodesy: advanced calculus, linear algebra, error theory, and analysis; Surveying applications: varies, but error theory and analysis usually central; Professional skills: pre-calculus; Measurement theory: graph theory, error theory, and analysis; and error theory and analysis: calculus, linear algebra, statistics.

- The advance survey mathematics will help seminar attendees to recognize solution formats for problems and then make correct and effective use of appropriate methods to solve complex survey problems. **Overview of advanced mathematics for land surveyors emphasize mathematical concepts and principles rather than computation.** This will include overview of college algebra, trigonometry, analytic geometry, differential and integral calculus, linear algebra, numerical analysis, probability and statistics, and advanced calculus. Surveyors will find math operations, fundamental and advanced mathematical skills addressed in this course very useful in the educational and professional career.

- It is important for land surveyors to have a developed understanding of the basic operations of arithmetic, algebra, geometry, and trigonometry. This presentation is not designed as a complete math course, but rather as an overview and guide to computation processes unique to surveying and mapping.

Why Mathematics is Important for Land Surveying Profession

Is it Necessary for Surveyors to Know Basic/Advanced Mathematical Concept

YES

As we know

Machine(Computer etc.): Garbage In-Garbage Out

This is applicable to any profession.

Needs to be manually checked some portion of Machine work

Which Portions of Mathematics are Critical for Land Surveyors

Basic or Advanced

In Era of Programmable Devices

?

[7]Mathematics of Life

$$Life = \int_{birth}^{death} \frac{happiness}{time} (\Delta time)$$

Relevant Topics in Basic Survey Mathematics

- **Logarithmic Functions**
- **Properties of Straight Lines**
- **Important Conversions in Surveying**
- **Concept of Derivatives**
- **Significant Figures**
- **Arithmetic, Geometric, and Harmonic Means**
- **Squares, Cubes, and Roots**
- **Area & Volume Calculation**
- **Pythagorean Theorem**
- **Trigonometric Functions**
- **Oblique Triangles**
- **Directions: Bearings and Azimuths**
- **Intersection of Straight Lines**
- **Intersection of Straight Line and Arc**
- **Horizontal Circular Curve**
- **Signed Numbers**
- **Equations**
- **Order of Operation**
- **Parenthesis**
- **Evaluating Equations and Combining Terms**
- **Solving Equations**
- **Quadratic Formulae**
- **Determinants**
- **Derivatives**
- **Survey Measurement Tape Corrections**

[1,2]Logarithmic Functions

- Logarithmic equations
  - $\log_b(x) = y \ where\ y = b^x$
- Logs that have a base of 10 are mostly written as $\log x$
- Logs that have a base of e are mostly written as $\ln x$
- $Base\ change\ formula$: $\log_b x = \frac{\log_a x}{\log_a b}$
- Rules of logarithms
  - $\log_b b = 1$
  - $\log 1 = 0$
  - Log(-ve number)= Not defined
  - $\log_b b^x = x$
  - $\log x^n = n\ \log x$
  - $\log xy = \log x + \log y$
  - $\log \frac{x}{y} = \log x - \log y$

Where symbols having their usual meanings

[1,2]Properties of Straight Lines

- Equations of straight lines
  - $Ax + By + C = 0$
  - $y = mx + b$
  - $y - y_1 = m(x - x_1)$
- $Slope = m = \frac{y_2 - y_1}{x_2 - x_1}$
- Parallel lines have the same slope
- Perpendicular lines have the negative reciprocal slope of each other
- Other important equations
  - $d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$
  - $\alpha = \tan^{-1} \frac{m_2 - m_1}{1 + m_2 \times m_1}$

  Where symbols having their usual meanings

[1,2]Important Relationships in Surveying (1 of 2)

- 1 US survey foot = $\frac{12}{39.37}meters$
- 1 international foot = 0.3048 meters
- 1 inch = 25.4 mm (international)
- 1 mile = 1.60935 km
- 1 acre = 43.560 feet^2 = 10 square chains
- 1 hectare = 10,000 meters^2 = 2.47104 acres
- 1 radian = $\frac{180°}{\pi}$
- 1 kilogram = 2.2046 pounds
- 1 liter = 0.2624 gallons
- 1 feet^3 = 7.481 gallons

Where symbols having their usual meanings

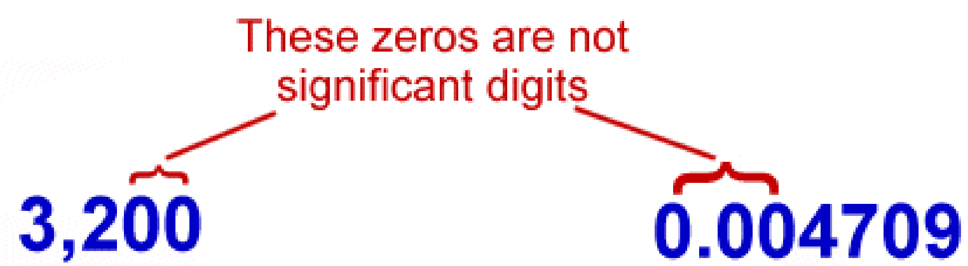
[1,2]Important Relationships in Surveying (2 of 2)

- Weight of 1 gallon of water = 8.34 pounds
- Weight of 1 feet^3 of water = 62.4 pounds
- 1 atm = 29.92 in-Hg = 14.696 psi
- Gravity of acceleration = 9.807 meters/second^2 = 32.174 feet/second^2
- Light's speed in a vacuum = 299,792,458 meters/second^2 = 186,282 miles/second
- Conversion between Celsius and Fahrenheit: $℃ = \frac{℉ - 32}{1.8}$
- 1 min of latitude = 1 nautical mile = 6,076 feet
- Mean of Earth's radius = 20,906,000 feet = 6,372,000 meters

Where symbols having their usual meanings

[10]Significant Figures

- Numbers that have some purpose/function in a measurement according to the measuring and computational precision
- Rules of significant figures
  - All digits that are not zeroes are significant
  - Zeroes between significant digits are significant
  - Zeroes to the left of the decimal point and the beginning of a number are not significant
  - Only trailing zeroes in the decimal portion are significant



[10]Significant Figures in Computations

- Addition and subtraction
  - The fewest decimal places in the measured values determine the amount of significant figures in the final answer
- Multiplication and division
  - Final answer has the same number of significant figures as the measured value that has the smallest significant figures
- Carry one extra digit than the number with the fewest significant figures in calculations to avoid round off mistakes

15.8 + 5.00 + 83	=	103.8	104
(3.7)(12.6)	=	46.62	47
63,000 - 2434	=	60,566	61,000
(26.00)(19.23)	=	499.98	500.0
.004 + 1.5 + 3.002	=	4.506	4.5
2829 / 943	=	3	3.00

[1,2] Arithmetic, Geometric, and Harmonic Means

- Arithmetic mean
  - Utilized when measuring lengths, distances, weights, etc.
  - $Arithmetic\ mean = \frac{Sum\ of\ values}{Total\ number\ of\ values}$
- Geometric mean
  - Applied when measuring speed, distance, resistance in a parallel circuit, etc.
  - $Geometric\ mean = \sqrt[n]{a_1 \times a_2 \times \ldots\ldots\ldots\ldots \times a_n}$
- Harmonic mean
  - Used when measuring percentages, growth rates, etc
  - To calculate the harmonic mean:
    1. Take the reciprocals of the measured values
    2. Obtain the arithmetic mean of those values
    3. Reciprocal of the obtained value is the harmonic mean

Where symbols having their usual meanings

[1,2] Squares, Cubes, and Roots

- Squares
  - $Squares = x^2$
  - A number multiplied by itself
  - Always positive
- Cubes
  - $Cubes = x^3$
  - A number multiplied by itself three times
  - Can be negative or positive
- nth root
  - $n^{th}\ root = \sqrt[n]{x}$
    - When n is even, there are two possible solutions for ever positive real number. There are no real solutions for negative numbers.
    - When n is odd, there is only possible solution.

