New Methods in Efficient Post-Tensioned Slab Design Using Topology Optimization

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ABSTRACT

Post-tensioned (PT) flat-plate gravity framing systems are highly efficient and reduce embodied carbon when compared to conventional reinforced concrete framing systems. Efficiency is especially apparent in multi-span applications with regular orthogonal support arrangements. Even though PT flat-plate gravity framing systems are less efficient in single-span or irregular support applications, they are still useful in reducing slab thickness, improving construction efficiency, and reducing seismic mass.

A novel approach to determining PT tendon arrangements has been applied to several buildings informed by topology optimization results. Topology optimization is an optimization method which determines optimal load paths in a finite element continuum. Thus, by orienting PT tendons along the optimal load paths suggested by topology optimization, it has been shown that 25% or more of PT quantities can be reduced while maintaining the same mild steel reinforcement. Many of the observed arrangements do not follow traditional uniform/banded arrangements. Also, the deflection performance is significantly more consistent since tendons are resisting load in a manner consistent with the load demands. This can help alleviate common issues with thin flat-plate gravity systems such as irregular floor flatness due to warping incited by PT systems and inconsistent deflection at the exterior wall.

This new design method has been applied to three buildings and coordinated with construction teams for efficient application. This presentation will discuss the entire design procedure from initial concepts through complete construction documents as applied to three buildings. This presentation will be of interest to academics and practicing structural engineers.

INTRODUCTION

Unbonded post-tensioned gravity framing systems are increasingly common in multi-story construction throughout the United States and increasingly in other countries. Buildings with irregular column arrangements are increasingly common given the densification of urban centers, common usage of 3D digital modeling, and advanced construction methods now common to the industry.

The concurrent advancement of these building characteristics has resulted in post-tensioning primarily being employed for reductions in slab thickness. However, some designs may not fully realize the material efficiencies once observed in orthogonal column arrangements more current in past decades. A new design methodology which is responsive to irregular support conditions is needed to fully realize the performance and material efficiency potential of post-tensioned gravity framing systems.

The proposed innovative design methodology enhances the structural designers understanding of gravity framing through the employment of topology optimization. This optimization method iteratively searches a continuous design space for the stiffest configuration of material given a set of support conditions and static loadings. Previously applied for in-plane investigation of lateral force resisting systems (Sarkisian, 2016), this method is reconsidered for out-of-plane demands common to gravity framing systems. Through these results, new load paths and corresponding tendon layout arrangements are identified. These arrangements avoid the inherent inefficiencies of traditional banded-distributed tendon configurations and more directly address the force demands of the gravity framing system. New geometries are explored for various levels of improvement with attention to both material efficiency and constructability.
Figure 1. Common Examples of Irregular Supports in Gravity Framing Systems

A series of case studies where this methodology has been applied are presented and associated materials savings identified. Consistently, through various design constraints, a minimum of 25% tendon material savings is realized without any increase in mild reinforcement. Additionally, more consistent deflection results are revealed improving floor flatness results. Each project has been developed with direct input from contractors and construction-related considerations are addressed.

**TOPOLOGY OPTIMIZATION**

Topology optimization with density methods is a numerical optimization method that enables the identification of optimal geometries for a variety of structural systems. The numerical process is based on the finite element method and assumes “densities” for each element. The densities are related to the stiffness of the element so that if an element has a significant contribution to the target structural performance (e.g.: the overall stiffness of the structure), it will have a high density. Otherwise the density will be reduced. The target volume of material to be utilized in defining the structure is limited to a prescribed percentage of the original design domain. For applications in architecture, the design domain is taken to be the outer skin or shell of the building or outer envelope of a bridge so that the resulting structural system is expressed in the exterior as an integral part of the architecture itself. Thus, the optimal layout problem in terms of an objective function $f$ can be stated using the design variables, $d$, and the displacements, $u$, as follows:

$$
\min_d f(d, u)
$$

subject to:

$$
g_1(d, u) = 0
$$

$$
g_1(d, u) \leq 0
$$

(1)

where, the design field, $d$, and the structural response, $u$, are related through the equality and inequality constraint functions, $g$. A common optimization problem solved in structural engineering applications is the minimum compliance problem (maximum overall stiffness):

$$
\min_d f(d, u) = u^T K(d) u
$$

subject to:

$$
g_1(d, u) = K(d) u - p
$$

$$
g_2(d, u) = V(d) - \bar{V}
$$

(2)
where $g_1$ represents the equilibrium equation constraint, while $g_2$ is the constraint on the available volume of material for the design, $V$. The global stiffness matrix is given by $K(d)$ which depends on the design variables, $d$, $u$ and $p$ are the vectors of nodal displacements and forces, respectively. The minimum compliance problem corresponds to the maximization of the system stiffness and it is employed to calculate the optimal material layout in the following examples.

