Optimal design of intermittently operated subsonic-supersonic ejectors

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The optimal design for maximum run duration of intermittently operated subsonic-supersonic air-air ejector pumps of fixed geometry was investigated using an inviscid one-dimensional model of the mixing chamber. The study was confined to the case of matched static pressures of the primary and secondary air streams. An empirical correction factor was applied to the ID results in order to take approximate account of the two-dimensional velocity distributions at the inlet and outlet of the mixing chamber and of viscous shear forces. Optimisation of both the primary and secondary Mach numbers was found to yield significantly longer run durations than when the secondary is flow is sonic and just the primary Mach number is optimised. The most pronounced advantages of optimising both the primary and secondary Mach numbers were obtained in the range of high ejector performance. The main advantage is that an optimised ejector is physically smaller, and requires a smaller storage vessel than one that is not optimised.

NOMENCLATURE

A area
c a sub-expression to simplify a long formula
D diameter
L length
m mass of stored air
M Mach number
P pressure
$P_a$ stagnation pressure
R gas constant
$R_e$ empirical correction factor of static pressure ratio $P_m/P_s$
$T$ temperature
$T_0$ stagnation temperature
t run duration
V storage capacity
W mass flow rate
$\gamma$ specific heat ratio
$\tau$ dimensionless coefficient of run duration

Subscripts

a ambient conditions
b outlet conditions of subsonic diffusor
c corrected conditions after mixing
f final conditions in storage vessel
i initial conditions in storage vessel
m mixed
p primary
s secondary
spec specified

INTRODUCTION

Supersonic ejector pumps have been intensively studied for many years by many researchers, for example, Dutton et al., who conducted a thorough theoretical and experimental investigation of a range of supersonic-supersonic air-air ejectors. Optimal supersonic ejector designs, however, seem not to have received that attention. The need for a procedure by which ejector designs can be optimised formally arose from a number of practical ejector applications. These include, amongst others, the ejector pump used in the hot air wind tunnel described by Hattingh et al., and the one used in the air bag system designed for the cushioning of the ground impact of a parachute’s pay load mentioned by Pienaarl. The present investigation was focused on the optimal design of subsonic-supersonic air-air ejectors with matched static pressures of the primary and secondary air streams. The theoretical model used, following Dutton et al., was based on one-dimensional gas dynamic theory. This is admittedly a very simple theory for a complex phenomenon such as the supersonic mixing of two air streams, but, judging by the experimental results from these researchers, it is accurate enough for design purposes. The alternative is to take viscous effects into account in a two-dimensional theory that can trap the shock patterns in the mixing chamber. This could, however, be prohibitively time-consuming in an analysis that is aimed at revealing basic optimisation trends for design purposes, instead of predicting the flow field and shock patterns precisely.

An optimal design procedure was formulated by Dutton & Carroll for ejector applications in which the primary mass flow rate, $W_p$, or primary stagnation pressure, $P_{op}$, needs to be minimised. The intermittently operated ejector, however, must be optimised for maximum run duration, which is not necessarily obtained with either minimum $W_p$ or minimum $P_{op}$. The purpose of the investigation reported here was to show that optimal configurations exist for this case too, to formulate and implement an op-

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timisation procedure and, finally, to show that significant reductions in run duration are suffered when the selected values of the ejector design parameters \( M_p \) and \( M_s \) are far from optimal. The optimisation procedure essentially involves a numerical search for the particular combination of the values of \( M_p \) and \( M_s \) that yields the maximum run duration.

**Design features and operating principle**

The main components of a subsonic-supersonic ejector pump are the plenum chamber, the primary nozzle, the secondary nozzle, the mixing chamber and the subsonic diffusor. A supersonic jet of high stagnation pressure enters the mixing chamber through the primary nozzle, inducing a secondary fluid of low stagnation pressure to flow at a high subsonic or even sonic Mach number into the mixing chamber. The velocity at the exit plane of the mixing chamber of a properly designed ejector is usually so high that a subsonic diffusor for the recovery of dynamic pressure is indispensable. An example with an annular primary nozzle and a central secondary nozzle is schematically illustrated with its supply system in Fig. 1. Its operating principle is the transfer of momentum from the primary (pumping) fluid to the secondary (pumped) fluid by the action of viscous shear. This induces the secondary fluid to flow from the region of low stagnation pressure in the exit plane of the secondary nozzle into the region of intermediate stagnation pressure in the exit plane of the mixing chamber.