Where symbols having their usual meanings

[10]AREA Calculation

Trapezoidal Rule

$$Area = Width \times \frac{h_1 + h_n}{2 + h_2 + h_3 \ldots\ldots\ldots + h_{n-1}}$$

Simpson's 1/3 Rule

$$= \frac{Width}{3} \times (h_0 + h_n + 4(h_1 + h_3 + \ldots) \ldots\ldots + 2(h_2 + h_4 \ldots))$$

Where symbols having their usual meanings

[10]VOLUME Calculation

Average End Area Method:

$$Volume = \frac{A_1 + A_2}{2}$$

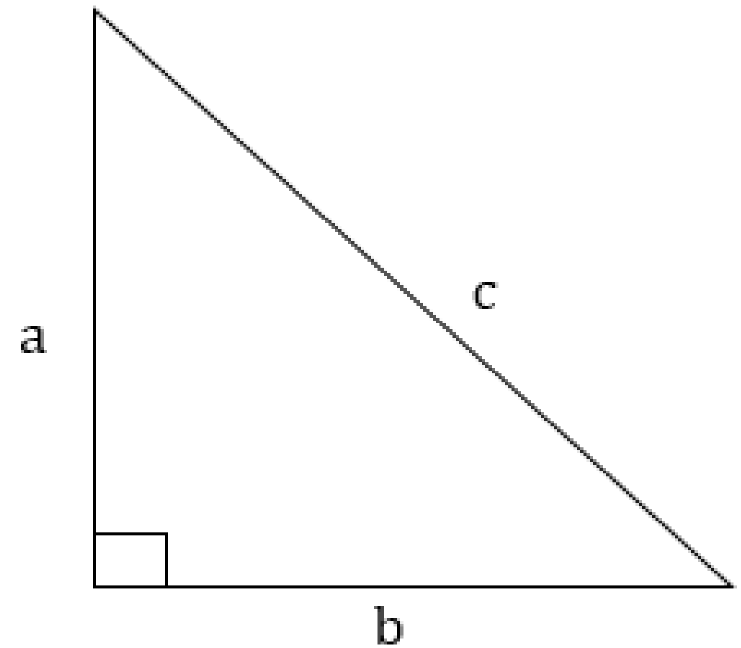
Prismoidal Method:

$$Volume = \frac{A_1 + 4A_m + A_2}{6}$$

Where symbols having their usual meanings

[1,2]Pythagorean Theorem

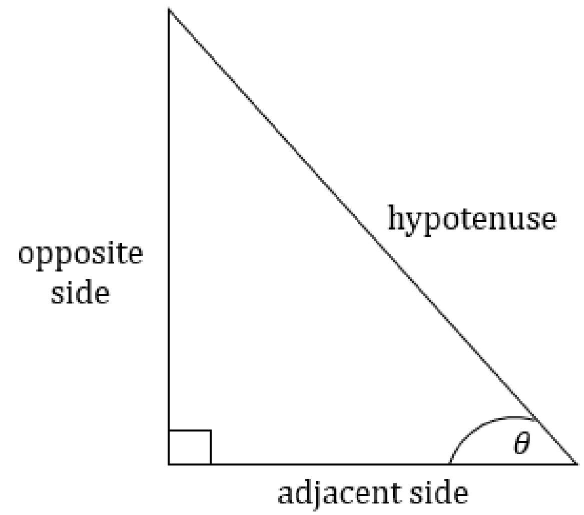
- Pythagorean theorem
  - $c^2 = a^2 + b^2$
  - Common Pythagorean triples
    - 3-4-5
    - 5-12-13
    - 8-15-17
    - 7-24-25
    - 9-40-41
    - Utilized by surveyors in order to obtain right angles with a tape measure



Where symbols having their usual meanings

[1,2] Trigonometric Functions

- Common trigonometric functions
  - $\sin\theta = \dfrac{opposite\ side}{hypotenuse},\ \csc\theta = \dfrac{1}{\sin\theta}$
  - $\cos\theta = \dfrac{adjacent\ side}{hypotenuse},\ \sec\theta = \dfrac{1}{\cos\theta}$
  - $\tan\theta = \dfrac{adjacent\ side}{opposite\ side},\ \cot\theta = \dfrac{1}{\tan\theta}$

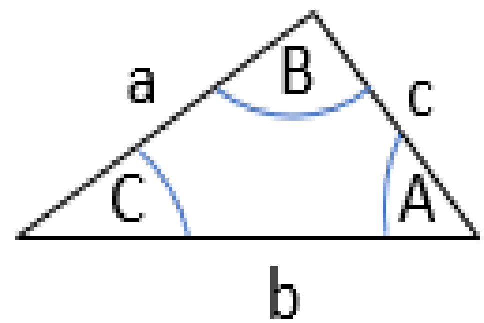


- Common trigonometric rules and identities
  - $\sin\alpha = \sin(180° - \alpha) = -\sin(180° + \alpha) = -\sin(360° - \alpha)$
  - $\cos\alpha = -\cos(180° - \alpha) = -\cos(180° + \alpha) = \cos(360° - \alpha)$
  - $\tan\alpha = -\tan(180° - \alpha) = \tan(180° + \alpha) = -\tan(360° - \alpha)$
  - $\sin^2\alpha + \cos^2\alpha = 1$
  - $\sin\alpha\ \cos\alpha = \frac{1}{2}\sin 2\alpha$
  - $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha}$

Where symbols having their usual meanings

[1,2] Oblique Triangles

- Oblique triangle
  - Triangle has either three acute angles or one obtuse angle and two acute angles
  - In order to find the missing angles and sides of such a triangle, the sine and cosine law should be used
    - Sine law: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
      - Two angles and an adjacent side
      - Two sides and a non-included angle
      - Two angles and a non-adjacent side
    - $Cosine\ law$: $c^2 = a^2 + b^2 - 2ab\cos C$
      - Two sides and the included angle
      - Three sides



Where symbols having their usual meanings

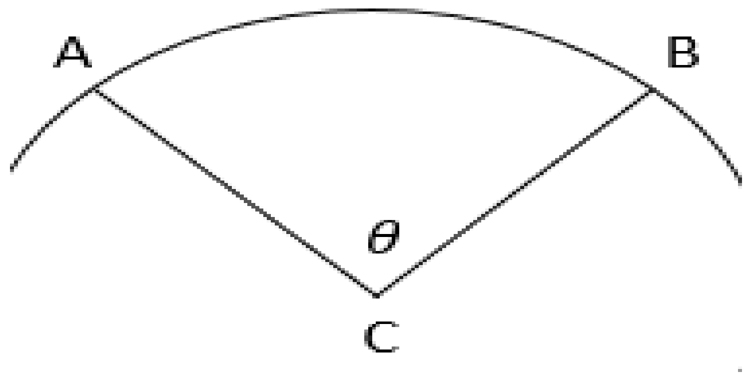
[1,2]Directions: Bearings and Azimuths

- In most cases, north is the 0° reference. But, south and any point could be assigned as the 0° reference.
  - Azimuths
    - Only measured clockwise from north
    - Can be anywhere from 0° to 360°
    - Need no quadrant designation
  - Bearings
    - Measured either counterclockwise or clockwise from the south or north
    - Cannot exceed 90°
    - Require a quadrant designation