In density methods, a void is signified by a null material density ($d = 0$), while $d = 1$ represents solid material. For regions of gray material, or intermediate densities, the commonly used Solid Isotropic Material with Penalization (SIMP) model is employed (Zhou and Rozvany (1991), Rozvany et al. (1992), Bendsoe (1989), Bendsoe and Sigmund (1999)):

$$E(x) = \rho(x)^p E_0$$

(3)

This power-law relationship uses the Young’s Modulus of solid material and the penalization power $p \geq 1$ to force the material to tend towards 0 or 1 (void or solid respectively) where the element density assumes a value somewhere in this range. The optimization process can also include continuation on the penalization power from 1 to 4 in steps of 0.5 until convergence.

An example of the aforementioned topology optimization framework is shown in Fig. 2. This example illustrates the optimal topology of a simply supported truss with an aspect ratio of 6 to 1 derived using the educational codes provided in Talischi et al. (2012). A great advantage in the use of this methodology is that the resulting solution can have any size, shape or geometry.

![Figure 2](image_url)  

Figure 2. Topology optimization of a simply supported truss using the educational code Polytop (Talischi et al., 2012).

Using topology optimization techniques, the resulting design solutions are usually unique and innovative with an organic aesthetics as illustrated by Fig. 2. The results show member lines intersecting at 90 degree angles reminiscent of the geometries of Michell frames (Michell, 1904). In particular, the left-most and right-most areas of the design domain give way to the development of a bounded Michell-like truss. Additional examples of application of topology optimization are indicated in Beghini et al. (2014).

The application of this technique to the design of the PT tendon layouts is described in the following sections.

**CASE STUDIES**

A series of case studies are presented which demonstrate this novel design methodology applied to flat plate PT design. Through these examples, it is shown the design methodology can be applied to a number of project types with consistent reductions in material quantities.

**Case Study 1: Center Core Floor Plan – 75 Howard**

This 220 ft., 20-story luxury high-rise residential development (see figure 3) is located in San Francisco, California and achieves quality indoor and outdoor living spaces through a well-integrated structural layout with unit configurations. The proposed superstructure consists of a special reinforced concrete shear wall core and perimeter gravity columns with two-way flat plate post-tensioned slab framing. The typical gravity framing system consists of an 8 in. post-tensioned flat plate slab. The slab typically cantilevers 8 ft. beyond the perimeter columns. At corner units where the slab cantilever extends up to 15 ft., a sculpted column capital is incorporated and integrated into the corner unit layout (see figure 4).
Three PT design layouts were studied. First, a conventional study followed a traditional banded-distributed tendon layout. Second, topology optimization was studied considering the self-weight of the slab. This revealed new insights that were impossible to acknowledge before, therefore, two more layouts were defined to test the findings. The layouts are as follows:

Layout 1. Traditional Banded-Distributed Layout: This design consists of banded groups of tendons running through the perimeter column lines, distributed tendons running in the north-south direction and some additional non-orthogonal tendons along diagonal lines reaching to the cantilevered corners (see figures 5). This layout requires 1.20 PSF (pounds per square foot) of tendon steel per floor plate. Designers consider this relatively inefficient.

Layout 2. Partially Optimized Tendon Layout: Results of the topology optimization are applied only to column banded tendons. The banded tendons going through the perimeter columns are found to be better distributed to the ends of the long rectangular columns rather than the center. Additionally, the layout at cantilevered corners is reconsidered for a more uniform distribution (see figure 6). These modifications provided a significant reduction in the amounts of tendons steel required to 1.00 PSF. Although improved, topology optimization results indicate areas of further investigation.

Layout 3. Fully Optimized Tendon Layout: Results of the topology optimization are applied to column banded and distributed tendons. The pattern of horizontal sweeps is correlated to the topology optimization results between the central shear wall core and perimeter column supports. Traditionally thought of in terms of one-way behavior, it is discovered that a more refined understanding of behavior indicates a different arrangement (see figure 7). This change provided additional savings bringing the quantities down to 0.75 PSF. Thus, a floor plate with highly irregular supports and significant cantilevers is designed to have tendon quantities equal to or less than orthogonal column configurations of similar spans.
In summary, for the 75 Howard project, three layouts of tendons were studied, each one resulted in different amounts of tendons material but similar mild reinforcement quantities and deflection/stress performance. Thus, the fully optimized tendon arrangement of Layout 3 is found to require 37% less tendon material than traditional orthogonal approaches with similar mild reinforcement requirements.

