ADVANCED SURVEY MATH

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Advanced Survey Mathematics

NYSAPLS Conference
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Purpose

• Brief review of traverse calculations, coordinates and areas
• The vector
• Curves
• Intersections (line-line, line-horizontal arc, arc-arc)
• [Manual] scientific calculator use
• Basic principles used in survey
• NO discussion of pre-programmed calculators or computers
What **YOU** need to do

- Participate (we are all friends here)
- Speak up
- No such thing as “wrong” answer (in this class)
- OR… the only wrong answer is the one you didn’t ask!
- Learning is the ultimate goal
- Use your calculator in class!

Class overview

- We will travel fast
- Some basic details may be glossed over (though some will be pointed out)
- This is only a small part of the body of knowledge you should know
- Develop skills in researching, reading, trying, applying and learning while you do
Resources to keep in mind for your future professional development

- Internet
- Colleagues (bosses, peers, others in the profession)
- Societies
- High School
- Community college
- Universities
- Books: read, read, read
- Check out CST program

Calculating lat and dep

\[
lat = \cos A \cdot \text{Length}
\]
\[
dep = \sin A \cdot \text{Length}
\]

If using bearings must apply signs manually!!
Calculate coordinates

- Use initial value that is given
- Or assume value
- Know how to “translate”
- Understand process of rotation
- Know how to scale [essential for doing state plane coordinates…but other things too, such as “localization”]

Areas

- Break up into triangles and calculate—NOT!
- Use DMD method or
- Use coordinate method
Simple area calculation

<table>
<thead>
<tr>
<th>Point</th>
<th>Northing</th>
<th>Easting</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>300</td>
<td>50</td>
</tr>
<tr>
<td>C</td>
<td>450</td>
<td>225</td>
</tr>
<tr>
<td>D</td>
<td>175</td>
<td>550</td>
</tr>
</tbody>
</table>

Simple area calculation / 2

<table>
<thead>
<tr>
<th>Line</th>
<th>Latitude</th>
<th>Departure</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>300</td>
<td>50</td>
</tr>
<tr>
<td>BC</td>
<td>150</td>
<td>175</td>
</tr>
<tr>
<td>CD</td>
<td>-275</td>
<td>325</td>
</tr>
<tr>
<td>DA</td>
<td>-175</td>
<td>-550</td>
</tr>
</tbody>
</table>
Inversing

• Just the opposite of breaking down a traverse leg
• Uses latitudes and departures [may be obtained by differencing coordinates]
• Pythagorean theorem for length
• \( \tan^{-1}[\text{dep/lat}] = \text{azimuth angle (or bearing angle)} \)

\[
\text{length} = \sqrt{(\text{dep}^2 + \text{lat}^2)} \\
\text{azimuth} = \tan^{-1} \left( \frac{\text{dep}}{\text{lat}} \right)
\]

If using bearings must determine quadrants manually!!
Sideshots

• Simple application of direction calculation
• Then break down into lat and dep
• Add lat to N-coord; dep to E-coord
• No check...unless measured as a sideshot from another traverse point also

Area by coordinates

• List coord in order
• Multiply one coordinate along one axis by next coordinate on other axis
• Proceed with this method along one column of coordinates and sum all products
• Then multiply in opposite direction along other column and sum all products
• Difference of the sums is \emph{twice} the area
Horizontal curves

EQUATIONS FOR THE ELEMENTS

\[ D = 5729.5780 \text{m/R} \quad \text{or} \quad D = 5729.5780 \text{ft/R} \]

\[ \Delta L = \frac{D}{100} \text{ft (m)} \]
\[ T = R\tan(\Delta/2) \]
\[ PT = PC + L \]
\[ LC = 2R\sin(\Delta/2) \]
\[ PC = PI - T \]
\[ E = T\tan(\Delta/4) \]
\[ M = E(\cos \Delta/2) \]
\[ L = 100m(\Delta/D) \]
\[ = 100\text{ft}(\Delta/D) \]
Vertical curves

Equations for vertical curves

\[ Y = VPC_y + Bx + \frac{(A)x^2}{200L} \]

Where:
- \( Y \) = Elevation of the curve at a distance \( x \) from the VPC (ft)
- \( VPC_y \) = Elevation of the VPC (ft)
- \( B \) = Slope of the approaching roadway
- \( A \) = The change in grade between the two slopes
  (From 2\% to -2\% would be a change of -4\% or -4)
- \( L \) = Length of the curve (ft)
- \( x \) = Horizontal distance from the VPC (ft)
  (Begins with 0 at VPC to L at VPT for graphing.)
Law of Sines

\[
\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}
\]

Law of Cosines

\[
c^2 = a^2 + b^2 - 2ab \cos C
\]
Triangle solution when three sides are known

\[ s = \frac{a + b + c}{2} \]

\[ \sin \frac{1}{2} A \sqrt{\frac{(s - b)(s - c)}{bc}} \]

Bearing-bearing intersections

Given: A, B, directions of AX and BX.

Find coordinates of X.

Solution:

1. Calculate length and direction of AB.

2. Calculate angles A and B by subtracting azimuth of AB from azimuth of AX and subtracting azimuth of BX from azimuth of BA. Angle \( x = 180 - A - B \).