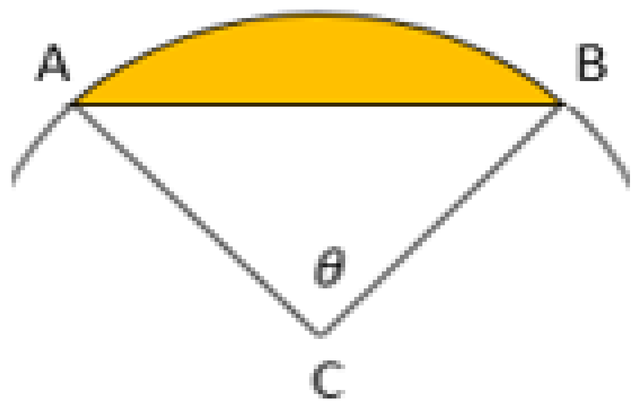
Intersection of Straight Lines

- The intersection of two straight lines that are non-parallel is always a point.
- Steps to find the point where two straight lines intersect
  1. Obtain the equations of each line
  2. Set the two equations equal to each other and solve for X
  3. Plug in the X value into one of the equations and solve for Y

Intersection of Straight Lines and Arcs



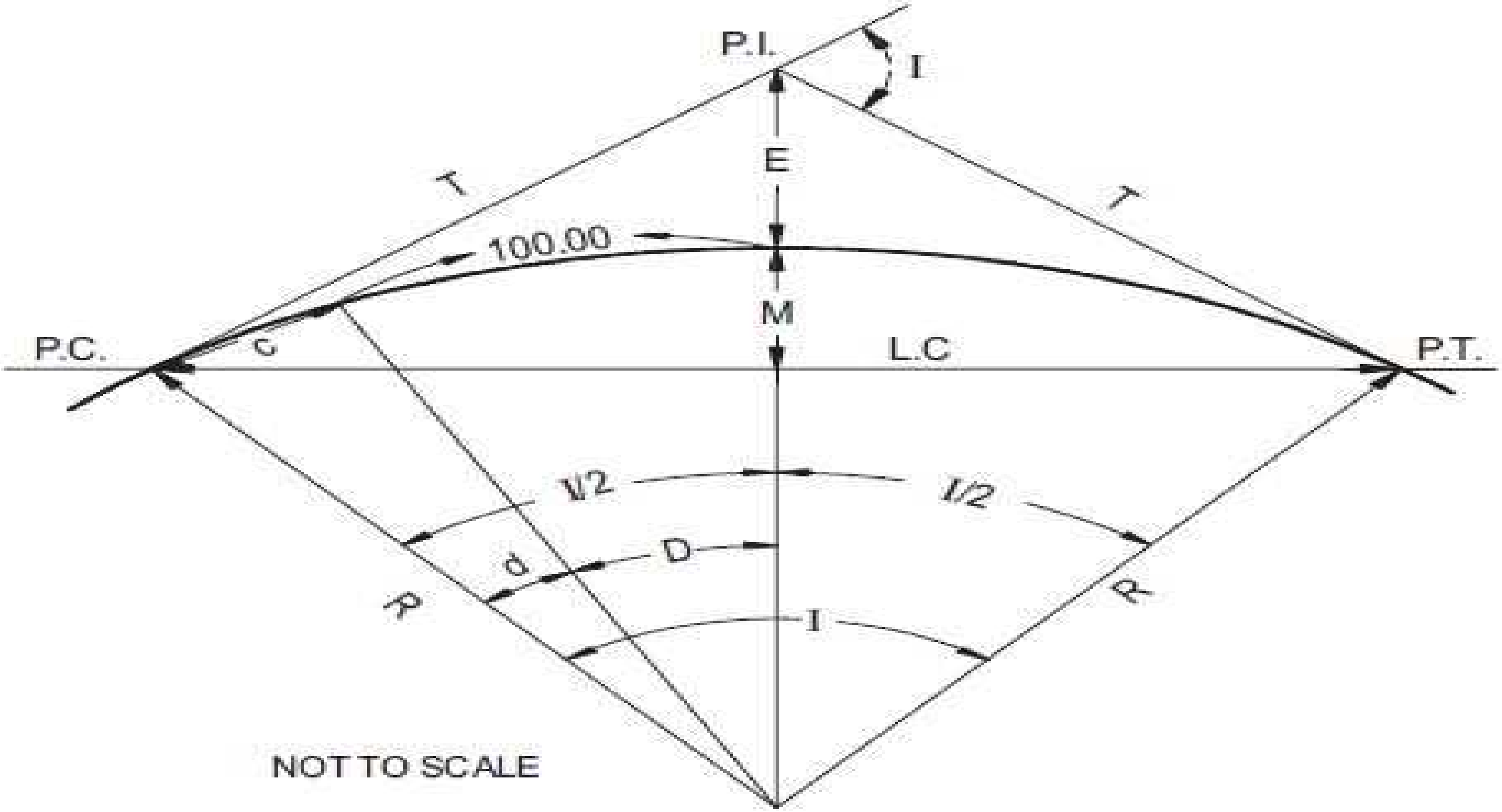
- $Area\ of\ a\ sector = \dfrac{radius^2 \times \theta\ (in\ radians)}{2}$



- $Area\ of\ a\ segment = Area\ of\ sector\ - area\ of\ triangle$

Where symbols having their usual meanings

[1,2]Horizontal Circular Curve (Sketch)



[1,2]Horizontal Circular Curve (Parameters)

- $\text{Length of circular curve (L)} = \text{Radius (R)} \times \text{I(in radians)}$
- $\text{Long tangent length (T)} = \text{Radius (R)} \times \tan\left(\frac{\text{I}}{2}\right)$
- $\text{Mid ordinate (M)} = \text{Radius (R)} \times (1 - \cos\left(\frac{\text{I}}{2}\right))$
- $\text{External distance (E)} = \text{Radius (R)} \times (\sec\left(\frac{\text{I}}{2}\right) - 1)$
- $\text{Long Chord length} = 2 \times \text{Radius (R)} \times \sin\left(\frac{\text{I}}{2}\right)$

[1,2]Signed Numbers

- Consist of both negative and positive numbers
- Sum of two positive numbers is always positive
- Sum of two negative numbers is always negative
- Sum of a positive and negative number Determine the difference in absolute value of the number
    - Sum is positive if the value of the absolute value of the positive number is larger
    - Sum is negative if the value of the absolute value of the negative number is smaller
- When subtracting one signed number from another, take the inverse of the second number and follow the above addition rules
- Product and quotient of two negative/positive numbers is always positive
- Product and quotient of a positive and negative number is always negative

[1,2]Equations

- Statement in which two expressions are set equal to each other
- Each side of an equation may be modified to solve for a variable
  - Each side has to be modified to the same degree and quantity
    - Anything done to the left side of the equation must be done to the right side of the equation as well
- Utilized to determine a variable's value from its relationship to the known values in the equation
- Formed from conditions given by a word problem

[1,2] Order of Operations

- Acronym: PEMDAS
  - Mnemonic phrase: “Please excuse my dear Aunt Sally”
  - P refers to parentheses
    - Innermost parentheses is solved first if there are many nested parentheses
  - E refers to exponents
  - M refers to multiplication and D refers to division
    - Performed from left to right
      - Example: $28 \div 7 \times 2$
  - A refers to addition and S refers to subtraction
    - Performed from left to right
      - Example: $12 - 7 + 6$

[1,2] Parentheses

- Group terms together
  - Indicate what parts of the equations should be evaluated first by grouping terms together
  - Clarify the order in which a confusing or very long equation should be solved
- Rules for removing parentheses from an equation
  - If a plus sign precedes the parentheses, the sign of the terms inside the parentheses do not have to be changed
  - If a minus sign precedes the parentheses, the sign of each term inside the parentheses have to changed
  - If a number precedes the parentheses, the number should be multiplied to each term inside the parentheses

[1,2]Evaluating Equations and Combining Terms

- When evaluating an equation, follow PEMDAS
  - While following PEMDAS, combine terms that have the same variable in them
    - Example: $3b - 3a - 5c + 4b$
  - Terms that have the same variable raised to the same power should also be combined while executing PEMDAS
    - Example: $11xy - 11x^2y - xy + xy^2 + 2x^2b$