Case Study 2: Rectangular Floor Plate – Long Beach Civic Center

The Long Beach Civic Center consists of a New City Hall and New Port Headquarters constructed over a combined two-level subterranean parking garage (Figure 9). The City Hall and Port Headquarters buildings each consist of 11-story office towers 162 ft tall above grade with matching typical floor plate sizes of approximately 275 ft x 85 ft. The structural system for each office tower utilizes 10” post-tensioned concrete flat plate slabs supported by reinforced concrete columns and two C-shaped special reinforced concrete shear wall cores. Stud rails are designed to strengthen the slab-to-column connections, and upturned PT beams support the 16 ft corner cantilevers of the floor plate. A raised access floor system conceals the upturned beams and MEP systems, revealing exposed concrete ceilings in the typical office space.

A baseline PT layout consisting of banded tendons in the E-W direction and distributed in the N-S direction was developed for the project (Figure 10). Despite the rectilinear geometry of the floor plan and structural grid, topology optimization studies (Figure 11) showed that the most efficient distribution of material followed a multi-banded curvilinear layout that could be configured by a series of overlapping S-shape tendons. It was determined that the optimized layout could reduce tendon quantity by more than 20% when compared with the baseline banded-distributed layout.
In order to ensure that the intersections of the multiple banded tendons were feasible from both an analysis and detailing perspective, visualization tools were developed through a Grasshopper definition linking SAFE® to Rhino®. It allowed for clash detection by direct coordination of tendons and rebar in 3D (figure 12), as well as direct tracking of tendon curvature in plan so the design geometry could be adjusted to avoid hairpin requirements for increased material savings and ease of construction.

Case Study 3: Reuleaux Triangular Floor Plate – Parkmerced Block 22

With advancement in technology, some unique forms of structures have emerged, both horizontally and vertically. The 14-story, 150ft twin residential towers of Parkmerced Block 22 (figure 13) is an example where a rounded triangular floor plate is used. The typical gravity system consists of 8-inch post-tensioned flat plate system supported by the central reinforced concrete shear wall core and a ring of gravity columns offset by 7ft from the slab edge. The maximum distance between any two columns is 32ft (figure 14).
During initial design phases, this floor plate was designed with a conventional, orthogonal layout of PT tendons in addition to employing a wide band of tendons along the ring of perimeter columns. For this conventional system, the PT tonnage for the floor was calculated to be 1.10 psf and the mild reinforcement was 2.7 psf. As the design continued, it was realized that this unique form of floor plate placed a variety of challenges for the orthogonal layout of post-tensioned system, namely; (1) The form does not allow for anchors for orthogonal PT tendons to be perpendicular to the slab edge; (2) The MEP shafts are not located orthogonally and often interrupts the distributed layout of the tendons; (3) The diagonal distance between the column at the corner and the shear wall was connected by distributed lines of PT, thus increasing the load path distance.

Topology optimization for this floor plate with dead loads only illustrated that the most efficient layout of the PT layout would be the curvilinear layout that is shown in the Figure 18. In general, the density of material concentrates in bands which forms the shortest path to the supports. Laying the PT in this optimum path results in PT savings of approximately 35% when compared to the baseline conventional system. The curvilinear path for the PT also allows for the PT anchors to be perpendicular to the slab edges. The horizontal radius of the
PT tendons was limited to slope of 1:12 to minimize the use of hairpins. The mild reinforcement was radially laid out. It has minimum spacing near the core which continuously increases as it approaches the slab edge.

**CONSTRUCTABILITY**

The efficiency of the unique layout of post-tensioned tendons must be developed along with the construction considerations. Specifically, the layering of mild steel reinforcement and post-tensioned tendons must be coordinated such that they can be coplanar to maximize efficiency, PT tendons are typically held high at support conditions for resistance of negative moment. The top mild reinforcement is arranged parallel to the PT tendon layout and between the tendons. The top mild reinforcement layer perpendicular to the PT is located just below the PT tendon. Similarly, at the mid-span of the slab, where the PT tendons located at the bottom, the bottom mild steel reinforcement is placed parallel to the PT tendons.

The PT layout was coordinated and slightly modified based on location of the slab penetrations required for MEP shafts. In addition, the PT layout was coordinated with the slab embeds for the curtain wall system.

To communicate the layout of the PT tendons in plan, the X and Y coordinates of selected points on the curve can be reported. The z elevation may be also reported. One construction concept could be to layout support chairs corresponding to the elevation required at each x, y point using a commonly available GPS equipment. The PT subcontractor could then simply “connect the dots” of the support chairs with each band of PT tendons.

**CONCLUSIONS**

The use of topology optimization analysis to obtain the PT layout is opening a new window of opportunity for flat plate slab design. The case studies shown present a clear indication that there is potential saving in material quantities thereby reducing cost and embodied carbon.

**REFERENCES**


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