3. Use law of sines to solve for either length AX or BX
Bearing-bearing solution

\[ \frac{AB}{\sin X} = \frac{BX}{\sin A} \]

\[ BX = \frac{AB}{\sin X} \cdot \sin A \]

Example for bearing-bearing

- A at N100, E200
- B at N400, E 600
- Az. AX = 135°
- Az. BX = 215°

- Calculate AB
  - Length = 500.00
  - Az. = 53°07′48″
Example for bearing-bearing

• Calculate X by subtracting azimuths
  \[ \frac{AB}{\sin X} = \frac{BX}{\sin A} \]
  \[ BX = \frac{AB}{\sin X} \sin A \]
  \[ AB = BX \sin X \sin A \]

  \[ AX \]
  \[ X \]
  \[ BX \]

\[ BX = \frac{500}{\sin 80} \sin 81^\circ 52' 12'' = 502.611 \]

• Calculate angle at A
  \[ 135^\circ 00' 00'' - 53^\circ 07' 48'' = 81^\circ 52' 12'' \]

Distance-distance intersections

Given: A, B, distances AX and BX.

Location of X?

Solution:
1. Calculate AB
2. With three sides known, use law of cosines to solve for angle A (or B)
3. By inspection determine which solution is applicable.
Distance-distance solution

\[ BX^2 = AX^2 + AB^2 - 2(AX)(AB)\cos A \]

\[ \cos A = -\frac{BX^2 - AX^2 - AB^2}{2 \cdot AX \cdot AB} \]

Example for distance-distance

- A at N100, E200
- B at N400, E 600
- Distance AX = 600
- Distance BX = 700

- Calculate AB
  - Length = 500.00
  - Az. = 53°07’48’’
Example for distance-distance

\[ 700^2 = 600^2 + 500^2 - 2(600)(700)\cos A \]
\[ \cos A = -\frac{700^2 - 600^2 - 500^2}{2 \cdot 600 \cdot 500} \]
\[ A = 78^\circ 27'47" \]

\[ BX^2 = AX^2 + AB^2 - 2(AX)(AB)\cos A \]
\[ \cos A = -\frac{BX^2 - AX^2 - AB^2}{2 \cdot AX \cdot AB} \]

Bearing-distance intersections

Given: A, B, azimuth of AX and length BX.
Where is X?

Solution:
1. Solve for angle A
2. Use law of sines to solve for angle X
3. Two solutions are possible so determine correct solution by inspection
Example for bearing-distance

- A at N100, E200
- B at N400, E 600
- Az. AX = 135°
- Distance BX = 600

- Calculate AB
  - Length = 500.00
  - Az. = 53°07'48"
Example for bearing-bearing

Calculate angle at A

\[ 135°00'00" - 53°07'48" = 81°52'12" \]

\[ \sin X = \frac{500}{600} \sin 81°52'12" = 0.824958 \]

\[ X = 55°35'03" \]

Perpendicular offset

Given: coordinates of A, B, (or A and azimuth of AB) and P.

What is perpendicular offset distance from P to AB?

Solution:

1. Calculate azimuth of AB (if needed)
2. Calculate azimuth and length of line AP
3. Calculate \( \alpha \)
4. Now solve right triangle for distance PX
Example for perpendicular offset

- P at N510, E1120
- A at N415, E865
- B at N670, E1550

- Calculate AP from coordinates
  - $\Delta E = 255$
  - $\Delta N = 95$
  - Length = 272.10
  - Az. = $69^\circ34'02''$

Example for perpendicular offset / 2

- Calculate AB from coordinates
  - $\Delta E = 685$
  - $\Delta N = 255$
  - Length = 730.925
  - Az. = $69^\circ34'54''$

Note sketch is revised!
Example for perpendicular offset / 3

• $\alpha = 0^\circ 00' 52''$
• $PX = AP \sin \alpha = 0.068$

Questions?
About the seminar presenter

Joseph V. R. Paiva is a consultant in the field of geomatics and general business, particularly to international developers, manufacturers and distributors of instrumentation and other geomatics tools. Prior to this he was managing director of Spatial Data Research, Inc., a GIS data collection, compilation and software development company. Immediately prior, he was at Trimble Navigation Ltd. His roles included senior scientist and technical advisor for Land Survey research & development, VP of the Land Survey group and director of business development for the Engineering and Construction Division. Previous to that, Paiva was vice president and a founder of Sokkia Technology, Inc., guiding development of GPS- and software-based products for surveying, mapping, measurement and positioning. He has also held senior technical management positions in The Lietz Co. and Sokkia Co. Ltd. Dr. Paiva was assistant professor of civil engineering at the University of Missouri-Columbia, and a partner in a surveying/civil engineering consulting firm. Dr. Paiva’s special areas of interest include interface development and design for software and hardware, errors analysis and survey instrumentation of all types. His key contributions in the development field are: design of software flow for the SDR2 and SDR33 Electronic Field Books and the software interface for the Trimble TTS500 total station. He is a member of several professional societies, has presented numerous papers and writes columns for P.O.B. magazine and The Empire State Surveyor. In May 2006 he completed an approximately three year stint as a columnist for Civil Engineering News. He is a Registered Professional Engineer, Registered Land Surveyor, is an ACSM representative to ABET, serving as a program evaluator and team chair on accreditation visits to surveying programs, and has more than 30 years experience working in civil engineering, surveying and mapping. Dr. Paiva is currently working on book to be published soon on the subject of total stations; it is intended to be a practitioner’s guide to help in the understanding, operation, testing and adjustment of these ubiquitous instruments.

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