[1,2]Solving Equations

- Solution to equations can be found by choosing an unknown that needs to be solved for, isolating it, and solving for it
- Processes that can be performed to simplify solving equations include
  - Clearing out any fractions by multiplying both sides of the equation by the denominator of the fraction
  - Adding, subtracting, multiplying, or dividing the same terms from both sides
  - Combining like terms
  - Factoring
  - Expanding
- Numbers should only be plugged into the equation when the value for the variable has been solved for and when the equation has been set up

[1,2]The Quadratic Formula

- Most quadratic equations are in the form of $ax^2 + bx + c = 0$
- Utilized mainly when factoring the equation is not possible
- $Quadratic\ Formula$: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
  - a, b, and c are numerical coefficients from "$ax^2 + bx + c$"
  - To use the quadratic formula, the equation must be set to zero
  - Can yield two, one, or no real solutions depending on the discriminant
    - Discriminant greater than 0, 2 real solutions
    - Discriminant equal to 0, 1 real solution
    - Discriminant less than 0, no real solutions

[1,2]Determinants (1 of 2)

- Consists of two vertical lines that enclose the elements of the determinant (which are in n rows and n columns)
  - Has an order of n
  - Elements of the determinant = numbers within vertical lines = n^2 numbers
- Minor of an element
  - Determinant that is produced from striking out the row and column that the given element is in
- Place signs
  - Change from positive and negative as one moves through the rows and columns
  - Place sign of the element in the first row and column is always positive

[1,2]Determinants (2 of 2)

- When the order of the determinant is greater than 1, the value of the determinant can be found by
  1. Choose any given row or column
  2. Multiply each element in the chosen row/column by its minor
  3. Take the sum of these products (known as expansion of the determinant)
     - Keep in mind the place sign of each element
- Second-order determinant
  - $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = a(d) - b(c)$
- Third-order determinant
  - $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a(ei - fh) - b(di - fg) + c(dh - eg)$

[1,2]Concept of Derivatives

- The derivative of y = f(x) is:
  - $D_x y = \frac{dy}{dx}$ = y'= $\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$ = slope of function
- x = a is a critical point on y = f(x) if
  - $f'(a) = 0 \ \ or\ f''(a)\ is\ undefined$
- x = a is the maximum of y = f(x) if
  - $f'(a)\ is\ a\ critical\ point\ and\ f''(a) < 0$
  - $f'(a)\ is\ a\ critical\ point\ and\ f'(x)\ changes\ from\ positive\ to\ negative\ at\ a$
- x = a is the minimum of y = f(x) if
  - $f'(a)\ is\ a\ critical\ point\ and\ f''(a) > 0$
  - $f'(a)\ is\ a\ critical\ point\ and\ f'(x)\ changes\ from\ negative\ to\ positve\ at\ a$
- x = a is an inflection point on y = f(x) if
  - f''(a)=0 or $f''(a)\ is\ undefined$
  - f''(x) changes from negtaive to positive at a or vice−versa

Where symbols having their usual meanings

[9,10]Survey Measurement Tape Corrections

A. Correction for Temperature
B. Correction for Pull/Tension
C. Correction for Sag
D. Other Misc. Corrections

[9,10]Survey Measurement Tape Corrections

Correction for Temperature:

$$C_T = L \times (T - T_s) \times Coefficient\ of\ linear\ expansion$$

Correction for Pull/Tension

$$C_P = \frac{L(P - P_s)}{Area \times Modulus\ of\ elasticity}$$

Correction for Sag

$$C_s = \frac{W^2 \times L^3}{24 \times P^2}$$

[9,10]Survey Measurement Tape Corrections

Where

L= Length of Tape (FT)

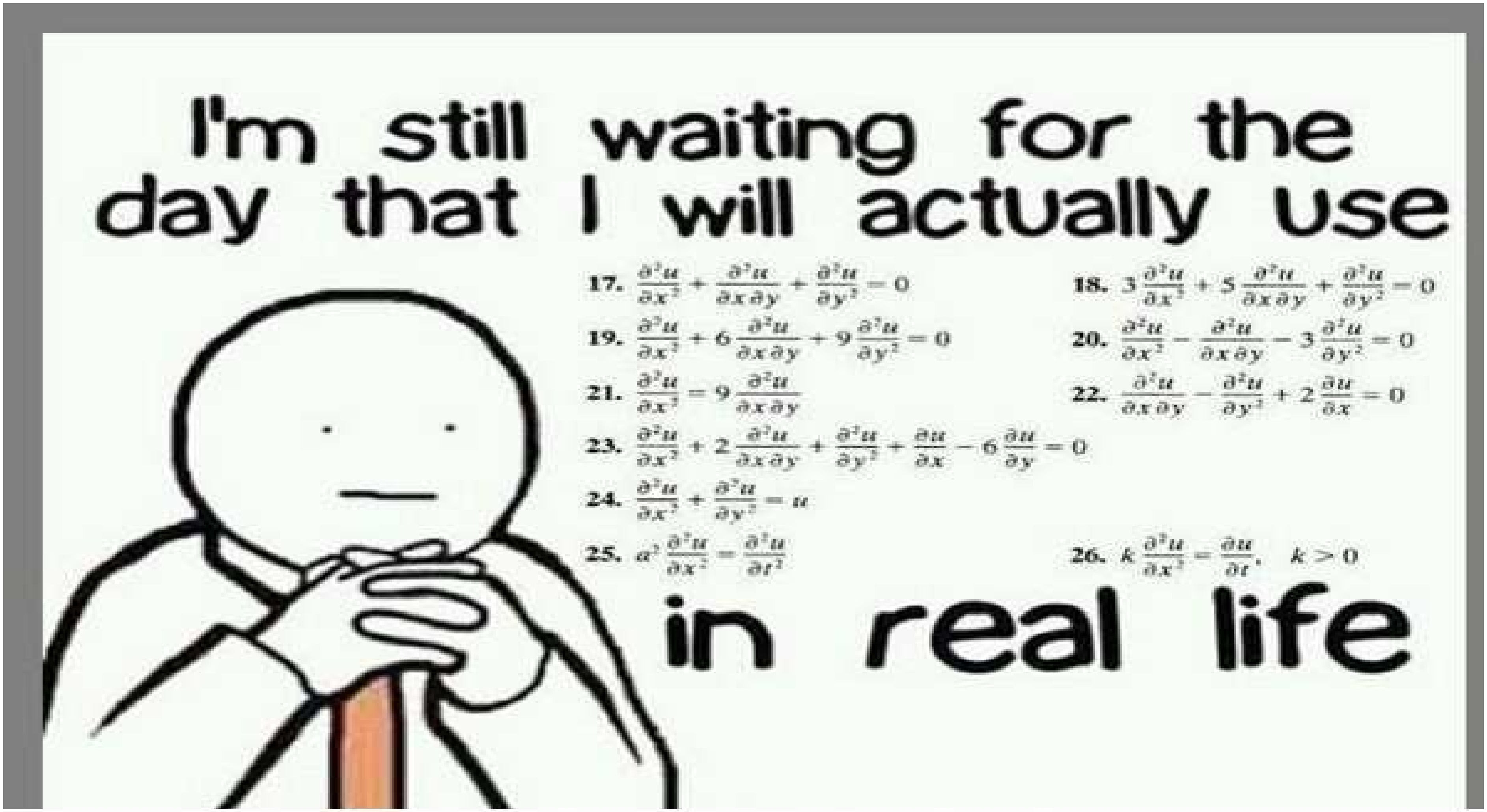
T_S= Standardization Temperature of Tape (Degree Celsius/Fahrenheit)

A= Cross-sectionType equation here. Area of Tape (FT^2)

E= Modulus of Elasticity of Tape Material(LB/FT^2)

W= Weight of Tape (LB/FT)

[6]Significance of Complex Mathematics in Real Life



Relevant Topics in Advanced Survey Mathematics

- Levelling
- Surveying Measurements and its corrections
- Tachometry
- Photogrammetry
- Complex circular curve with transition
- Vertical Curve
- Astronomy
- Other Miscellaneous Topics

Surveying Topic Illustration
Levelling Basic Concept

- To measure the level difference of Height (Delta H) between two points P and Q, two vertical measuring rods are set up at these two points and a level/total station near (preferably between) two points.
- The difference in height between points P and Q is the difference of staff reading.