![Fig. 1. Schematic representation of ejector system](image)

**Intermittent operation**

The storage vessel of the ejector system in Fig. 1 is equipped with a capacitive (passive) heat source that limits the decrease in stagnation temperature during discharge. Not much heat transfer from the walls of the storage vessel and supply line occurs during discharge, provided that the \( L/D \) ratios of the vessel and supply line are not very large. The stagnation temperature upstream of the heat source will then decrease approximately as if the expansion process were isentropic. This decrease, however, is largely compensated for by the passive heat source at the downstream end of the storage vessel, which heats up the passing air stream again, providing an air supply of nearly constant stagnation temperature, \( T_{0p} \). The regulating valve maintains a constant primary stagnation pressure, \( P_{0p} \), thus compensating for the decreasing vessel pressure, and along with the nearly constant \( P_{0p} \), this yields a nearly constant \( W_p \).

**Theory**

The viscous shear forces between the two jets entering the mixing chamber (and the resulting momentum transfer) arise from the large velocity gradients of the two-dimensional velocity distribution in the entrance region of the mixing region. However, fairly accurate predictions of ejector performance can be made by a one-dimensional flow model in which the principles of conservation of mass, momentum, and energy are applied to the constant area mixing chamber. The main assumptions of the one-dimensional flow model of compressible mixing at constant area are that:

a) the velocity distributions in the exit planes of the primary and secondary nozzles, the mixing chamber and the subsonic diffusor are uniform,

b) deviations from these uniform velocity profiles, along with viscous wall shear forces in the mixing chamber can be accounted for to fair approximation, as was shown by Dutton et al.\(^1\) by applying an empirical correction factor to the pressure recovery ratio \( P_{m}/P_s \) and that
c) the flow through the mixing chamber and subsonic diffusor can be regarded as adiabatic by virtue of insufficient time for significant heat transfer to or from the duct walls.

The stagnation pressure loss in the subsonic diffusor downstream of the mixing chamber is assumed to be negligible compared to the shock losses in the mixing chamber.

**Supply system**

Assuming that the expansion in the storage vessel upstream of the heat source is adiabatic and isentropic, and that \( W_p \) is constant, it follows, as shown in Appendix A, that the maximum available run duration obtained with an initial mass \( m_i \) of stored air is:

\[
t = \frac{m_i}{W_p} \left[ 1 - \left( \frac{P_s}{P_i} \right)^{1/\gamma} \right]
\]

(1)

The run duration is non-dimensionalised with respect to the time \( t_{\text{max}} = m_i/W_s \) that it would have taken to evacuate the storage vessel completely, had its rate of mass discharge been equal to the secondary mass flow rate, \( W_s \). This is done merely so that the storage capacity \( V \) need not be known \textit{a priori}. The resulting \textit{coefficient of run duration}:

\[
\tau = \left( \frac{W_p}{W_s} \right)^{-1} \left[ 1 - \left( \frac{P_s}{P_i} \right)^{1/\gamma} \right]
\]

(2)

is a suitable \textit{dimensionless} object function for the optimisation procedure, because its maximum implies that the \textit{dimensional} run duration is a maximum for given secondary flow conditions. The expansion ratio \( P_s/P_i \) in the storage vessel (and therefore the available run duration for given
$W_p/W_s$ is clearly related to the initial pressure ratio $P_i/P_a$ of the supply system, the pressure loss ratio $P_f/P_{op}$ of the supply line and the required primary stagnation pressure ratio, $P_{op}/P_a$, by:

$$\frac{P_f}{P_i} = \frac{P_f}{P_{op}} \left( \frac{P_i}{P_a} \right)^{-1}$$

Note that $P_f/P_{op}$ is a design constant of the supply line, one that is determined uniquely by the layout and dimensions of the supply line, the regulating valve, the plenum chamber and the primary nozzle, whereas the ratio $P_i/P_a$ is a design constant of the storage vessel and its compressor system. The reason for constant $P_f/P_{op}$ is threefold. Firstly, according to Colebrook,\(^6\) the influence of varying Reynolds number on pressure losses in pipes is negligible in the hydraulically rough region of the Moody chart. Secondly, a design study of Pienaar\(^7\) showed that the flow in typical wind tunnel supply lines falls in this region. Thirdly, the regulating valve can be assumed to be choked when fully open in the last instant of a run, which is not an unduly restrictive assumption. A value of $P_f/P_{op} = 2$ has been found by Pienaar\(^7\) to make adequate provision for the losses in supply lines of reasonable $L/D$ ratio if its regulating valve is properly selected.