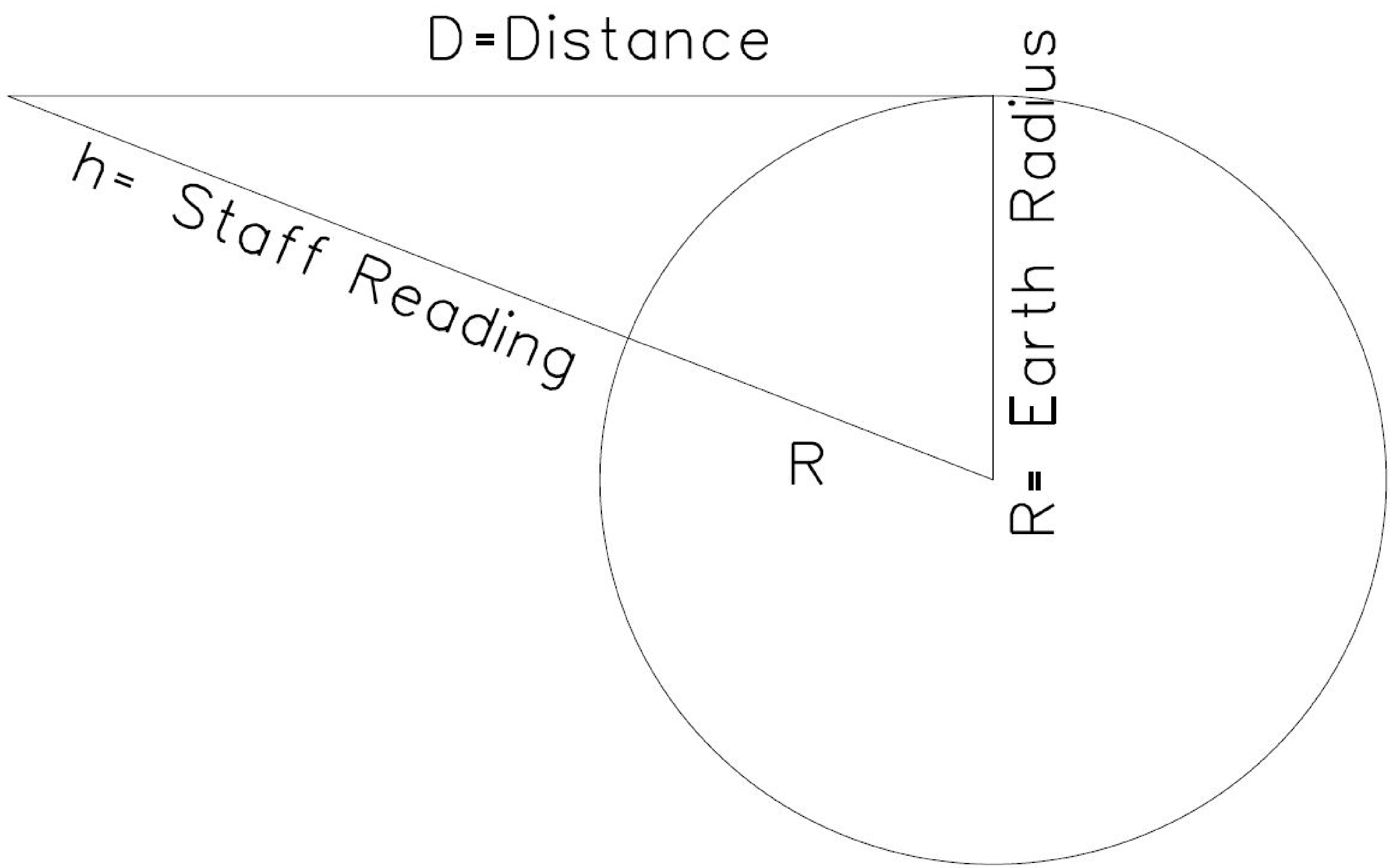
[9,10]Corrections in Levelling

Correction for Earth Curvature and Refraction:

$$Curvature\ correction = \frac{(Distance ** 2)}{2 \times Earth\, radius}$$

$$Combined\ correction = \frac{6 \times (Distance ** 2)}{7 \times Earth\, radius}$$

CURVATURE CORRECTION



Important Terminology in Levelling

- Backshot (BS)
- Foreshot (FS)
- Intermediate Shot (IS)
- Turning Point (TP)
- Bench Mark (BM)
- Instrument Height (HI)

Levelling Basic Concept

Observation Point	Instrument Location	BS	IS	FS	Level of various Points	Height of Instrument	Level of Instrument Station
BM	set up - A	X1'=HI1			BM	HI1	BM + IH1
1	set up - A		X1		BM+HI1-X1	HI1	BM + IH1
2	set up - A		X2		BM+HI1-X2	HI1	BM + IH1
3	set up - A			X3	BM+HI1-X3	HI1	BM + IH1
3	set up - B	X3'=H12			BM+HI1-X3	HI2	BM + IH1
4	set up - B		X4		BM + IH1- X3 + HI2 - X4	HI2	BM + IH1- X3 + HI2
5	set up - B		X5		BM + IH1- X3 + HI2 - X5	HI2	BM + IH1- X3 + HI2
6	set up - B			X6	BM + IH1- X3 + HI2 - X6	HI2	BM + IH1- X3 + HI2

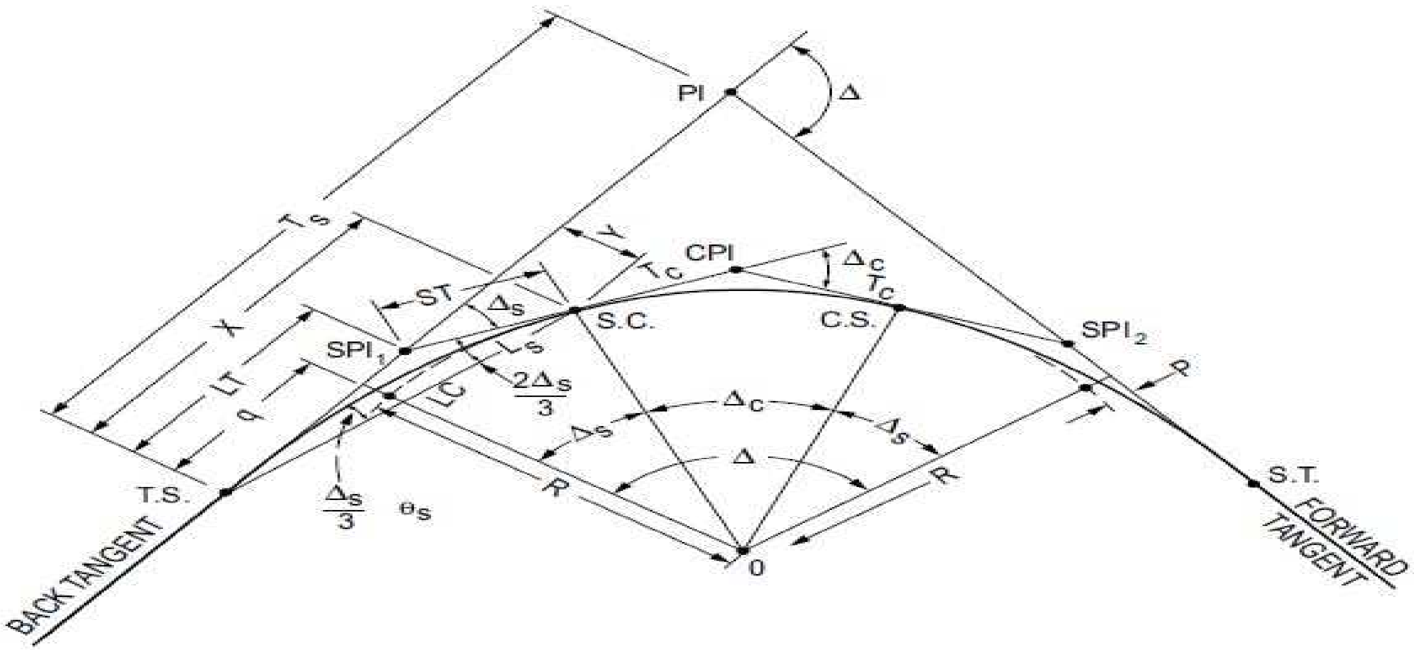
LEVEL CALCULATION CHECK:
[SUM (Foreshot) – SUM (Backshot)] = [First Bench Mark Level - Last Bench Mark Level]

Levelling Basic Concept

LEVEL CALCULATION CHECK:

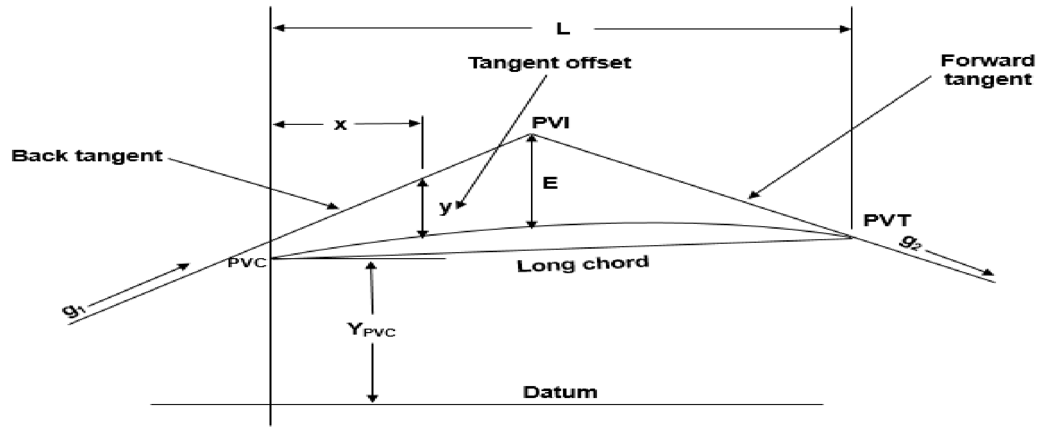
[SUM (Foreshot) – SUM (Backshot)] = [First Bench Mark Level - Last Bench Mark Level]

[2,9,10]Circular With Transition Curve



- T.S. – Tangent to spiral
- S.C.– Spiral to curve
- C.S.– Curve to spiral
- S.T. – Spiral to tangent
- T_S – Spiral tangent
- X – Distance along tangent from T.S. to point at right angle to S.C.
- Y – Right angle distance from tangent to S.C.
- LT – Long tangent (spiral)
- ST – Short tangent (spiral)
- L_S – Length of spiral (arc)
- LC – Long chord
- q – Distance along tangent to a point at right angle to ghost bc (marginally less than $L_S/2$)
- P – distance from tangent that the curve (ghost bc) has been offset
- T_C – Circular curve tangent
- CPI– Circular curve PI
- SPI – Spiral curve PI
- P.I. – Point of intersection of curve tangents
- L_C – Length of circular curve
- L – Length of curve system – T.S. to S.T.

[2,9,10]Vertical Curve



- $Tangent\ elevation = Y_{PVC} + g_1 x = Y_{PVI} + g_2(x - \frac{L}{2})$
- $Curve\ elevation = Y_{PVC} + g_1 x + ax^2 = Y_{PVC} + g_1 x + (\frac{g_2 - g_1}{2L})x^2$
- $y = ax^2$
- $a = \frac{g_2 - g_1}{2L}$
- $E = a(\frac{L}{2})^2$
- $Rate\ of\ change\ of\ grade\ (r) = \frac{g_2 - g_1}{L}$
- $Horizontal\ distance\ to\ the\ minimum\ or\ maximum\ elvation$

$$on\ the\ curve = -\frac{g_1}{2a} - \frac{g_1 L}{g_1 - g_2}$$

Legend

- L = horizontal length of curve
- PVC = point of vertical curvature
- PVI = point of vertical intersection
- PVT = point of vertical tangency
- g_1 = grade of back tangent
- g_2 = grade of forward tangent
- x = horizontal distance from either PVC or PVT to a point on the curve
- a = parabola constant
- y = tangent offset
- E = tangent offset at PVI

[2,8-10]State Plane Coordinates

The plane–rectangular coordinate system established by United States Coast and Geodetic Survey (National Ocean Survey) for use in defining position of geodetic stations in terms of plane rectangular (X & Y) coordinates. The **State Plane Coordinate** System (SPS or SPCS) is a set of 124 geographic zones or **coordinate** systems designed for specific regions of the United **States**. Each **state** contains one or more **state plane** zones, the boundaries of which usually follow county lines.

Two Map Projection systems

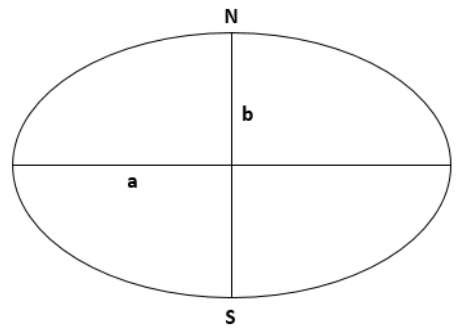
1. Lambert Conformal Conic Map Projection system
2. Transverse Mercator Map Projection system

- Scale factor $= \dfrac{\text{Grid distance}}{\text{Geodetic (ellipsoidal) distance}}$
- Elevation factor $= \dfrac{\text{Ellipsoid radius (R)}}{\text{Ellipsoid radius (R)} + \text{Orthometric height (H)} + \text{Geoid height (N)}}$
- When the level of precision is less than $\dfrac{1}{200{,}000}$
  - Ellipsoid radius (R) = 20,906,000 feet
  - Orthometric height (H) is above sea level
  - Geoid height (N) = 0

[2, 8-10] Geodesy

It is a science that treats mathematically of the figure and size of earth.

- $f = \frac{a-b}{a}$
- $e^2 = \frac{a^2-b^2}{a^2}$
- $M = \frac{a\,(1-e^2)}{(1-e^2\sin^2\phi)^{3/2}}$
- $N = \frac{a}{(1-e^2\sin^2\phi)^{1/2}}$



Legend

- f = flattening
- a = semi-major axis
- b = semi-minor axis
- e = eccentricity

- $Angular\ convergence\ of\ meridians\ (in\ radians) = \frac{d\tan\phi(1-e^2\sin^2\phi)^{1/2}}{a}$
- $Linear\ convergance\ of\ meridians = \frac{ld\tan(1-e^2\sin^2\phi)^{1/2}}{a}$
- GRS80
  - a = 6,378,173.0 meters
  - 1/f = 298.25722101
- Clark 1866
  - a = 6,378,206.4 meters
  - 1/f = 294.97869821

Legend (continued)

- M = radius in meridian
- ϕ = latitude
- N = radius in prime vertical
- d = distance along parallel at latitude ϕ
- l = length along meridians separated by d

[2,8-10]Tachometry

Surveying method used to denote the procedure for obtaining horizontal distances and differences in elevation by indirect methods, which are based on the optical geometry of the instruments employed.