The value of $P_{op}/P_a$, on the other hand, and the value of $W_p/W_s$, must be determined by solving the gas dynamic equations that model the ejector. The solution procedure is based on the theoretical formulations that follow.

**Inviscid compressible mixing at constant area**

The one-dimensional flow model of inviscid, compressible mixing at constant area as expounded by Dutton et al.\(^1\) can be reduced, for the case of identical primary and secondary fluids considered here, to the form:

$$\frac{W_p}{W_s} = \frac{P_p A_p M_p}{P_s A_s M_s} \left( \frac{1 + \frac{\gamma - 1}{2} M_p^2}{1 + \frac{\gamma - 1}{2} M_s^2} \right)^{1/2}$$

and

$$M_m = \left[ \frac{(1 - 2\gamma c_m + \sqrt{1 - 2\gamma c_m - 2c_m})}{(2\gamma c_m - \gamma + 1)} \right]^{1/2}$$

in which

$$c_m = \left( \frac{1 + \frac{\gamma - 1}{2} M_p^2}{1 + \frac{\gamma - 1}{2} M_s^2} \right)^{2}$$

The negative inner root in equation (5) yields a real subsonic solution for $M_m$, whereas the positive root yields a real supersonic solution, provided that $1 - 2\gamma c_m - 2c_m$ is positive.

**Primary to secondary static pressure ratio**

Preliminary results of the present investigation have shown that the longest run duration is obtained with ejectors designed for the highest possible primary to secondary static pressure ratio, $P_p/P_s$. It was decided, however, to restrict the optimisation to the case of $P_p/P_s = 1$, partly because the one-dimensional model of the subsonic-supersonic ejector can reasonably be expected to be fairly reliable in this case, but also because operation near this matched static pressure point was shown by Dutton et al.\(^1\) to improve ejector pressure recovery. Another motivation is that this yields a conservative design, in the sense that, according to experimental results of these authors, even better performance can be expected if one were to increase the primary stagnation pressure of a given ejector slightly in order to obtain a value of $P_p/P_s$ slightly higher than unity. This pressure ratio, however, must not become so high that a significant risk of aerodynamic choking in the mixing chamber or of a reverse flow region in the secondary stream arises.

**Recovery ratio and run duration**

The wall shear force in the mixing chamber and non-uniformities of inlet and outlet velocity distributions are approximately accounted for by an empirical correction factor $R_E = 0.8$, based on the experimental results of Dutton et al.\(^1\). This factor represents the ratio of measured $P_m/P_s$ to its theoretically predicted value. The experiments of these authors were conducted, admittedly, with supersonic-supersonic ejectors, but it does not seem unreasonable to use the same value of $R_E$ in a design analysis of supersonic-to-subsonic ejectors too. One should, however, perform an experimental check of its applicability to supersonic-to-subsonic ejectors too.

Following Dutton & Carroll\(^4,8\) the corrected Mach number after mixing is found from:

$$M_c = \left[ \frac{(\sqrt{1 + 2\gamma c_c} - 2c_c - 1)}{(\gamma - 1)} \right]^{1/2}$$

in which:

$$c_c = M_m^2 \left( 1 + \frac{\gamma - 1}{2} M_m^2 \right) / R_E^2$$

The corrected static recompression ratio of the mixing chamber then follows directly from:

$$\frac{P_c}{P_s} = \left( \frac{1 + \frac{W_p}{W_s}}{1 + \frac{A_p}{A_s}} \right)^{-1} \times \frac{M_s}{M_c} \left( 1 + \frac{\gamma - 1}{2} M_c^2 \right) / \left( 1 + \frac{\gamma - 1}{2} M_c^2 \right)^2$$