- $Horizontal\ distance = K \times S \times \cos^2 \alpha$
  - K is the Stadia interval factor (in most cases, it is 100)
  - S is the rod intercept
  - α is the slope angle with respect to the horizontal
- $Vertical\ distance = K \times S \times \sin \alpha \cos \alpha$
  - K is the Stadia interval factor (in most cases, it is 100)
  - S is the rod intercept
  - α is the slope angle with respect to the horizontal

[2,8-10]Photogrammetry

The science or art of obtaining reliable measurements by photographs

- Vertical photograph
  - Scale $= \frac{ab}{AB} = \frac{f}{H-h}$
  - Relief displacement $= \frac{rh}{H}$
- Parallax equations
  - $p = x - x'$
  - $X = \frac{xB}{p}$
  - $Y = \frac{yB}{p}$
  - $h = H - \frac{fB}{p}$
  - $h_2 = h_1 + \frac{p_2 - p_1}{p_2}(H - h_1)$

Legend

- B = airbase of stereo pair
- f = focal length
- H = flying height above datum
- h = height above datum
- r = radial distance from principal point
- p = parallax measured on stereo pair
- x = coordinate measured on left photo
- x' = coordinate measured on right photo
- X = ground coordinate
- Y = ground coordinate
- y = coordinate measured on left photo

[2,8-10]Astronomy

In Surveying, azimuths or geographical positions determined by direct observation on the sun or a star (or stars)

Astronomical Triangle: The triangle on the celestial sphere formed by arcs of great circles connecting the celestial pole, the zenith and a celestial body.

The Three angles of astronomical triangle (PZS) are :

- At the Pole-The Hour Angle,
- At the Celestial body-The Parallactic Angle,
- At the Zenith-The Azimuth Angle;

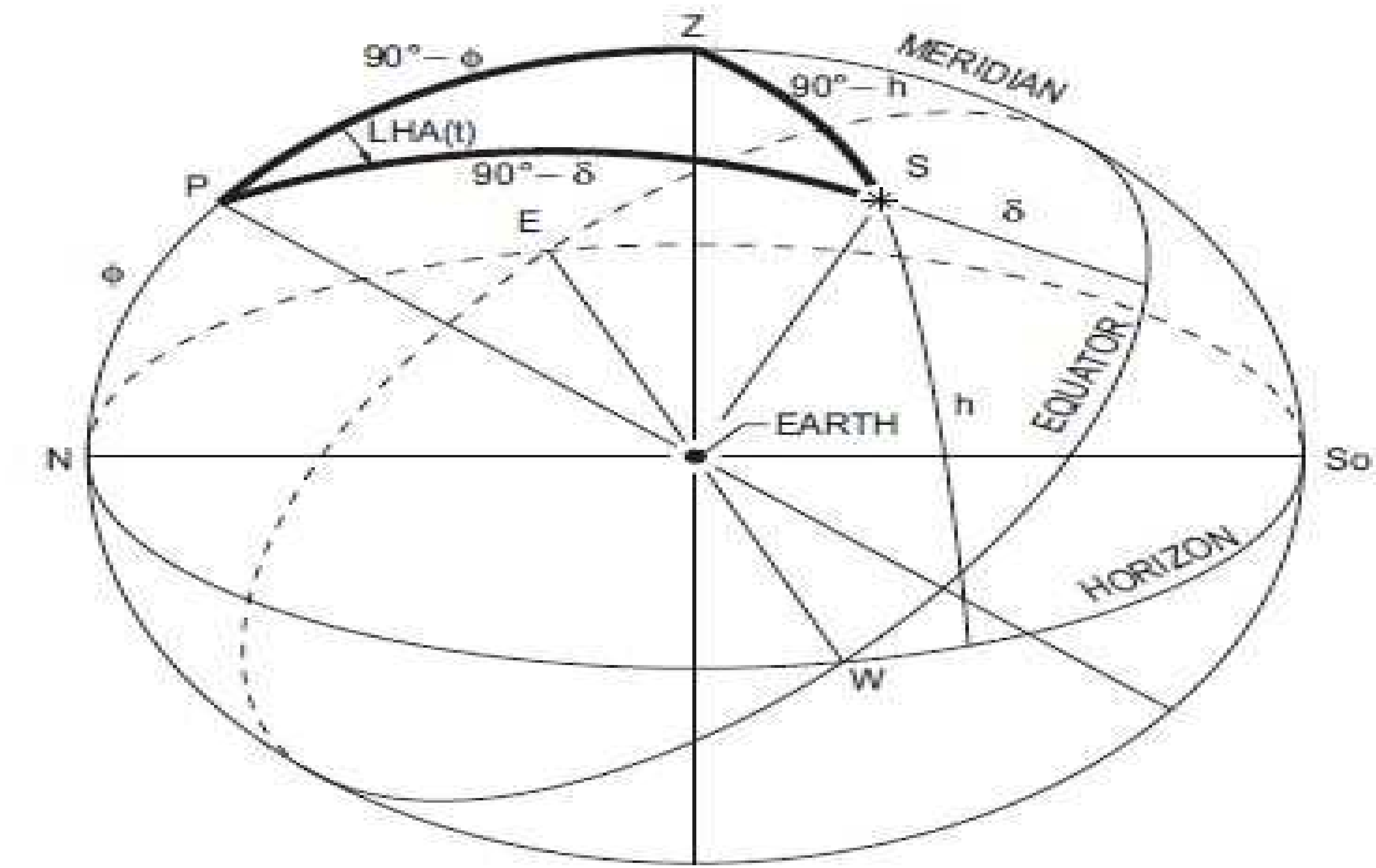
The Three Sides of Astronomical triangles (PZS) are :

- Zenith Distance- Zenith to Celestial Body,
- Polar Distance- Pole to Celestial Body,
- Co-latitude- Pole to Zenith;

[2,8-10]Basic Astronomic Terminology

- Celestial Sphere
- Zenith/Nadir
- Altitude
- Latitude/Longitude
- Declination
- Horizon
- Equator/Pole
- Northern/Southern Hemi-sphere
- and others

[2,8-10]Astronomical Triangle



[2,8-10]Astronomy

Az = Azimuth (from north) to sun/star

δ = Declination

Ø = Latitude

h = Altitude of sun/star

LHA = Local hour angle (sometimes referred to as "t" or "hour angle")

SD = Arc length of sun's semi-diameter

p = Polar distance

[18]How To Use The LEICA TPS1105 Total Station

1. Setting Up The Machine
2. Taking Measurements
3. To Machine Turn “Off”
4. To Change bet. IR and Reflector less Measurements
5. To Illuminate Display
6. To Turn on Laser Pointer
7. Setting Up the Job for Recording Data Point
8. Recording Data Points and Taking Measurements
9. How to Convert Data from the Leica Total Station to Micro station or Any CAD software

[3]The Nature of Measurement: Mistakes and Errors

Cause and Prevention of Mistakes

A mistake is a "blunder," a "goof," a "slip-up." Mistakes in measurement can be traced to carelessness, inattention, improper training, bad habits, lack of innate ability, poor judgment, adverse measuring or observing conditions, and various negative attitudes, emotions and perceptions that plague humans. Mistakes can be caused by making a decision without sufficient information or evidence. Land surveyors, for example, often make decisions about the location of property corners without sufficient evidence. Mistakes happen through transposing numbers, striking wrong keys on calculators, misreading scales, etc.

Mistakes will never be completely eliminated from measurements, but they can be reduced in most cases by developing the measurer in a way that he or she learns to be more careful, attentive, conscientious and proud of the work being done. Through proper training and development of good work habits, development and maintenance of positive attitudes, and understanding the theory and practice involved with the variable being measured, mistakes can be controlled and practically eliminated. We will always need to confront them, however, because human imperfections make them inevitable.

[3]Errors and Their Sources

Natural errors. Measurements are usually made in an environment that is essentially uncontrollable (outdoors). Effects on instruments and processes from such factors as temperature, atmospheric pressure, atmospheric refraction, humidity, solar and other heat, wind, gravity, and earth's curvature must be measured, and readings must be corrected for these variables if accurate results are to be expected.

Instrumental errors. All measurements employ instruments, from the simple plumb line to the most sophisticated electronic apparatus. Some error is always present in the measurements due to imperfection in manufacture, adjustment or basic characteristics of the instrument. Even when "in adjustment" there is error, since the adjusting process usually must involve human manipulation and judgment with no perfection in the procedure used.

Personal errors. Since humans are directly involved with all measurements, and since humans are imperfect, errors are inevitable in measurements. Automation and electronics have reduced personal errors in measurements, but not eliminated them. People still perform centering and alignment judgments, for example, even when readings are digital.

Calculation errors. Unless sufficient digits are recorded and carried through all computation steps, and unless conversion factors and constants contain sufficient digits, round-off errors occur. Significant figures in measurements directly affect the significant figures in computed results. Significant figures and round-off errors, and the broader subject of precision, are subjects for other parts in this series.

[3]Errors and Their Classification

Systematic Errors

Systematic errors are those that generally obey or follow some mathematical or physical law. The cause of such errors can usually be traced to instrument maladjustment, lack of calibration, or the environment. If they are discovered, they can be quantified through instrument tests or calibration and through understanding the various effects of nature. Since they are discoverable, they essentially can be corrected. Systematic errors occur when the cross-hairs of a surveying instrument get out of adjustment, or when a surveyor's steel tape or a tailor's cloth tape become stretched. They occur when the tires on a vehicle are not of a diameter which would yield a true distance consistent with what the odometer reads. They occur in electronic distance measurements because the measurements are affected by changes in the temperature and atmospheric pressure. They occur because of manufacturing errors in graduations of any type of scale.

Random Errors

Random errors are unavoidable. They follow random patterns, or the laws of "chance." They have unknown signs; thus, they are expressed as "plus or minus." The magnitude of such an error is unknown, but it can be estimated. These errors are caused by human and instrument imperfections as well as uncertainties in determining the effects of the environment on measurements.

[3]Mistakes Vs. Random Errors

Mistakes occur because of negligence, while random errors occur due to imperfection. Negligence can be defined as either deliberate or willful deviation from accepted practices or adopted standards (whether what is accepted or adopted is known to the individual or not), or an occurrence caused by carelessness, acting with insufficient or faulty information, etc. Mistakes are either deliberate, as in fraud or tampering with data, or they occur because someone is unwilling to study, learn and employ correct procedures, to control emotions, to keep in practice, to focus on the task at hand, or to think. By contrast, random errors occur naturally, even when the individual is attempting to perform the procedure correctly.

It is true that there is a "gray area" between random errors and mistakes. If a person is too hasty in some mechanical measuring procedure, for example, the large random errors that occur start to look a lot like mistakes. Since it is poor judgment to hurry in such an instance, the results have a lot of scatter, and some might say that they contain small mistakes. Whether they are small mistakes or large random errors does not matter as much as knowing how to deal with the data.

[4]Types of Errors

The theory of errors discussed here deals only with the accidental errors after all the known errors are eliminated and accounted for.

Errors of measurement are of three kinds:

- Mistakes,
- Systematic errors, and
- Accidental errors.

4Mistakes

Mistakes are errors that arise from inattention, inexperience, carelessness and poor judgment or confusion in the mind of the observer.

If a mistake is undetected, it produces a serious effect on the final result. Hence every value to be recorded in the field must be checked by some independent field observation.

[4]Systematic Error

A systematic error is an error that under the same conditions will always be of the same size and sign. A systematic error always follows some definite mathematical or physical law, and a correction can be determined and applied. Such errors are of constant character and are regarded as positive or negative according as they make the result too great or too small. Their effect is therefore, cumulative.

If undetected, systematic errors are very serious. Therefore:

(1) All the surveying equipments must be designed and used so that whenever possible systematic errors will be automatically eliminated and

(2) All systematic errors that cannot be surely eliminated by this means must be evaluated and their relationship to the conditions that cause them must be determined. For example, in ordinary levelling, the levelling instrument must first be adjusted so that the line of sight is as nearly horizontal as possible when bubble is centered. Also the horizontal lengths for back sight and foresight from each instrument position should be kept as nearly equal as possible. In precise levelling, everyday, the actual error of the instrument must be determined by careful peg test, the length of each sight is measured by stadia and a correction to the result is applied.

[4]Accidental Error

Accidental errors are those which remain after mistakes and systematic errors have been eliminated and are caused by a combination of reasons beyond the ability of the observer to control. They tend sometimes in one direction and some times in the other, i.e., they are equally likely to make the apparent result too large or too small.

An accidental error of a single determination is the difference between

(1) the true value of the quantity and (2) a determination that is free from mistakes and systematic errors. Accidental error represents limit of precision in the determination of a value. They obey the laws of chance and therefore, must be handled according to the mathematical laws of probability.

[4]The Law of Accidental Errors

Investigations of observations of various types show that accidental errors follow a definite law, the law of probability. This law defines the occurrence of errors and can be expressed in the form of equation which is used to compute the probable value or the probable precision of a quantity. The most important features of accidental errors which usually occur are:

- Small errors tend to be more frequent than the large ones ; that is they are the most probable.
- Positive and negative errors of the same size happen with equal frequency ; that is, they are equally probable.
- Large errors occur infrequently and are impossible.

[5]Precision and Accuracy

[

Precision is the degree of closeness or conformity of repeated measurements of the same quantity to each other

Where as

The accuracy is the degree of conformity of a measurement to its true value.

[1&2]Discounting Factors

$$\frac{P}{F} = (1+i)^{-n}$$

$$\frac{P}{A} = \frac{[-1+(1+i)^n]}{i \times (1+i)^n}$$

- Where the variables stand for the following:
  - P = Present worth
  - A = Annualized cost
  - F = Future cost
  - i = Interest rate
  - n= Number of years
- It is to be noted that other Discounting Factors e.g. F/P, A/P, F/A, A/F can be derived by manipulating above two formula.

[1&2]Discounting Factors

Factor Name	Converts	Symbol	Formula
Single Payment Compound Amount	to F given P	$(F/P, i\%, n)$	$(1+i)^n$
Single Payment Present Worth	to P given F	$(P/F, i\%, n)$	$(1+i)^{-n}$
Uniform Series Sinking Fund	to A given F	$(A/F, i\%, n)$	$\frac{i}{(1+i)^n-1}$
Capital Recovery	to A given P	$(A/P, i\%, n)$	$\frac{i(1+i)^n}{(1+i)^n-1}$
Uniform Series Compound Amount	to F given A	$(F/A, i\%, n)$	$\frac{(1+i)^n-1}{i}$
Uniform Series Present Worth	to P given A	$(P/A, i\%, n)$	$\frac{(1+i)^n-1}{i(1+i)^n}$
Uniform Gradient Present Worth	to P given G	$(P/G, i\%, n)$	$\frac{(1+i)^n-1}{i^2(1+i)^n}-\frac{n}{i(1+i)^n}$
Uniform Gradient † Future Worth	to F given G	$(F/G, i\%, n)$	$\frac{(1+i)^n-1}{i^2}-\frac{n}{i}$
Uniform Gradient Uniform Series	to A given G	$(A/G, i\%, n)$	$\frac{1}{i}-\frac{n}{(1+i)^n-1}$

How to Visualize Immediate Preliminary Solution to Complex Engineering and Survey Mathematical Problems

- Conventional Methods

- Shortcut Methods with use basic/advances engineering & Surveying concept

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Future Course Profile

1. Basic Survey Mathematics

Overview of basic undergraduate mathematics for land surveyors emphasize mathematical concepts and principles rather than computation. Emphasis will be given more on practical use of basic mathematics in surveying day to day working.

2. Advanced Survey Mathematics

Overview of advanced undergraduate mathematics for land surveyors emphasize mathematical concepts and principles rather than computation. Emphasis will be given more on practical use of advanced mathematics in surveying day to day working.

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Questions

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THANK YOU!!!