For the purpose of the current investigation, however, the stagnation recompression ratio:

$$\frac{P_b}{P_{b0}} \approx \frac{P_{bc}}{P_{b0}} = \frac{P_c}{P_s} \left( 1 + \frac{\gamma - 1}{2} M_s^2 \right)^{\frac{\gamma}{\gamma - 1}}$$

is regarded as the more appropriate measure of ejector performance (or load). It is, after all, a low secondary stagnation pressure that really loads an ejector, not a low static.
pressure, and the ratio \( P_c/P_s \) implicitly disregards the substantial contribution of a high secondary dynamic pressure to the overall recovery ratio, whereas the alternative ratio \( P_c/P_0s \) would disregard the important advantage of subsonic diffusion after mixing.

The required primary stagnation pressure ratio \( P_{op}/P_a \) in equation (3) may now be expressed in terms of the recovery ratio by:

\[
\frac{P_{op}}{P_a} = \frac{P_{op}}{P_{os}} \left( \frac{P_o}{P_{os}} \right)^{-1} \frac{P_o}{P_a}
\]

(11)

in which

\[
\frac{P_{op}}{P_{os}} = \frac{P_{op}}{P_s} \left( \frac{1 + \frac{1}{2} M_p^2}{1 + \frac{1}{2} M_s^2} \right)
\]

(12)

Therefore the coefficient of run duration can finally be computed using equations (2) to (12).

**Computational procedures**

The preceding theoretical formulation can be used in various computational procedures, depending on the problem at hand. An *ejector performance calculation*, firstly, involves the direct computation of the recompression ratio, \( P_{oc}/P_{os} \), once the parameters \( \gamma, P_1/P_{os}, P_f/P_{op}, P_b/P_{as}, R_E \), the operating point \( P_a/P_1 = 1 \), suitable primary and secondary Mach numbers, \( M_p \) and \( M_s \), and the primary to secondary ratio of nozzle exit plane areas, \( A_p/A_s \), have been specified. Such a calculation can be performed explicitly by solving equations (4), (6), (5), (8), (7), (9), (10), (12), and (11) in this order, after which \( \tau \) follows from equations (3) and (2).

An *ejector design calculation*, secondly, requires the computation of \( M_\infty \) that, for given values of \( M_s \) and \( A_p/A_s \), will yield some given recompression ratio \( P_{oc}/P_{os} \). This is accomplished by estimating the value of \( M_\infty \) initially, which will generally not yield the required \( P_{oc}/P_{os} \) directly, and then finding the actually required value of \( M_\infty \) iteratively, by repeated performance calculations, each with an improved estimation of \( M_\infty \). (Alternatively one could select \( M_p \) and then compute the required value of \( A_p/A_s \) iteratively.) This effectively constitutes the iterative zeroing of a scalar function \( P_{oc}/P_{os} - (P_{oc}/P_{os})_{spec} = f_1(M_p) \).

Thirdly, a *constrained optimal design calculation*, i.e. constrained to a preselected value of \( M_s \), involves not only the computation of the value of \( M_p \) that would yield a specified value of \( P_{oc}/P_{os} \) for given \( M_s \) in an inner iteration loop, but also the computation, in an outer iteration loop, of the value of \( A_p/A_s \) with which this is achieved optimally, in other words with the maximum possible run duration. This effectively constitutes the maximization of a function of the form \( \tau = f_2(A_p/A_s) \), a function in the computation of which the zero point of said function \( P_{oc}/P_{os} - (P_{oc}/P_{os})_{spec} = f_1(M_p) \) must clearly be found at each iteration step.

Finally, an *unconstrained optimal design calculation*, i.e. one in which \( M_s \) is optimised too (other design considerations permitting), involves the same inner iteration loop as in the previous procedure, but now it is the combination of \( A_p/A_s \) and \( M_\infty \) with which this is achieved optimally that must be found in the outer iteration loop. This effectively constitutes the maximization of a function of the form \( \tau = f_3(M_\infty, M_p) \), one that also requires the function \( P_{oc}/P_{os} - (P_{oc}/P_{os})_{spec} = f_1(M_p) \) to be zeroed at each iteration step.

If one experiments numerically with a representative number of combinations of ejector design parameters, then an ejector design that is probably not very far from optimal can be obtained, as shown by Hattingh et al.² The present investigation, however, supplies a formal procedure for obtaining an optimised design.

**Discussion of results**

These numerical procedures were applied to an air-air ejector with \( P_b/P_1 = 1, P_c/P_{oc} \approx 1, P_f/P_a = 1 \) and \( R_E = 0.8 \), and of which the air supply system is specified by \( \gamma = 1.4, P_1/P_\infty = 40, P_f/P_\infty = 2 \). Selected optimisation results for this case are plotted against \( P_{oc}/P_a \) in Figs. 2-7 and discussed below.

**Existence of an optimal secondary Mach number**

In the limit of \( M_s = 0 \) only an infinitely large \( A_s \) can accommodate the secondary mass flow rate required in a given application. However, a finite ratio of \( A_p/A_s \) is needed to ensure that the required \( P_{oc}/P_{os} \) is obtained, which implies that an infinitely large \( A_p \) is needed. This, in turn, will require an infinitely large \( W_p \) and, consequently, zero run duration will be obtained with a vessel of finite storage capacity. Consideration of the limit of infinitely large \( M_s \) (allowing for supersonic \( M_s \) in this argument), along with the implied (diverging) secondary nozzle shows that it too would require an infinitely large \( A_s \). This line of thought again leads to the conclusion that zero run duration will be available. It is therefore not surprising that an optimal \( M_s \) must exist, other design parameters permitting.

The optimal value of \( M_s \) is seen in Fig. 2 to be high subsonic throughout the selected range of \( P_{oc}/P_{os} \) which is in qualitative agreement with the preceding inductive argument against very low subsonic or very high supersonic \( M_s \). The reason why the optimum values of \( M_s \) are relatively close to unity over such a wide range of \( P_{oc}/P_{os} \) as shown in Fig. 2, however, is not so clear. One may be surprised, at first, by the implication that a subsonic diffuser upstream of the secondary inlet of the mixing chamber is undesirable, even though an advantage is apparently to be gained from the higher \( P_\infty \) afforded by subsonic pressure recovery. The explanation for this, however, is clearly given by the argument against very low \( M_s \) presented here.

**Existence of an optimal primary Mach number**

For a given recovery ratio \( P_{oc}/P_{os} \), the required value of \( A_p/A_s \) increases as the selected value of \( M_\infty \) decreases, because low \( M_\infty \) is evidently associated with low pumping capacity, and high \( A_p/A_s \) with high pumping capacity. A lower limit on \( M_\infty \), for which only an infinitely large value of \( A_p/A_s \) will yield the required recovery ratio, can be shown
to exist, again implying an infinitely large $W_p$ and zero run duration. In addition, the starting requirement of the supersonic primary nozzle places an upper limit on $M_p$, one at which the required primary stagnation pressure for starting the nozzle will be equal to the storage pressure less the minimum supply line losses (i.e. with fully opened regulating valve), again reducing the available run duration to zero. Provided that this upper limit of $M_p$ exceeds its lower limit (which it will *not* if the required recovery ratio is higher than allowable by the limitation of the available maximum storage pressure), then an intermediate, optimal value of $M_p$ will exist at which the maximum run duration is available.

The optimal value of $M_p$ is seen in Fig. 3 to be in the intermediate supersonic range which agrees with the qualitative argument above. The largest percentage difference, roughly 38%, between the optimal values of $A_p/A_s$ for $M_s = 1$ and $M_s = M_{s, opt}$ shown in Fig. 4 is significantly larger than the largest percentage difference, roughly 20%, between the optimal values of $P_{0p}/P_a$ in Fig. 5. The largest percentage difference between the coefficients of run duration in Fig. 6 is roughly 20%. The largest percentage difference in primary mass flow requirement reflected by $W_p/W_s$ in Fig. 7, however, is only 9%, which is not as much as one would expect at first, judging by the differences of 38% and 20% in $A_p/A_s$ in Fig. 4 and $P_{0p}/P_a$ in Fig. 6, respectively. Careful scrutiny of the mass flow ratio $W_p/W_s$ expressed in terms of stagnation pressure ratio
The optimisation of the intermittently operated ejector involves the maximisation of its coefficient of run duration, which is jointly determined by its mass flow ratio and the relevant pressure ratios in equation (2). It should be designed for and operated at the highest expansion ratio, \( P_p/P_a \), at which the required upstream conditions of the secondary flow can prevail. The Mach numbers \( M_e \) in the exit plane of the mixing chambers of optimally designed ejectors are significantly lower than the optimal values of \( M_s \), but are still high subsonic, which emphasises the importance of a properly designed subsonic diffuser downstream of the mixing chamber. Subsonic diffusion to low Mach numbers in the secondary inlet of the mixing chamber, on the other hand, reduces the maximum obtainable run durations. Ideally both the primary and secondary nozzles should be designed for the optimal \( M_p \) and \( M_s \), at which the maximum run duration for the required recovery ratio is obtained, but if other design considerations preclude the free choice of \( M_s \), then \( M_p \) can and should still be optimised for maximum run duration. One should note, however, that even though an ejector with a sonic secondary nozzle (with \( M_s = 1 \)) suffers a significant reduction in run duration compared to the maximum obtainable with an optimum \( M_s \), especially in the range of high recompression ratios, it will be close to optimal in the range of lower recompression ratios.

The results presented in Figs. 2–7 can be regarded as ejector design curves for the particular supply system considered here. This means that, if the required ejector performance \( P_{bc}/P_{ba} \) in a specific application is known, one can use these curves to find not only the values of \( M_s \) and \( M_p \) by which this performance would be optimally achieved, but also the corresponding values of \( A_p/A_s \), \( P_{bp}/P_a \) and \( W_p/W_s \). The values of \( M_s \), \( M_p \) and \( A_p/A_s \) allow one to design the primary and secondary ejector nozzles. The values of \( P_{bp}/P_a \) and \( W_p/W_s \), on the other hand, along with the specified pressure ratios \( P_1/P_2 \) and \( P_1/P_{bp} \) of a given compressor and supply line, respectively, allow one to find the required storage capacity for a given run duration quite easily by using equations (1) and (2). The higher the required ejector performance, the larger the required ejector and the larger the required capacity of the storage vessel for given run duration will be.

### References


### Figures

Fig. 6. Increase in run time due to optimisation of secondary Mach number.

Fig. 7. Decrease in primary mass flow rate \( W_p \) due to optimisation of secondary Mach number.
APPENDIX A

Conservation of mass in an air storage vessel from which the air flows at a constant mass flow rate, \( W_p \), requires that the difference between the initial mass \( m_i \) and the final mass \( m_f \) of air in the vessel is related to \( W_p \) by

\[
m_i - m_f = W_p t
\]

Solving for \( t \) one obtains

\[
t = \frac{m_i}{W_p} \left( 1 - \frac{m_f}{m_i} \right)
\]

from which it follows for a vessel of capacity \( V \), on substitution of \( m_i = \rho_i V \) and \( m_f = \rho_f V \), that the run time is related to the ratio of final to initial air density in the vessel by

\[
t = \frac{m_i}{W_p} \left( 1 - \frac{\rho_f}{\rho_i} \right)
\]

The expansion of the air in the vessel is approximately adiabatic if the heat transfer from the vessel wall to the expanding air is negligible. The process is approximately isentropic too, because the exceedingly low flow velocity inside the vessel implies that the friction losses due to the shear stress at the vessel wall are negligible. The density \( \rho \) and pressure \( P \) of a perfect gas in an adiabatic and isentropic process, furthermore, are related to one another by \( \rho^{-\gamma} P = \text{constant} \), a well-known result from basic thermodynamic theory. It follows from this relationship that \( \rho_f^{-\gamma} P_f = \rho_i^{-\gamma} P_i \), upon rearrangement of which one obtains

\[
\frac{\rho_f}{\rho_i} = \left( \frac{P_f}{P_i} \right)^{1/\gamma}
\]

Substitution from equation (A.4) into equation (A.3) finally yields the result that the run duration \( t \) is related to the ratio of final pressure \( P_f \) to initial pressure \( P_i \) by

\[
t = \frac{m_i}{W_p} \left[ 1 - \left( \frac{P_f}{P_i} \right)^{1/\gamma} \right]
\]